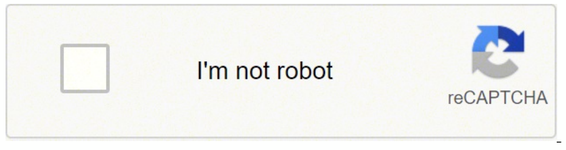

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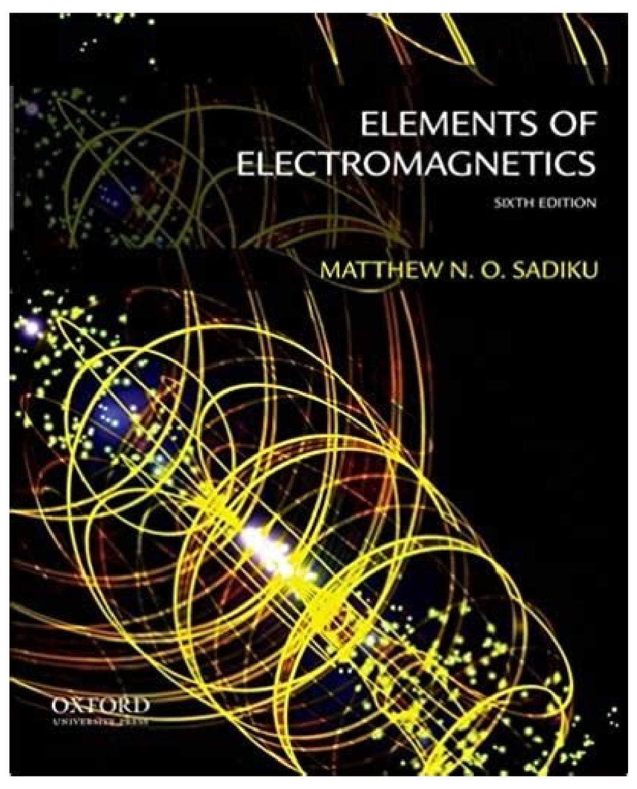

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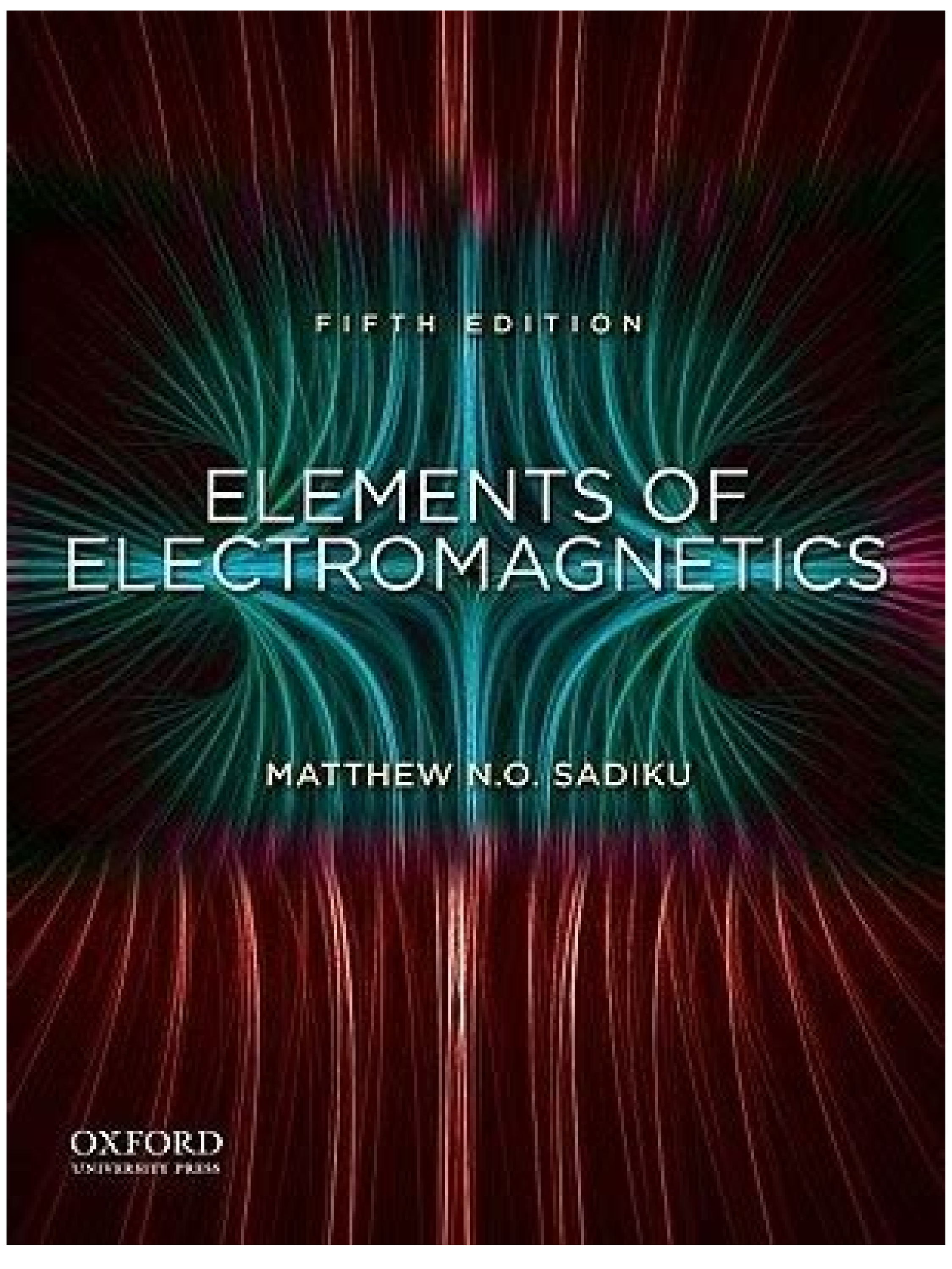

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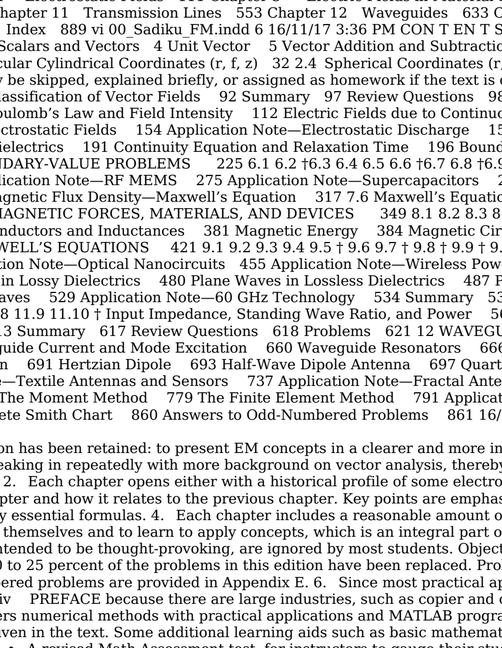
Although the new edition improves on the previous editions, the core of the subject of EM has not changed. The fundamental objective of the first edition has been retained: to present EM concepts in a clearer and more interesting manner than other texts. This objective is achieved in the following ways: 1. To avoid complicating matters by covering EM and mathematical concepts simultane ously, vector analysis is covered at the beginning of the text and applied gradually. This approach avoids breaking in repeatedly with more background on vector analysis, thereby creating discontinuity in the flow of thought. It also separates mathematical theorems from physical concepts and makes it easier for the student to grasp the generality of those theorems. Vector analysis is the backbone of the mathematical formulation of EM problems. 2. Each chapter opens either with a historical profile of some electromagnetic pioneers or with a discussion of a modern topic related to the chapter. The chapter starts with a brief introduction that serves as a guide to the whole chapter and also links the chapter to the rest of the book. The introduction helps the students see the need for the chapter and how it relates to the previous chapter. Key points are emphasized to draw the reader's attention. A brief summary of the major concepts is discussed toward the end of the chapter. 3. To ensure that students clearly get the gist of the matter, key terms are defined and highlighted. Important formulas are boxed to help students identify essential formulas. 4. Each chapter includes a reasonable amount of solved examples. Since the examples are part of the text, they are clearly explained without asking the reader to fill in missing steps. In writing out the solution, we aim for clarity rather than efficiency. Thoroughly worked out examples give students confidence to solve problems themselves and to learn to apply concepts, which is an integral part of engineering education. Each illustrative example is followed by a problem in the form of a Practice Exercise, with the answer provided. 5. At the end of each chapter are ten review questions in the form of multiple-choice objective items. Open-ended questions, although they are intended to be thought-provoking, are ignored by most students. Objective review questions with answers immediately following them provide encouragement for students to do the problems and gain immediate feedback. A large number of problems are provided and are presented in the same order as the material in the main text. Approximately 20 to 25 percent of the problems in this edition have been replaced. Problems of intermediate difficulty are identified by a single asterisk; the most difficult problems are marked with a double asterisk. Enough problems are provided to allow the instructor to choose some as examples and assign some as homework problems. Answers to odd-numbered problems are provided in Appendix E. 6. Since most practical applications involve time-varying fields, six chapters are devoted to such fields. However, static fields are given proper emphasis because they are special cases of dynamic fields. Ignorance of electrostatics is no longer acceptable xiii 00_Sadiku_FM.indd 13 16/11/17 3:36 PM xiv PREFACE because there are large industries, such as copier and computer peripheral manufacturing, that rely on a clear understanding of electrostatics. 7. The last section in each chapter is devoted to applications of the concepts covered in the chapter. This helps students see how concepts apply to real-life situations. 8. The last chapter covers numerical methods with practical applications and MATLAB programs. This chapter is of paramount importance because most practical problems are only solvable using numerical techniques. Since MATLAB is used throughout the book, an introduction to MATLAB is provided in Appendix C. 9. Over 130 illustrative examples and 300 figures are given in the text. Some additional learning aids such as basic mathematical formulas and identities are included in Appendix A. Another guide is a special note to students, which follows this preface. NEW TO THE SIXTH EDITION • Five new Application Notes designed to explain the real-world connections between the concepts discussed in the text. • A revised Math Assessment test, for instructors to gauge their students' mathematical knowledge and preparedness for the course.
• New and updated end-of-chapter problems. Solutions to the end-of-chapter problems and the Math Assessment, as well as PowerPoint slides of all figures in the text, can be found at the Oxford University Press Ancillary Resource Center. Students and professors can view Application Notes from previous editions of the text on the book's companion website www.oup.com/us/sadiku. Although this book is intended to be self-explanatory and useful for self-instruction, the personal contact that is always needed in teaching is not forgotten. The actual choice of course topics, as well as emphasis, depends on the preference of the individual instructor. For example, an instructor who feels that too much space is devoted to vector analysis or static fields may skip some of the materials; however, the students may use them as reference. Also, having covered Chapters 1 to 3, it is possible to explore Chapters 9 to 14. Instructors who disagree with the vector-calculus-first approach may proceed with Chapters 1 and 2, then skip to Chapter 4, and refer to Chapter 3 as needed. Enough material is covered for two-semester courses. If the text is to be covered in one semester, covering Chapters 1 to 9 is recommended; some sections may be skipped, explained briefly, or assigned as homework. Sections marked with the dagger sign (†) may be in this category. ACKNOWLEDGMENTS I thank Dr. Sudarshan Nelatury of Penn State University for providing the new Application Notes and the Math Assessment test. It would not be possible to prepare this edition without the efforts of Executive Editor Dan Kaveney, Associate Editor Christine Mahon, Assistant Editor Megan Carlson, Marketing Manager David Jurman, Marketing Assistant Colleen Rowe, Production Editor Claudia Dukeshire, and Designer Michele Laseau at Oxford University Press, as well as Susan Brown and Betty Pessagno. 00_Sadiku_FM.indd 14 16/11/17 3:36 PM PREFACE xv I thank the reviewers who provided helpful feedback for this edition: Mohammadreza (Reza) Barzegaran Lamar University Sudarshan Nelatury Penn State Erie Sharif M. A. Bhuiyan Tuskegee University Sima Noghanian University of North Dakota Muhammad Dawood New Mexico State University Vladimir Rakov University of Florida Robert Gauthier Carleton University Lisa Shatz Suffolk University Jesmin Khan Tuskegee University Edwin Marengo Northeastern University Kyle Sundqvist Texas A&M University Lili H.



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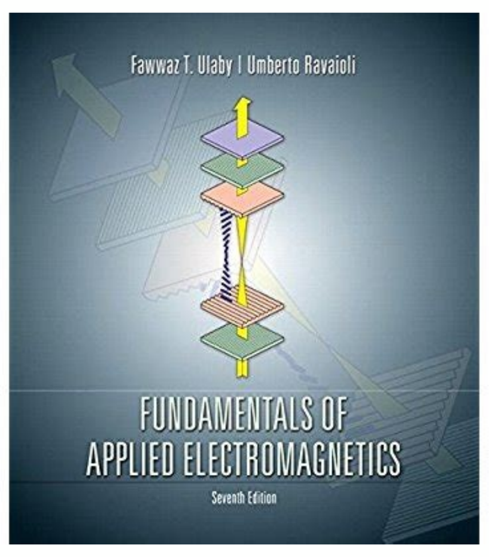

O. SADIKU Prairie View A&M University New York • Oxford OXFORD UNIVERSITY PRESS 00_Sadiku_FM.indd 3 16/11/17 3:36 PM Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trade mark of Oxford University Press in the UK and certain other countries. Published in the United States of America by Oxford University Press 198 Madison Avenue, New York, NY 10016, United States of America. © 2018, 2015, 2010, 2007, 2000 by Oxford University Press © 1994, 1989 by Holt, Rinehart, & Winston, Inc For titles covered by Section 112 of the US Higher Education Opportunity Act, please visit www.oup.com/us/he for the latest information about pricing and alternate formats. All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior permission in writing of Oxford University Press, or as expressly permitted by law, by license, or under terms agreed with the appropriate reproduction rights organization. Inquiries concerning reproduction outside the scope of the above should be sent to the Rights Department, Oxford University Press, at the address above. You must not circulate this work in any other form and you must impose this same condition on any acquirer. Library of Congress Cataloging-in-Publication Data Names: Sadiku, Matthew N. O., author. Title: Elements of Electromagnetics / Matthew N.O. Sadiku, Prairie View A&M University. Description: Seventh edition.
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Printed by LSC Communications, United States of America 00_Sadiku_FM.indd 4 16/11/17 3:36 PM To my wife, Kikelomo 00_Sadiku_FM.indd 5 16/11/17 3:36 PM BRIEF TABLE OF CONTENTS 16/11/17 3:36 PM PR EFACE This new edition is intended to provide an introduction to engineering electromagnetics (EM) at the junior or senior level. Although the new edition improves on the previous editions, the core of the subject of EM has not changed. The fundamental objective of the first edition has been retained: to present EM concepts in a clearer and more interesting manner than other texts. This objective is achieved in the following ways: 1. To avoid complicating matters by covering EM and mathematical concepts simultane ously, vector analysis is covered at the beginning of the text and applied gradually. This approach avoids breaking in repeatedly with more background on vector analysis, thereby creating discontinuity in the flow of thought. It also separates mathematical theorems from physical concepts and makes it easier for the student to grasp the generality of those theorems. Vector analysis is the backbone of the mathematical formulation of EM problems.

2. Each chapter opens either with a historical profile of some electromagnetic pioneers or with a discussion of a modern topic related to the chapter. The chapter starts with a brief introduction that serves as a guide to the whole chapter and also links the chapter to the rest of the book. The introduction helps the students see the need for the chapter and how it relates to the previous chapter. Key points are emphasized to draw the reader's attention. A brief summary of the major concepts is discussed toward the end of the chapter. 3. To ensure that students clearly get the gist of the matter, key terms are defined and highlighted. Important formulas are boxed to help students identify essential formulas. 4. Each chapter includes a reasonable amount of solved examples. Since the examples are part of the text, they are clearly explained without asking the reader to fill in missing steps. In writing out the solution, we aim for clarity rather than efficiency. Thoroughly worked out examples give students confidence to solve problems themselves and to learn to apply concepts, which is an integral part of engineering education. Each illustrative example is followed by a problem in the form of a Practice Exercise, with the answer provided. 5. At the end of each chapter are ten review questions in the form of multiple-choice objective items.

Open-ended questions, although they are intended to be thought-provoking, are ignored by most students. Objective review questions with answers immediately following them provide encouragement for students to do the problems and gain immediate feedback. A large number of problems are provided and are presented in the same order as the material in the main text. Approximately 20 to 25 percent of the problems in this edition have been replaced. Problems of intermediate difficulty are identified by a single asterisk; the most difficult problems are marked with a double asterisk. Enough problems are provided to allow the instructor to choose some as examples and assign some as homework problems. Answers to odd-numbered problems are provided in Appendix E. 6. Since most practical applications involve time-varying fields, six chapters are devoted to such fields. However, static fields are given proper emphasis because they are special cases of dynamic fields. Ignorance of electrostatics is no longer acceptable xiii 00_Sadiku_FM.indd 13 16/11/17 3:36 PM xiv PREFACE because there are large industries, such as copier and computer peripheral manufacturing, that rely on a clear understanding of electrostatics. 7. The last section in each chapter is devoted to applications of the concepts covered in the chapter. This helps students see how concepts apply to real-life situations. 8. The last chapter covers numerical methods with practical applications and MATLAB programs. This chapter is of paramount importance because most practical problems are only solvable using numerical techniques. Since MATLAB is used throughout the book, an introduction to MATLAB is provided in Appendix C. 9. Over 130 illustrative examples and 300 figures are given in the text. Some additional learning aids such as basic mathematical formulas and identities are included in Appendix A. Another guide is a special note to students, which follows this preface. NEW TO THE SIXTH EDITION • Five new Application Notes designed to explain the real-world connections between the concepts discussed in the text. • A revised Math Assessment test, for instructors to gauge their students' mathematical knowledge and preparedness for the course. • New and updated end-of-chapter problems. Solutions to the end-of-chapter problems and the Math Assessment, as well as PowerPoint slides of all figures in the text, can be found at the Oxford University Press Ancillary Resource Center. Students and professors can view Application Notes from previous editions of the text on the book's companion website www.oup.com/us/sadiku. Although this book is intended to be self-explanatory and useful for self-instruction, the personal contact that is always needed in teaching is not forgotten. The actual choice of course topics, as well as emphasis, depends on the preference of the individual instructor. For example, an instructor who feels that too much space is devoted to vector analysis or static fields may skip some of the materials; however, the students may use them as reference. Also, having covered Chapters 1 to 3, it is possible to explore Chapters 9 to 14. Instructors who disagree with the vector-calculus-first approach may proceed with Chapters 1 and 2, then skip to Chapter 4, and refer to Chapter 3 as needed. Enough material is covered for two-semester courses. If the text is to be covered in one semester, covering Chapters 1 to 9 is recommended; some sections may be skipped, explained briefly, or assigned as homework.

Sections marked with the dagger sign (†) may be in this category. ACKNOWLEDGMENTS I thank Dr. Sudarshan Nelatury of Penn State University for providing the new Application Notes and the Math Assessment test. It would not be possible to prepare this edition without the efforts of Executive Editor Dan Kaveney, Associate Editor Christine Mahon, Assistant Editor Megan Carlson, Marketing Manager David Jurman, Marketing Assistant Colleen Rowe, Production Editor Claudia Dukeshire, and Designer Michele Laseau at Oxford University Press, as well as Susan Brown and Betty Pessagno. 00_Sadiku_FM.indd 14 16/11/17 3:36 PM PREFACE xv I thank the reviewers who provided helpful feedback for this edition: Mohammadreza (Reza) Barzegaran Lamar University Sudarshan Nelatury Penn State Erie Sharif M. A. Bhuiyan Tuskegee University Sima Noghanian University of North Dakota Muhammad Dawood New Mexico State University Vladimir Rakov University of Florida Robert Gauthier Carleton University Lisa Shatz Suffolk University Jesmin Khan Tuskegee University Edwin Marengo Northeastern University Kyle Sundqvist Texas A&M University Lili H.
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Please keep sending those errors directly to the publisher or to me at . —Matthew N.O. Sadiku Prairie View, Texas 00_Sadiku_FM.indd 16 16/11/17 3:36 PM A NOTE TO THE S TUDENT Electromagnetic theory is generally regarded by students as one of the most difficult courses in physics or the electrical engineering curriculum. But this conception may be proved wrong if you take some precautions.
From experience, the following ideas are provided to help you perform to the best of your ability with the aid of this textbook: 1. Pay particular attention to Part 1 on vector analysis, the mathematical tool for this course. Without a clear understanding of this section, you may have problems with the rest of the book. 2. Do not attempt to memorize too many formulas. Memorize only the basic ones, which are usually boxed, and try to derive others from these.
Try to understand how formulas are related. There is nothing like a general formula for solving all problems. Each formula has limitations owing to the assumptions made in obtaining it. Be aware of those assumptions and use the formula accordingly. 3. Try to identify the key words or terms in a given definition or law. Knowing the meaning of these key words is essential for proper application of the definition or law. 4. Attempt to solve as many problems as you can. Practice is the best way to gain skill.
The best way to understand the formulas and assimilate the material is by solving problems. It is recommended that you solve at least the problems in the Practice Exercise immediately following each illustrative example. Sketch a diagram illustrating the problem before attempting to solve it mathematically. Sketching the diagram not only makes the problem easier to solve, but also helps you understand the problem by simplifying and organizing your thinking process. Note that unless otherwise stated, all distances are in meters. For example (2, 1, 5) actually means (2 m, 1 m, 5 m). You may use MATLAB to do number crunching and plotting. A brief introduction to MATLAB is provided in Appendix C. A list of the powers of 10 and Greek letters commonly used throughout this text is provided in the tables located on the inside cover. Important formulas in calculus, vectors, and complex analysis are provided in Appendix A. Answers to odd-numbered problems are in Appendix E. xvii 00_Sadiku_FM.indd 17 xvii 16/11/17 3:36 PM ABOU T T HE AU T HOR Matthew N.
O.
Sadiku received his BSc degree in 1978 from Ahmadu Bello University, Zaria, Nigeria, and his MSc and PhD degrees from Tennessee Technological University, Cookeville, Tennessee, in 1982 and 1984, respectively. From 1984 to 1988, he was an assistant professor at Florida Atlantic University, Boca Raton, Florida, where he did graduate work in computer science. From 1988 to 2000, he was at Temple University, Philadelphia, Pennsylvania, where he became a full professor. From 2000 to 2002, he was with Lucent/ Avaya, Holmdel, New Jersey, as a system engineer and with Boeing Satellite Systems, Los Angeles, California, as a senior scientist. He is currently a professor of electrical and computer engineering at Prairie View A&M University, Prairie View, Texas. He is the author of over 370 professional papers and over 70 books, including Elements of Electromagnetics (Oxford University Press, 7th ed., 2018), Fundamentals of Electric Circuits (McGraw-Hill, 6th ed., 2017, with C. Alexander), Computational Electromagnetics with MATLAB (CRC, 4th ed., 2018), Metropolitan Area Networks (CRC Press, 1995), and Principles of Modern Communication Systems (Cambridge University Press, 2017, with S. O. Agbo). In addition to the engineering books, he has written Christian books including Secrets of Successful Marriages, How to Discover God's Will for Your Life, and commentaries on all the books of the New Testament Bible. Some of his books have been translated into French, Korean, Chinese (and Chinese Long Form in Taiwan), Italian, Portuguese, and Spanish. He was the recipient of the 2000 McGraw-Hill/Jacob Millman Award for outstanding contributions in the field of electrical engineering. He was also the recipient of Regents Professor award for 2012–2013 by the Texas A&M University System. He is a registered professional engineer and a fellow of the Institute of Electrical and Electronics Engineers (IEEE) "for contributions to computational electromagnetics and engineering education." He was the IEEE Region 2 Student Activities Committee Chairman. He was an associate editor for IEEE Transactions on Education. He is also a member of the Association for Computing Machinery (ACM) and the American Society of Engineering Education (ASEE). His current research interests are in the areas of computational electromagnetics, computer networks, and engineering education.
His works can be found in his autobiography, My Life and Work (Trafford Publishing, 2017) or on his website, www.matthewsadiku.com. He currently resides with his wife Kikelomo in Hockley, Texas. He can be reached via email at . xviii 00_Sadiku_FM.indd 18 16/11/17 3:36 PM M AT H A SSE SSMEN T 1.1 Let u be the angle between the vectors A and B. What can be said about u if (i) |A +B| , |A 2 B|, (ii) |A 1 B| 5 |A 2 B|, (iii) |A 1 B| . |A 2 B|? 1.2 Two sides of a parallelogram ABCD denoted as p = 5ax and q = 3ax + 4ay are shown in Figure MA-1 Let the diagonals intersect at O and make an angle a. Find the coordinates of O and the magnitude of a. Based on the value of a, what can we call ABCD? D C O A p = 5ax FIGURE MA-1 Parallelogram ABCD. q = 3ax + 4ay α B 1.3 What is the distance R between the two points A(3, 5, 1) and B(5, 7, 2)? Also find 1 its reciprocal, . R 1.4 What is the distance vector RAB from A(3, 7, 1) to B(8, 19, 2) and a unit vector aAB in the direction of RAB? 1.5 Find the interval of values x takes so that a unit vector u satisfies |(x 2 2)u| , |3u|. 1.6 There are four charges in space at four points A, B, C, and D, each 1 m from every other. You are asked to make a selection of coordinates for these charges. How do you place them in space and select their coordinates? There is no unique way. 1.7 A man driving a car starts at point O, drives in the following pattern 15 km northeast to point A, 20 km southwest to point B, 25 km north to C, 10 km southeast to D, 15 km west to E, and stops. How far is he from his starting point, and in what direction? MA-1 00_Sadiku_FM.indd 1 16/11/17 3:36 PM MA-2 MATH ASSESSMENT 1.8 A unit vector an makes angles , , and with the x-, y-, and z-axes, respectively. > Express an in the rectangular coordinate system. Also express a nonunit vector OP of length parallel to an. 1.9 Three vectors p, q, and r sum to a zero vector and have the magnitude of 10, 11, and 15, respectively. Determine the value of p ? q 1 q ? r 1 r ? p.
1.10 An experiment revealed that the point Q(x, y, z) is 4 m from P(2, 1, 4) and that the > vector QP makes 45.5225, 59.4003, and 60 with the x-, y-, and z-axes, respectively.
Determine the location of Q. 1.11 In a certain frame of reference with x-, y-, and z-axes, imagine the first octant to be a room with a door. Suppose that the height of the door is h and its width is . The top-right corner P of the door when it is shut has the rectangular coordinates (, 0, h). Now if the door is turned by angle , so we can enter the room, what are the coordinates of P? What is the length of its diagonal r 5 OP in terms of and z? > Suppose the vector OP makes an angle with the z-axis; express and h in terms of r and . > > > 1.12 Consider two vectors p 5 OP and q=OQ in Figure MA-2. Express the vector GR in terms of p and q. Assume that /ORQ 5 90°.
Q O R FIGURE MA-2 Orthogonal projection of one vector over another. P 1.13 Consider the equations of two planes: 3x 2 2y 2 z 5 8 2x 1 y 1 4z 5 3 Let them intersect along the straight line =0 and E2 the field in the region z 0... > '); if isempty(er1); er1 = 1; elseif er1 < 1; er1 = 1; end % check if dielectric is physical disp('Enter the relative permittivity in the region '); er2 = input(' z < 0...
> '); if isempty(er2); er2 = 1; elseif er2 < 1; er2 = 1; end % check if dielectric is physical % prompt the user for the region disp('Enter the side of the interface where the electric'); side = input('field is known (given)... > '); % if user entered something other than "r" "c" or "s" % set default as "r" if isempty(side); side = 1; elseif side > 2; side = 2; end % check if dielectric is physical if side == 1; % prompt the user for the field disp('Enter the electric field in side 1 in the '); E1 = input(' form [Ex Ey Ez]...
>'); E1n = E1(3)*[0 0 1]; % normal direction is +z E2n = E1n*er1/er2; % e-field boundary condition % for normal component E1t = E1 - E1n; % tangential component of E1 E2t = E1t; % e-field boundary condition for % tangential component E2 = E2t + E2n; elseif side == 2; % prompt the user for the field disp('Enter the electric field in side 2 in the '); E2 = input(' form [Ex Ey Ez]... >'); E2n = E2(3)*[0 0 1]; % normal direction is +z E1n = E2n*er2/er1; % e-field boundary condition % for normal component E2t = E2 - E2n; % tangential component of E2 23/09/17 1:15 PM 214 CHAPTER 5 ELECTRIC FIELDS IN MATERIAL SPACE else E1t = E2t; % e-field boundary condition for %tangential component E1 = E1t + E1n; disp('Invalid specification, please re-try '); end % Display results disp(sprintf('The electric fields are ')); disp(sprintf(' E1 = (%d, %d, %d) V/m',E1(1), E1(2), E1(3))); disp(sprintf(' E2 = (%d, %d, %d) V/m',E2(1), E2(2), E2(3))); SUMMARY 1. Materials can be classified roughly as conductors 1 s W 1, er 5 1 2 and dielectrics 1 s V 1, er $ 1 2 in terms of their electrical properties and r, where is the conductivity and r is the dielectric constant or relative permittivity. 2. Electric current is the flux of electric current density through a surface; that is, I 5 3 J # dS 3. The resistance of a conductor of uniform cross section is R5 , sS 4. The macroscopic effect of polarization on a given volume of a dielectric material is to "paint" its surface with a bound charge Qb 5 AS rps dS and leave within it an accumulation of bound charge Qb 5 ev rpv dv, where rps 5 P # an and rpv 5 2= # P. 5. In a dielectric medium, the D and E fields are related as D 5 eE, where e 5 eoer is the permittivity of the medium while E and P are related as P 5 xe eoE. 6. The electric susceptibility xe 15 er 2 1 2 of a dielectric measures the sensitivity of the material to an electric field. 7. A dielectric material is linear if D 5 eE holds, that is, if is independent of E. It is homogeneous if is independent of position. It is isotropic if is a scalar. 8. The principle of charge conservation, the basis of Kirchhoff 's current law, is stated in the continuity equation 'rv 50 't 9. The relaxation time, Tr 5 e/s, of a material is the time taken by a charge placed in its interior to decrease by a factor of e21 or to .37% of its original magnitude. 10. Boundary conditions must be satisfied by an electric field existing in two different media separated by an interface. For a dielectric–dielectric interface =#J1 E1t 5 E2t D1n 2 D2n 5 rS 05_Sadiku_Ch05.indd 214 or D1n 5 D2n if rS 5 0 23/09/17 1:15 PM Review Questions 215 For a dielectric–conductor interface, Et 5 0, Dn 5 eEn 5 rS because E 5 0 inside the conductor. 11. Materials of high dielectric constant are of great interest for high-performance electronic devices. REVIEW QUESTIONS 5.1 Which is not an example of convection current? (a) A moving charged belt (b) Electronic movement in a vacuum tube (c) An electron beam in a television tube (d) Electric current flowing in a copper wire 5.2 What happens when a steady potential difference is applied across the ends of a conducting wire? (a) All electrons move with a constant velocity. (b) All electrons move with a constant acceleration. (c) The random electronic motion will, on the average, be equivalent to a constant velocity of each electron. (d) The random electronic motion will, on the average, be equivalent to a nonzero constant acceleration of each electron. 5.3 The formula R 5 ,/ 1 sS 2 is for thin wires. (a) True (c) Not necessarily (b) False 5.4 Seawater has er 5 80. Its permittivity is (a) 81 (c) 5.162 3 10210 F/m (b) 79 (d) 7.074 3 10210 F/m 5.5 Both o and xe are dimensionless. (a) True (b) False 5.6 If = # D 5 e= # E and = # J 5 s= # E in a given material, the material is said to be 05_Sadiku_Ch05.indd 215 (a) Linear (d) Linear and homogeneous (b) Homogeneous (e) Linear and isotropic (c) Isotropic (f) Isotropic and homogeneous 23/09/17 1:15 PM 216 CHAPTER 5 ELECTRIC FIELDS IN MATERIAL SPACE 5.7 The relaxation time of mica 1 s 5 10215 S/m, er 5 6 2 is (a) 5 3 10210 s (b) 1026 s (c) 5 hr (d) 10 hr (e) 15 hr 5.8 The uniform fields shown in Figure 5.18 are near a dielectric–dielectric boundary but on opposite sides of it. Which configurations are correct? Assume that the boundary is charge free and that 2 > 1.
5.9 Which of the following statements are incorrect?
(a) The conductivities of conductors and insulators vary with temperature and frequency.
(b) A conductor is an equipotential body in steady state, and E is always tangential to the conductor. (c) Nonpolar molecules have no permanent dipoles. (d) In a linear dielectric, P varies linearly with E. FIGURE 5.18 For Review Question 5.8. 05_Sadiku_Ch05.indd 216 23/09/17 1:16 PM Problems 217 5.10 The electric conditions (charge and potential) inside and outside an electric screening are completely independent of one another. (a) True (b) False Answers: 5.1d, 5.2c, 5.3c, 5.4d, 5.5b, 5.6d, 5.7e, 5.8e, 5.9b, 5.10a. PROBLEMS Section 5.3—Convection and Conduction Currents 5.1 Let the current density be J 5 e2x cos 4 yax 1 e2x sin 4 yay A/m2. Determine the current crossing the surface x 5 2, 0 y p/3, 0 z 4. 10 2103t 5.2 In a certain region, J 5 e ar A/m2 . Determine how much current is crossing r surface r 5 4 m at t 5 2 ms. 10 sin f ar A/m2, determine the current flowing through the surface r r 5 2, 0 , f , p, 0 , z , 5 m. 5.3 Given that J 5 5.4 In a cylindrical conductor of radius 4 mm, the current density is J 5 5e210raz A/m2. Find the current through the conductor. 5.5 The current density is J5 20 cosu ar A/m2 r13 Determine the current through the surface r 5 3, p/4 u p/2, 0 f 2p. Section 5.4—Conductors 5.6 A 1 MV resistor is formed by a cylinder of graphite–clay mixture having a length of 2 cm and a radius of 4 mm. Determine the conductivity of the resistor. 5.7 If the ends of a cylindrical bar of carbon 1 s 5 3 3 104 S/m 2 of radius 5 mm and length 8 cm are maintained at a potential difference of 9 V, find (a) the resistance of the bar, (b) the current through the bar, (c) the power dissipated in the bar. 5.8 A conducting wire is 2 mm in radius and 100 m in length. When a dc voltage of 9 V is applied to the wire, it results in a current of 0.3 A. Find: (a) the E-field in the wire, (b) the conductivity of the wire. 5.9 Two wires have the same diameter and same resistance. If one is made of copper, and the other is of silver, which wire is longer? 5.10 A long wire with circular cross section has a diameter of 4 mm. The wire is 5 m long and it carries 2 A when a 12 V voltage is applied across its ends. Determine the conductivity of the wire.
05_Sadiku_Ch05.indd 217 16/11/17 3:18 PM 218 CHAPTER 5 ELECTRIC FIELDS IN MATERIAL SPACE a b ρ1 c ρ2 FIGURE 5.19 For Problem 5.12. 5.11 A composite conductor 10 m long consists of an inner core of steel of radius 1.5 cm and an outer sheath of copper whose thickness is 0.5 cm. Take the resistivities of copper and steel as 1.77 3 1028 and 11.8 3 1028 V # m, respectively. (a) Determine the resistance of the conductor.
(b) If the total current in the conductor is 60 A, what current flows in each metal? (c) Find the resistance of a solid copper conductor of the same length and cross-sectional areas as the sheath. 5.12 The cross section of a conductor made with two materials with resistivities 1 and 2 is shown in Figure 5.19. Find the resistance of length of the conductor. 5.13 A 12 V voltage is applied across the ends of a silver wire of length 12.4 m and radius 0.84 mm. Determine the current through the wire. Sections 5.5–5.7—Polarization and Dielectric Constant 5.14 At a particular temperature and pressure, a helium gas contains 5 3 1025 atoms/m3. If a 10 kV/m field applied to the gas causes an average electron cloud shift of 10218 m, find the dielectric constant of helium. 5.15 A dielectric material contains 2 3 1019 polar molecules/m3, each of dipole moment 1.8 3 10227 C # m.
Assuming that all the dipoles are aligned in the direction of the electric field E 5 105ax V/m, find P and r. 5.16 A 10 mC point charge is embedded in wood, which has e 5 4.0. Assuming that the charge is located at the origin, find P at r = 1 m. 100 5.17 In a certain dielectric for which er 5 3.5, given that P 5 a nC/m2, find E and D at r r r 5 2 m. 5.18 A cylindrical slab has a polarization given by P 5 po a. Find the polarization charge density pv inside the slab and its surface charge density ps. 5.19 A spherical shell has r 5 1.2 cm and r 5 2.6 cm as inner and outer radii, respectively. If P 5 4rar pC/m2, determine (a) the total bound surface charge on the inner surface, (b) the total bound surface charge on the outer surface, (c) the total bound volume charge. 05_Sadiku_Ch05.indd 218 16/11/17 1:18 PM Problems 219 ε = ε0 ε = 5ε0 ε = 2ε0 ε = ε0 a b c FIGURE 5.20 For Problem 5.23. 5.20 In a slab of Teflon 1 e 5 2.1 eo 2 , E 5 6ax 1 12ay 2 20az V/m, find D and P. 5.21 The potential distribution in a dielectric material (e 5 8eo 2 is V 5 4x2 yz3 V. Find V, E, and P at point (–2, 5, 3). 5.22 In a dielectric material 1 e 5 5eo 2 , the potential field V 5 10x2yz 2 5z2 V, determine (a) E, (b) D, (c) P, (d) rv . 5.23 Concentric spheres r 5 a, r 5 b, and r 5 c have charges 4 C, 26 C, and 10 C, respectively, placed on them. If the regions separating them are filled with different dielectrics as shown in Figure 5.20, find E, D, and P everywhere. 5.24 Consider Figure 5.21 as a spherical dielectric shell so that e 5 eoer for a , r , b and e 5 eo for 0 , r , a.
If a charge Q is placed at the center of the shell, find (a) P for a , r , b (b) rpv for a , r , b (c) rps at r 5 a and r 5 b 5.25 Two point charges in free space are separated by distance d and exert a force 2.6 nN on each other. The force becomes 1.5 nN when the free space is replaced by a homogeneous dielectric material. Calculate the dielectric constant of the material. *5.26 A conducting sphere of radius a has a total charge Q uniformly distributed on its surface. (a) If the sphere is embedded in a medium with permittivity , find the energy stored. a 2 (b) Repeat part (a) if the permittivity varies as « 5 «o a1 1 b . r 05_Sadiku_Ch05.indd 219 16/11/17 3:18 PM 220 CHAPTER 5 ELECTRIC FIELDS IN MATERIAL SPACE FIGURE 5.21 For Problem 5.24. 5.27 A solid sphere of radius a and dielectric constant r has a uniform volume charge density of ro. (a) At the center of the sphere, show that V5 roa2 1 2er 1 1 2 6eoer (b) Find the potential at the surface of the sphere. 5.28 In an anisotropic medium, D is related to E as 4 1 1 Ex Dx £ Dy § 5 eo £ 1 3 1 § £ Ey § Dz 1 1 2 Ez Find D due to E 5 Eo 1 ax 1 ay 2 az 2 V/m. Section 5.8—Continuity Equation and Relaxation Time 5.29 For static (time-independent) fields, which of the following current densities are possible? (a) J 5 2x3yax 1 4x2z2ay 2 6x2yzaz (c) J 5 z2 a 1 z cos f az r r (b) J 5 xyax 1 y z 1 1 2 ay 1 2yaz (d) J 5 sin u ar r2 5.30 If J 5 e22y sin 2xax 1 e22y cos 2xay 1 zaz A/m2, find the rate of change of the electric charge density. 100 ar A/m2, find (a) the time rate of increase in the volume charge density, (b) the r2 total current passing through surface defined by r 5 2, 0 , z , 1, 0 , f , 2p. 5.31 If J 5 5.32 An excess charge placed within a conducting medium becomes one-half of its initial value in 80 ms. Calculate the conductivity of the medium and the relaxation time. Assume that its dielectric constant is 7.5. 05_Sadiku_Ch05.indd 220 16/11/17 1:18 PM Problems 221 5.33 Let rv be the volume charge density of charges in motion. If u is their velocity, show that (u # = 2 rv 1 rv= # u 1 'rv 5 0. 't 5.34 The current density is given by J 5 0.5 sin xax A/m2. Determine the time rate of increase of the charge density (i.e., dv/dt) at point (2, 4, 23). 5.35 Determine the relaxation time for each of the following media: (a) Hard rubber 1 s 5 10215 S/m, e 5 3.1eo 2 (b) Mica 1 s 5 10215 S/m, e 5 6eo 2 (c) Distilled water 1 s 5 1024 S/m, e 5 80eo 2 5.36 Lightning strikes a dielectric sphere of radius 20 mm for which er 5 2.5, s 5 5 3 1026 S/m and deposits uniformly a charge of 1 C. Determine the initial volume charge density and the volume charge density 2 ms later. Section 5.9—Boundary Conditions 5.37 Show that the normal and tangential components of the current density J at the interface between two media with conductivities s1 and s2 satisfy J1n 5 J2n, J1t s 5 1 s2 J2t 5.38 Let z < 0 be region 1 with dielectric constant er1 5 4, while z 0 is region 2 with er2 5 7.5. Given that E1 5 60ax 2 100ay 1 40az V/m, (a) find P1, (b) calculate D2. 5.39 Region 1 is x 0 with, e1 5 4eo, while region 2 is x 0 with e 5 2eo. If E2 5 6ax 2 10ay 1 8az V/m, (a) find P1, and P2, (b) calculate the energy densities in both regions. 5.40 A dielectric interface is defined by 4x 1 3y 5 10 m. The region including the origin is free space, where D1 5 2ax 2 4ay 1 6.5az nC/m2. In the other region, er2 5 2.5. Find D2 and the angle u 2 that D2 makes with the normal. 5.41 Regions 1 and 2 have permittivities e1 5 2eo and e2 5 5eo. The regions are separated by a plane whose equation is x + 2y + z = 1 such that x + 2y + z 1 is region 1. If E1 5 20ax 2 10ay 1 40az V/m, find: (a) the normal and tangential components of E1, (b) E2. 5.42 Given that E1 5 10ax 2 6ay 1 12az V/m in Figure 5.22, find (a) P1, (b) E2 and the angle E2 makes with the y-axis, (c) the energy density in each region. 5.43 Two homogeneous dielectric regions 1 1 r # 4 cm 2 and 2 1 r $ 4 cm 2 have dielectric constants 3.5 and 1.5, respectively. If D2 5 12ar 2 6af 1 9az nC/m2, calculate (a) E1 and D1, (b) P2 and rpv2, (c) the energy density for each region. 05_Sadiku_Ch05.indd 221 16/11/17 3:18 PM 222 CHAPTER 5 ELECTRIC FIELDS IN MATERIAL SPACE y ε 1 = 3ε 0 x 2 ε 2 = 4.5ε0 FIGURE 5.22 For Problem 5.42. FIGURE 5.23 For Problem 5.44. 5.44 A conducting sphere of radius a is half-embedded in a liquid dielectric medium of permittivity 1 as in Figure 5.23. The region above the liquid is a gas of permittivity 2. If the total free charge on the sphere is Q, determine the electric field intensity everywhere. 5.45 A dielectric sphere e1 5 2eo is buried in a medium with e2 5 6eo. Given that E2 5 10sinuar 1 5cosuau in the medium, calculate E1 and D1 in the dielectric sphere. *5.46 Two parallel sheets of glass 1 er 5 8.5 2 mounted vertically are separated by a uniform air gap between their inner surface.
The sheets, properly sealed, are immersed in oil 1 er 5 3.0 2 as shown in Figure 5.24. A uniform electric field of strength 2 kV/m in the horizontal direction exists in the oil. Calculate the magnitude and direction of the electric field in the glass and in the enclosed air gap when (a) the field is normal to the glass surfaces and (b) the field in the oil makes an angle of 75° with a normal to the glass surfaces. Ignore edge effects. 05_Sadiku_Ch05.indd 222 16/11/17 1:18 PM Problems 223 Glass z E0 30° ε0 Oil Oil ε1 = 2ε 0 ε 2 = 3ε 0 ε0 Air FIGURE 5.24 For Problem 5.46.
FIGURE 5.25 For Problem 5.49. 5.47 At a point on a conducting surface, E 5 30ax 2 40ay 1 20az mV/m. Calculate the surface charge density at that point. 5.48 (a) Given that E 5 15ax 2 8az V/m at a point on a conductor surface, what is the surface charge density at that point? Assume e 5 eo. (b) Region y $ 2 is occupied by a conductor. If the surface charge on the conductor is 220 nC/m2, find D just outside the conductor. 5.49 Two planar slabs of equal thickness but with different dielectric constants are shown in Figure 5.25. Eo in air makes an angle of 30° with the z-axis. Calculate the angle that E makes with the z-axis in each of the two dielectric layers. 05_Sadiku_Ch05.indd 223 16/11/17 3:18 PM Pierre-Simon de Laplace (1749–1827), a French astronomer and mathematician, discovered the Laplace transform and Laplace's equation, to be discussed in this chapter. He believed the world was entirely deterministic. To Laplace, the universe was nothing but a giant problem in calculus. Born of humble origins in Beaumont-en-Auge, Normandy, Laplace became a p rofessor of mathematics at the age of 20. His mathematical abilities inspired the famous mathematician Siméon Poisson, who called Laplace the Isaac Newton of France. Laplace made important contributions in potential theory, probability theory, astronomy, and celestial m echanics. He was widely known for his work Traité de Mécanique Céleste (Celestial Mechanics), which supplemented the work of Newton on astronomy. Laplace is one of the few giants in the history of probability and statistics. He was born and died a Catholic. Siméon-Denis Poisson (1781–1840), a French mathematical physicist whose name is attached to a wide area of ideas: Poisson's integral, Poisson's equation in potential theory (to be discussed in this chapter), Poisson brackets in differential equations, Poisson's ratio in elasticity, the Poisson distribution in probability theory, and Poisson's c onstant in e lectricity. Born at Pithviers, south of Paris, the son of a retired soldier, Siméon Poisson was originally forced to study medicine by his family, but he began to study mathematics in 1798 at the École Polytechnique at the age of 17. His abilities excited the interest of his teachers Lagrange and Laplace, whose friendship he retained to the end of their lives. A paper on finite differences, written when Poisson was 18, attracted the attention of Legendre. Poisson's chief interest lay in the application of mathematics to physics, e specially in electrostatics and magnetism. Poisson made important contributions to mechanics, theory of elasticity, optics, calculus, differential geometry, and probability theory. He published between 300 and 400 m athematical works. 06_Sadiku_Ch06.indd 224 23/09/17 1:39 PM CHAPTER 6 ELECTROSTATIC BOUNDARYVALUE PROBLEMS Wise men profit more from fools than fools from wise men; for the wise men shun the mistakes of the fools, but fools do not imitate the successes of the wise. —MARCUS P. CATO 6.1 INTRODUCTION The procedure for determining the electric field E in the preceding chapters has generally been to use either Coulomb's law or Gauss's law when the charge distribution is known, or E 5 2=V when the potential V is known throughout the region. In most practical situations, however, neither the charge distribution nor the potential distribution is known. In this chapter, we shall consider practical electrostatic problems where only electrostatic conditions (charge and potential) at some boundaries are known and it is desired to find E and V throughout the region. Such problems are usually tackled using Poisson's or Laplace's equation or the method of images, and they are usually referred to as boundaryvalue problems. The concepts of resistance and capacitance will be covered. We shall use Laplace's equation in deriving the resistance of an object and the capacitance of a capacitor. Example 6.5 should be given special attention because we will refer to it often in the remaining part of the text. 6.2 POISSON'S AND LAPLACE'S EQUATIONS Poisson's and Laplace's equations are easily derived from Gauss's law (for a linear, isotropic material medium): = # D 5 = # eE 5 rv (6.1) and E 5 2=V (6.2) 225 06_Sadiku_Ch06.indd 225 23/09/17 1:39 PM 226 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS Substituting eq. (6.2) into eq. (6.1) gives = # 1 2e=V 2 5 rv(6.3) for an inhomogeneous medium. For a homogeneous medium, eq. (6.3) becomes rv =2V 5 2 (6.4) e This is known as Poisson's equation. A special case of this equation occurs when rv 5 0 (i.e., for a charge-free region).
Equation (6.4) then becomes =2V 5 0(6.5) which is known as Laplace's equation. Note that in taking « out of the left-hand side of eq. (6.3) to obtain eq. (6.4), we have assumed that « is constant throughout the region in which V is defined; for an inhomogeneous region, « is not constant and eq. (6.4) does not follow eq. (6.3). Equation (6.3) is Poisson's equation for an inhomogeneous medium; it becomes Laplace's equation for an inhomogeneous medium when rv 5 0. Recall that the Laplacian operator 2 was derived in Section 3.8. Thus Laplace's equation in Cartesian, cylindrical, or spherical coordinates, respectively, is given by '2V '2V '2V 1 1 5 0(6.6) 'x2 'y2 'z2 1 ' 'V 1 '2V '2V ar b 1 2 2 1 2 5 0 (6.7) r 'r 'r r 'f 'z 1 ' 2 'V 1 ' 'V 1 '2V 1 u 1 5 0 (6.8) ar b asin b 'r 'u r2 'r r2sin u 'u r2sin2 u 'f2 depending on the coordinate variables used to express V, that is, V 1 x, y, z 2 , V 1 r, f, z 2 , or V 1 r, u, f 2 . Poisson's equation in those coordinate systems may be obtained by simply replacing zero on the right-hand side of eqs.
(6.6), (6.7), and (6.8) with 2rv /e. Laplace's equation is of primary importance in solving electrostatic problems involving a set of conductors maintained at different potentials.
Examples of such problems include capacitors and vacuum tube diodes.
Laplace's and Poisson's equations are not only useful in solving electrostatic field problem; they are used in various other field problems. For example, V would be interpreted as magnetic potential in magnetostatics, as temperature in heat conduction, as stress function in fluid flow, and as pressure head in seepage. 06_Sadiku_Ch06.indd 226 23/09/17 1:39 PM 6.3 Uniqueness Theorem 227 † 6.3 UNIQUENESS THEOREM Since there are several methods (analytical, graphical, numerical, experimental, etc.) of solving a given problem, we may wonder whether solving Laplace's equation in different ways gives different solutions. Therefore, before we begin to solve Laplace's equation, we should answer this question: if a solution of Laplace's equation satisfies a given set of boundary conditions, is this the only possible solution? The answer is yes: there is only one solution. We say that the solution is unique. Thus any solution of Laplace's equation that satisfies the same boundary conditions must be the only solution regardless of the method used. This is known as the uniqueness theorem. The theorem applies to any solution of Poisson's or Laplace's equation in a given region or closed surface. The theorem is proved by contradiction. We assume that there are two solutions V1 and V2 of Laplace's equation, both of which satisfy the prescribed boundary conditions. Thus =2V1 5 0, =2V2 5 0(6.9a) V1 5 V2 on the boundary (6.9b) We consider their difference Vd 5 V2 2 V1(6.10) which obeys =2Vd 5 =2V2 2 =2V1 5 0 (6.11a) Vd 5 0 on the boundary (6.11b) according to eq. (6.9). From the divergence theorem # # 3 = A dv 5 C A dS(6.12) v S where S is the surface surrounding volume v and is the boundary of the original problem. We let A 5 Vd =Vd and use a vector identity = # A 5 = # 1 Vd=Vd 2 5 Vd=2Vd 1 =Vd # =Vd But =2Vd 5 0 according to eq. (6.11a), so = # A 5 =Vd # =Vd(6.13) Substituting eq. (6.13) into eq. (6.12) gives # # 3 =Vd =Vd dv 5 C Vd =Vd dS(6.14) v S From eqs. (6.9) and (6.11), it is evident that the right-hand side of eq. (6.14) vanishes.
06_Sadiku_Ch06.indd 227 23/09/17 1:39 PM 228 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS Hence, 2 3 0 =Vd 0 dv 5 0 v Since the integrand is everywhere positive, or 0 =V 0 d 5 0(6.15a) Vd 5 V2 2 V1 5 constant everywhere in v(6.15b) But eq. (6.15) must be consistent with eq. (6.9b). Hence, Vd 5 0 or V1 5 V2 everywhere, showing that V1 and V2 cannot be different solutions of the same problem.
This is the uniqueness theorem: If a solution to Laplace's equation can be found that satisfies the boundary conditions, then the solution is unique.
Similar steps can be taken to show that the theorem applies to Poisson's equation and to prove the theorem for the case where the electric field (potential gradient) is specified on the boundary.
Before we begin to solve boundary-value problems, we should bear in mind the three things that uniquely describe a problem: 1. The appropriate differential equation (Laplace's or Poisson's equation in this chapter) 2.
The solution region 3. The prescribed boundary conditions A problem does not have a unique solution and cannot be solved completely if any of the three items is missing. 6.4 GENERAL PROCEDURES FOR SOLVING POISSON'S OR LAPLACE'S EQUATION The following general procedure may be taken in solving a given boundary-value problem

involving Poisson's or Laplace's equation: 1. Solve Laplace's (if rv 5 0) or Poisson's (if rv 2 0) equation using either (a) direct integration when V is a function of one variable or (b) separation of variables if V is a function of more than one variable. The solution at this point is not unique but is expressed in terms of unknown integration constants to be determined. 2. Apply the boundary conditions to determine a unique solution for V. Imposing the given boundary conditions makes the solution unique. 3.

Having obtained V, find E using E 5 2=V , D from D 5 eE, and J from J 5 sE. 06_Sadiku_Ch06.indd 228 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 229 4. If required, find the charge Q induced on a conductor using Q 5 eS rS dS, where rS 5 Dn and Dn is the component of D normal to the conductor. If necessary, the capacitance of two conductors can be found using C 5 Q/V or the resistance of an object can be found by using R 5 V/I , where I 5 eS J # dS. Solving Laplace's (or Poisson's) equation, as in step 1, is not always as complicated as it may seem. In some cases, the solution may be obtained by mere inspection of the problem. Also a solution may be checked by going backward and finding out if it satisfies both Laplace's (or Poisson's) equation and the prescribed boundary condition. EXAMPLE 6.1 Current-carrying components in high-voltage power equipment can be cooled to carry away the heat caused by ohmic losses.

A means of pumping is based on the force transmitted to the cooling fluid by charges in an electric field. Electrohydrodynamic (EHD) pumping is modeled in Figure 6.1. The region between the electrodes contains a uniform charge ro, which is generated at the left electrode and collected at the right electrode. Calculate the pressure of the pump if ro 5 25 mC/m3 and Vo 5 22 kV. Solution: Since rv 2 0, we apply Poisson's equation =2V 5 2 rv e The boundary conditions V 1 z 5 0 2 5 Vo and V 1 z 5 d 2 5 0 show that V depends only on z (there is no r or f dependence). Hence, 2ro d2V 5 e dz2 Integrating once gives 2roz dV 5 1A e dz Integrating again yields V52 0V 06_Sadiku_Ch06.indd 229 roz2 1 Az 1 B 2e FIGURE 6.1 An electrohydrodynamic pump; for Example 6.1. 23/09/17 1:39 PM 230 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS where A and B are integration constants to be determined by applying the boundary conditions. When z 5 0, V 5 Vo, Vo 5 20 1 0 1 B S B 5 Vo When z 5 d, V 5 0, 052 rod2 1 Ad 1 Vo 2e or A5 rod Vo 2 2e d The electric field is given by E 5 2=V 5 2 5 c The net force is roz dV a 5a 2 Abaz e dz z ro Vo d 1 az 2 b d az e d 2 F 5 3 rvE dv 5 ro 3 dS 3 v 5 roS c d z50 E dz d Voz ro 2 1 z 2 dz 2 d ` az 1 d 2e 0 F 5 roSVoaz The force per unit area or pressure is r5 F 5 roVo 5 25 3 1023 3 22 3 103 5 550 N/m2 S PRACTICE EXERCISE 6.1 In a one-dimensional device, the charge density is given by rv 5 rox/a.

If E 5 0 at x 5 0 and V 5 0 at x 5 a, find V and E. ro rox2 Answer: 1 a3 2 x3 2 , a. 6ea 2ae x 06_Sadiku_Ch06.indd 230 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 231 EXAMPLE 6.2 L A P Recombination FIGURE 6.2 For Example 6.2. The xerographic copying machine is an important application of electrostatics. The surface of the photoconductor is initially charged uniformly as in Figure 6.2(a). When light from the document to be copied is focused on the photoconductor, the charges on the lower surface combine with those on the upper surface to neutralize each other. The image is developed by pouring a charged black powder over the surface of the photoconductor. The electric field attracts the charged powder, which is later transferred to paper and melted to form a permanent image. We want to determine the electric field below and above the surface of the photoconductor. Solution: Consider the modeled version of Figure 6.2(a) shown in Figure 6.2(b). Since rv 5 0 in this case, we apply Laplace's equation. Also the potential depends only on x.

Thus =2V 5 d2V 50 dx2 Integrating twice gives V 5 Ax 1 B Let the potentials above and below x 5 a be V1 and V2, respectively: 06_Sadiku_Ch06.indd 231 V1 5 A1x 1 B1, x .

a(6.2.1a) V2 5 A2x 1 B2 , x , a (6.2.1b) 23/09/17 1:39 PM 232 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS The boundary conditions at the grounded electrodes are V1 1 x 5 d 2 5 0(6.2.2a) V2 1 x 5 0 2 5 0 (6.2.2b) At the surface of the photoconductor, V1 1 x 5 a 2 5 V2 1 x 5 a 2 (6.2.3a) D1n 2 D2n 5 rS ` x5a (6.2.3b) We use the four conditions in eqs. (6.2.2) and (6.2.3) to determine the four unknown constants A1, A2, B1, and B2. From eqs. (6.2.1) and (6.2.2), 0 5 A1d 1 B1 S B1 5 2A1d(6.2.4a) 0 5 0 1 B2 S B2 50 (6.2.4b) From eqs. (6.2.1) and (6.2.3a), A1a 1 B1 5 A2a(6.2.5) To apply eq.

(6.2.3b), recall that D 5 eE 5 2e=V so that rS 5 D1n 2 D2n 5 e1E1n 2 e2E2n 5 2e1 or dV1 dV2 1 e2 dx dx rS 5 2e1A1 1 e2A2(6.2.6) Solving for A1 and A2 in eqs. (6.2.4) to (6.2.6), we obtain rSax E1 5 2A1ax 5 , a#x#d e2 d e2 e1 c 1 1 2 d e1 a e1 d 2rS a 2 1b ax a E2 5 2A2ax 5 , e2 d e2 e1 c 1 1 2 d e1 a e1 0#x#a PRACTICE EXERCISE 6.2 For the model of Figure 6.2(b), if rS 5 0 and the upper electrode is maintained at Vo while the lower electrode is grounded, show that 2Vo ax 2Vo ax E1 5 , E2 5 e1 e2 e2 d2a1 a a1 d2 a e2 e1 e1 Answer: Proof. 06_Sadiku_Ch06.indd 232 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 233 EXAMPLE 6.3 Semi-infinite conducting planes at f 5 0 and f 5 p/6 are separated by an infinitesimal insulating gap as shown in Figure 6.3. If V 1 f 5 0 2 5 0 and V 1 f 5 p/6 2 5 100 V, calculate V and E in the region between the planes. Solution: Since V depends only on f, Laplace's equation in cylindrical coordinates becomes =2V 5 1 d2V 50 r2 df2 Since r 5 0 is excluded owing to the insulating gap, we can multiply by r2 to obtain d2V 50 df2 which is integrated twice to give V 5 Af 1 B We apply the boundary conditions to determine constants A and B. When f 5 0, V 5 0, 0501B S B50 When f 5 fo, V 5 Vo, Vo 5 Afo S A 5 Vo fo Hence, V5 G 06_Sadiku_Ch06.indd 233 Vo f fo FIGURE 6.3 Potential V 1 f 2 due to semi-infinite conducting planes. 23/09/17 1:39 PM 234 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS and E 5 2=V 5 2 Vo 1 dV af 5 2 a r df rfo f Substituting Vo 5 100 and fo 5 p/6 gives V5 600 f and p E5 2600 a pr f Check: =2V 5 0, V 1 f 5 0 2 5 0, V 1 f 5 p/6 2 5 100. PRACTICE EXERCISE 6.3 Two conducting plates of size 1 3 5 m are inclined at 45 to each other with a gap of width 4 mm separating them as shown in Figure 6.4. Determine an approximate value of the charge per plate if the plates are maintained at a potential difference of 50 V. Assume that the medium between them has er 5 1.5. Answer: 22.2 nC.

EXAMPLE 6.4 Two conducting cones 1 u 5 p/10 and u 5 p/6 2 of infinite extent are separated by an infinitesimal gap at r 5 0. If V 1 u 5 p/10 2 5 0 and V 1 u 5 p/6 2 5 50 V, find V and E between the cones.

Solution: Consider the coaxial cone of Figure 6.5, where the gap serves as an insulator between the two conducting cones. Here V depends only on , so Laplace's equation in spherical coordinates becomes =2V 5 1 d dV csin u d 50 du r2 sin u du FIGURE 6.4 For Practice Exercise 6.3. G 06_Sadiku_Ch06.indd 234 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 235 FIGURE 6.5 Potential V(u) due to conducting cones; for Example 6.4. Gap Since r 5 0 and u 5 0, p are excluded, we can multiply by r2 sin u to get d dV csin u d 50 du du Integrating once gives sin u dV 5A du or dV A 5 du sin u Integrating this results in V 5 A3 5 A3 5 A3 du du 5 A3 sin u 2 cos u/2 sin u/2 1/2 sec2 u/2 du tan u/2 d 1 tan u/2 2 tan u/2 5 A ln 1 tan u/2 2 1 B We now apply the boundary conditions to determine the integration constants A and B. or 06_Sadiku_Ch06.indd 235 V 1 u 5 u 1 2 5 0 S 0 5 A ln 1 tan u 1 /2 2 1 B B 5 2A ln 1 tan u 1 /2 2 23/09/17 1:39 PM 236 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS Hence, V 5 A ln c Also tan u/2 d tan u 1 /2 V 1 u 5 u 2 2 5 Vo S Vo 5 A ln c tan u 2 /2 d tan u 1 /2 or A5 Thus Vo tan u 2 /2 ln c d tan u 1 /2 tan u/2 d tan u 1 /2 V5 tan u 2 /2 ln c d tan u 1 /2 Vo ln c E 5 2=V 5 2 1 dV A a 52 a r du u r sin u u 52 Vo a tan u 2 /2 u r sin u ln c d tan u 1 /2 Taking u 1 5 p/10, u 2 5 p/6, and Vo 5 50 gives tan u/2 d tan p/20 tan u/2 V5 5 95.1 ln c dV tan p/12 0.1584 ln c d tan p/20 50 ln c and E52 95.1 a V/m r sin u u Check: =2V 5 0, V 1 u 5 p/10 2 5 0, V 1 u 5 p/6 2 5 Vo. 06_Sadiku_Ch06.indd 236 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 237 Gap FIGURE 6.6 For Practice Exercise 6.4. PRACTICE EXERCISE 6.4 A large conducting cone 1 u 5 45° 2 is placed on a conducting plane with a tiny gap separating it from the plane as shown in Figure 6.6. If the cone is connected to a 50 V source, find V and E at 1 23, 4, 2 2 . Answer: 27.87 V, 11.35a V/m. EXAMPLE 6.5 (a) Determine the potential function for the region inside the rectangular trough of infinite length whose cross section is shown in Figure 6.7. (b) For Vo 5 100 V and b 5 2a, find the potential at x 5 a/2, y 5 3a/4. Solution: (a) The potential V in this case depends on x and y. Laplace's equation becomes =2V 5 G 06_Sadiku_Ch06.indd 237 G '2V '2V 5 0(6.5.1) 2 1 'x 'y2 FIGURE 6.7 Potential V 1 x, y 2 due to a conducting rectangular trough; for Example 6.5. 23/09/17 1:39 PM 238 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS We have to solve this equation subject to the following boundary conditions: V 1 x 5 0, 0 # y , a 2 5 0(6.5.2a) V 1 x 5 b, 0 # y , a 2 5 0(6.5.2b) V 1 0 # x # b, y 5 0 2 5 0(6.5.2c) V 1 0 , x , b, y 5 a 2 5 Vo(6.5.2d) We solve eq. (6.5.1) by the method of separation of variables; that is, we seek a product solution of V. Let V 1 x, y 2 5 X 1 x 2 Y 1 y 2 (6.5.3) where X is a function of x only and Y is a function of y only.

Substituting eq. (6.5.3) into eq. (6.5.1) yields XsY 1 YsX 5 0 Dividing through by XY and separating X from Y gives 2 Xs Ys 5 (6.5.4a) X Y Since the left-hand side of this equation is a function of x only and the right-hand side is a function of y only, for the equality to hold, both sides must be equal to a constant l; that is, 2 Xs Ys 5 5 l(6.5.4b) X Y The constant l is known as the separation constant. From eq. (6.5.4b), we obtain Xs 1 lX 5 0(6.5.5a) and Ys 2 lY 5 0(6.5.5b) Thus the variables have been separated at this point and we refer to eq. (6.5.5) as separated equations. We can solve for X(x) and Y(y) separately and then substitute our solutions into eq. (6.5.3). To do this requires that the boundary conditions in eq. (6.5.2) be separated, if possible. We separate them as follows: V 1 0, y 2 5 X 1 0 2 Y 1 y 2 5 0 S X 1 0 2 5 0(6.5.6a) V 1 b, y 2 5 X 1 b 2 Y 1 y 2 5 0 S X 1 b 2 5 0(6.5.6b) V 1 x, 0 2 5 X 1 x 2 Y 1 0 2 5 0 S Y 1 0 2 5 0(6.5.6c) V 1 x, a 2 5 X 1 x 2 Y 1 a 2 5 Vo 1 inseparable 2 (6.5.6d) To solve for X(x) and Y(y) in eq. (6.5.5), we impose the boundary conditions in eq.

(6.5.6). We consider possible values of l that will satisfy both the separated equations in eq. (6.5.5) and the conditions in eq. (6.5.6). 06_Sadiku_Ch06.indd 238 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 239 CASE 1. If l 5 0, then eq. (6.5.5a) becomes Xs 5 0 or d2X 50 dx2 which, upon integrating twice, yields X 5 Ax 1 B(6.5.7) The boundary conditions in eqs. (6.5.6a) and (6.5.6b) imply that X 1 x 5 0 2 5 0 S 0 5 0 1 B or B 5 0 and X 1 x 5 b 2 5 0 S 0 5 A # b 1 0 or A 5 0 because b 2 0. Hence our solution for X in eq. (6.5.7) becomes X1x2 5 0 which makes V 5 0 in eq. (6.5.3). Thus we regard X 1 x 2 5 0 as a trivial solution and we conclude that l 2 0.
CASE 2. If l , 0, say l 5 2a2, then eq. (6.5.5a) becomes Xs 2 a2X 5 0 or where D 5 d , that is, dx 1 D 2 2 a2 2 X 5 0 DX 5 6aX(6.5.8) showing that we have two possible solutions corresponding to the plus and minus signs. For the plus sign, eq. (6.5.8) becomes dX 5 aX or dx Hence, 3 dX 5 a dx X dX 5 3 a dx or ln X 5 ax 1 ln A1 X where ln A1 is a constant of integration. Thus 06_Sadiku_Ch06.indd 239 X 5 A1eax(6.5.9a) 23/09/17 1:39 PM 240 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS FIGURE 6.8 Sketch of cosh x and sinh x showing that sinh x 5 0 if and only if x 5 0; for Case 2 of Example 6.5. Similarly, for the minus sign, solving eq. (6.5.8) gives X 5 A2e2ax(6.5.9b) The total solution consists of what we have in eqs. (6.5.9a) and (6.5.9b); that is, X 1 x 2 5 A1eax 1 A2e2ax(6.5.10) Since cosh ax 5 1 eax 1 e2ax 2 /2 and sinh ax 5 1 eax 2 e2ax 2 /2 or eax 5 cosh ax 1 sinh ax and e2ax 5 cosh ax 2 sinh ax, eq. (6.5.10) can be written as X 1 x 2 5 B1 cosh ax 1 B2 sinh ax(6.5.11) where B1 5 A1 1 A2 and B2 5 A1 2 A2. In view of the given boundary conditions, we prefer eq. (6.5.11) to eq. (6.5.10) as the solution. Again, eqs. (6.5.6a) and (6.5.6b) require that and X 1 x 5 0 2 5 0 S 0 5 B1 # 1 1 2 1 B2 # 1 0 2 or B1 5 0 X 1 x 5 b 2 5 0 S 0 5 0 1 B2 sinh ab Since a 2 0 and b 2 0, sinh ab cannot be zero. This is due to the fact that sinh x 5 0 if and only if x 5 0 as shown in Figure 6.8. Hence B2 5 0 and X1x2 5 0 This is also a trivial solution and we conclude that l cannot be less than zero. CASE 3. If l . 0, say l 5 b2, then eq. (6.5.5a) becomes Xs 1 b2X 5 0 06_Sadiku_Ch06.indd 240 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 241 that is, 1 D2 1 b2 2 X 5 0 or DX 5 6jbX(6.5.12) where j 5 !21. From eqs. (6.5.8) and (6.5.12), we notice that the difference between Cases 2 and 3 is the replacement of a by jb. By taking the same procedure as in Case 2, we obtain the solution as X 1 x 2 5 Coe jbx 1 C1e2jbx(6.5.13a) Since e jbx 5 cos bx 1 j sin bx and e2jbx 5 cos bx 2 j sin bx, eq. (6.5.13a) can be written as X 1 x 2 5 go cos bx 1 g1 sin bx(6.5.13b) where go 5 Co 1 C1 and g1 5 j 1 Co 2 C1 2 . In view of the given boundary conditions, we prefer to use eq. (6.5.13b). Imposing the conditions in eqs.

(6.5.6a) and (6.5.6b) yields and X 1 x 5 0 2 5 0 S 0 5 go # 1 1 2 1 0 or go 5 0 X 1 x 5 b 2 5 0 S 0 5 0 1 g1 sin bb Suppose g1 2 0 (otherwise we get a trivial solution), then sin bb 5 0 5 sin np S bb 5 np np b5 , n 5 1, 2, 3, 4, . . . (6.5.14) b Note that, unlike sinh x, which is zero only when x 5 0, sin x is zero at an infinite n umber of points as shown in Figure 6.9. It should also be noted that n 2 0 because b 2 0; we have already considered the possibility b 5 0 in Case 1, where we ended up with a trivial solution. Also we do not need to consider n 5 21, 22, 23, 24, . . .

because l 5 b2 FIGURE 6.9 Sketch of sin x showing that sin x 5 0 at infinite number of points; for Case 3 of Example 6.5. 06_Sadiku_Ch06.indd 241 23/09/17 1:39 PM 242 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS would remain the same for positive and negative integer values of n. Thus for a given n, eq. (6.5.13b) becomes npx Xn 1 x 2 5 gn sin (6.5.15) b Having found X(x) and l 5 b2 5 n2p2 (6.5.16) b2 we solve eq. (6.5.5b), which is now Ys 2 b2Y 5 0 The solution to this is similar to eq. (6.5.11) obtained in Case 2; that is, Y 1 y 2 5 ho cosh by 1 h1 sinh by The boundary condition in eq. (6.5.6c) implies that Y 1 y 5 0 2 5 0 S 0 5 ho # 1 1 2 1 0 or ho 5 0 Hence our solution for Y(y) becomes Yn 1 y 2 5 hn sinh npy b (6.5.17) Substituting eqs. (6.5.15) and (6.5.17), which are the solutions to the separated equations in eq. (6.5.5), into the product solution in eq. (6.5.3) gives Vn 1 x, y 2 5 gnhn sin npy npx sinh b b This shows that there are many possible solutions V1, V2, V3, V4, and so on, for n 5 1, 2, 3, 4, and so on. By the superposition theorem, if V1, V2, V3, . . . , Vn are solutions of Laplace's equation, the linear combination V 5 c1V1 1 c2V2 1 c3V3 1 . .

. 1 cnVn (where c1, c2, c3, . . . , cn are constants) is also a solution of Laplace's equation. Thus the solution to eq.

(6.5.1) is ` npy npx V 1 x, y 2 5 a cn sin sinh (6.5.18) b b n51 where cn 5 gnhn are the coefficients to be determined from the boundary condition in eq. (6.5.6d). Imposing this condition gives 06_Sadiku_Ch06.indd 242 ` npx npa V 1 x, y 5 a 2 5 Vo 5 a cn sin sinh (6.5.19) b b n51 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 243 which is a Fourier series expansion of Vo. Multiplying both sides of eq. (6.5.19) by sin mpx/b and integrating over 0 , x , b gives 3 Vo sin b 0 ` mpx npa mpx npx dx 5 a cn sinh sin dx(6.5.20) 3 sin b b 0 b b n51 b By the orthogonality property of the sine or cosine function (see Appendix A.9). 3 sin mx sin nx dx 5 c p/2, 0 p m2n m5n 0, Incorporating this property in eq. (6.5.20) means that all terms on the right-hand side of eq. (6.5.20) will vanish except one term in which m 5 n. Thus eq. (6.5.20) reduces to npx npa 2 npx dx 5 cn sinh dx 3 Vo sin 3 sin b b 0 b 0 b b b npx b npa 1 2npx cos ` 5 cn sinh a1 2 cos b dx np b 0 b 2 30 b b 2Vo or Vob npa # b 1 1 2 cos np 2 5 cn sinh np b 2 cn sinh that is, 2Vo npa 1 1 2 cos np 2 5 np b 4Vo , n 5 1, 3, 5, . . . 5 • np 0, n 5 2, 4, 6, .

.

. 4Vo cn 5 µ np sinh 0, npa b , n 5 odd n 5 even (6.5.21) Substituting this into eq. (6.5.18) gives the complete solution as V 1 x, y 2 5 4Vo a p n51,3,5, ... ` sin npy npx sinh b b (6.5.22) npa n sinh b Check: =2V 5 0, V 1 x 5 0, y 2 5 0 5 V 1 x 5 b, y 2 5 V 1 x, y 5 0 2 , V 1 x, y 5 a 2 5 Vo. The solution in eq. (6.5.22) should not be a surprise; it can be guessed by mere observation of the potential system in Figure 6.7. From this figure, we notice that along x, V varies 06_Sadiku_Ch06.indd 243 23/09/17 1:39 PM 244 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS from 0 1 at x 5 0 2 to 0 1 at x 5 b 2 and only a sine function can satisfy this requirement. Similarly, along y, V varies from 0 1 at y 5 0 2 to Vo 1 at y 5 a 2 and only a hyperbolic sine function can satisfy this. Thus we should expect the solution as in eq. (6.5.22). To determine the potential for each point 1 x, y 2 in the trough, we take the first few terms of the convergent infinite series in eq.

(6.5.22). Taking four or five terms may be sufficient. (b) For x 5 a/2 and y 5 3a/4, where b 5 2a, we have ` 4Vo a 3a sin np/4 sinh 3np/8 Va , b 5 a p n51,3,5, . .

. 2 4 n sinh np/2 5 4Vo sin p/4 sinh 3p/8 sin 3p/4 sinh 9p/8 1 c p sinh p/2 3 sinh 3p/2 1 sin 5p/4 sinh 15p/8 . . . 1 d 5 sinh 5p/2 4Vo 1 0.4517 1 0.0725 2 0.01985 2 0.00645 1 0.00229 1 . . . 2 p 5 0.6374Vo 5 It is instructive to consider a special case of a 5 b 5 1 m and Vo 5 100 V. The potentials at some specific points are calculated by using eq. (6.5.22), and the result is displayed in Figure 6.10(a). The corresponding flux lines and equipotential lines are shown in Figure 6.10(b). A simple MATLAB program based on eq. (6.5.22) is displayed in Figure 6.11. This self-explanatory program can be used to calculate V 1 x, y 2 at any point within the trough. In Figure 6.11, V 1 x 5 b/4, y 5 3a/4 2 is typically calculated and found to be 43.2 V. FIGURE 6.10 For Example 6.5: (a) V 1 x, y 2 calculated at some points, (b) sketch of flux lines and equipotential lines. 06_Sadiku_Ch06.indd 244 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 245 (a) h=0.1; vo=100; a=1.0; b=a; c=4*vo/pi; IMAX = a/h; JMAX = b/h; NMAX = 13; for I=1:IMAX x=h*I; for J=1:JMAX y=h*J; sum=0.0; for n =1:2:NMAX a1=sin(n*pi*x/b); a2=sinh(n*pi*y/b); a3=n*sinh(n*pi*a/b); sum= sum + c*a1*a2/a3; end V(I,J)=sum; end end mesh(V); (b) 120 100 80 60 40 20 0 10 5 0 0 2 4 6 8 10 FIGURE 6.11 (a) MATLAB program for Example 6.5, (b) the output of the MATLAB program. PRACTICE EXERCISE 6.5 For the problem in Example 6.5, take Vo 5 100 V, b 5 2a 5 2 m, and find V and E at (a) 1 x, y 2 5 1 a, a/2 2 (b) 1 x, y 2 5 1 3a/2, a/4 2 Answer: (a) 44.51 V, 299.25ay V/m, (b) 16.5 V, 20.6ax 2 70.34ay V/m.
06_Sadiku_Ch06.indd 245 23/09/17 1:39 PM 246 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS EXAMPLE 6.6 Find the potential distribution in Example 6.5 if Vo is not constant but (a) Vo 5 10 sin 3px/b, y 5 a, 0 # x # b px 1 5px (b) Vo 5 2 sin , y 5 a, 0 # x # b 1 sin b 10 b Solution: (a) In Example 6.5, every step before eq. (6.5.19) remains the same; that is, the solution is of the form ` npy npx (6.6.1) V 1 x, y 2 5 a cn sin sinh b b n51 in accordance with eq. (6.5.18). But instead of eq. (6.5.19), we now have V 1 y 5 a 2 5 Vo 5 10 sin ` 3px npx npa 5 a cn sin sinh b b b n51 By equating the coefficients of the sine terms on both sides, we obtain cn 5 0, n23 For n 5 3, 10 5 c3 sinh or c3 5 3pa b 10 3pa sinh b Thus the solution in eq. (6.6.1) becomes 3px V 1 x, y 2 5 10 sin b sinh 3py b 3pa sinh b (b) Similarly, instead of eq.

(6.5.19), we have Vo 5 V 1 y 5 a 2 or 2 sin ` px 1 5px npx npa 1 sin 5 a cn sin sinh b 10 b b b n51 Equating the coefficient of the sine terms: cn 5 0, 06_Sadiku_Ch06.indd 246 n 2 1, 5 23/09/17 1:39 PM 6.4 General Procedures for Solving Poisson's or Laplace's Equation 247 For n 5 1, 2 5 c1 sinh pa b or c1 5 2 sinh pa b For n 5 5, 1 5pa 5 c5 sinh 10 b or c5 5 1 10 sinh 5pa b Hence, py 5py px 5px sinh sin sinh b b b b 1 pa 5pa sinh 10 sinh b b 2 sin V 1 x, y 2 5 PRACTICE EXERCISE 6.6 In Example 6.5, suppose everything remains the same except that Vo is replaced by 7px , 0 # x # b, y 5 a. Find V 1 x, y 2 . Vo sin b 7py 7px Vo sin sinh b b Answer: . 7pa sinh b EXAMPLE 6.7 Obtain the separated differential equations for potential distribution V 1 r, f, z 2 in a charge-free region.

Solution: This example, like Example 6.5, further illustrates the method of separation of variables. Since the region is free of charge, we need to solve Laplace's equation in cylindrical coordinates; that is, =2V 5 1 ' 'V 1 '2V '2V ar b 1 2 2 1 2 5 0(6.7.1) r 'r 'r r 'f 'z We let 06_Sadiku_Ch06.indd 247 V 1 r, f, z 2 5 R 1 r 2 F 1 f 2 Z 1 z 2 (6.7.2) 23/09/17 1:39 PM 248 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS where R, ϕ, and Z are, respectively, functions of , , and z. Substituting eq. (6.7.2) into eq. (6.7.1) gives RZ d2ϕ d2Z ϕZ d r dR 1 Rϕ 50 a b1 2 r dr dr r df2 dz2 (6.7.3) We divide through by RFZ to obtain 1 d2ϕ 1 d2Z 1 d r dR a b1 2 2 5 2 rR dr dr Z dz2 r ϕ df (6.7.4) The right-hand side of this equation is solely a function of z, whereas the left-hand side does not depend on z. For the two sides to be equal, they must be constant; that is, 1 d2ϕ 1 d2Z 1 d r dR 5 2 5 2l2 a b1 2 rR dr dr Z dz2 r ϕ df2 (6.7.5) where 2l2 is a separation constant. Equation (6.7.5) can be separated into two parts: 1 d2Z 5 l2 Z dz2 (6.7.6) Z s 2 l 2Z 5 0 (6.7.7) r d r dR 1 d2ϕ 50 a b 1 l2r2 1 R dr dr ϕ df2 (6.7.8) or and Equation (6.7.8) can be written as r dR r2 d2R 1 d2ϕ 1 l2r2 5 2 5 m2 2 1 R dr R dr ϕ df2 (6.7.9) where 2 is another separation constant. Equation (6.7.9) is separated as ϕs 1 m2ϕ 5 0 (6.7.10) r2Rs 1 rRr 1 1 r2l2 2 m2 2 R 5 0 (6.7.11) and 06_Sadiku_Ch06.indd 248 23/09/17 1:39 PM 6.5 Resistance and Capacitance 249 Equations (6.7.7), (6.7.10), and (6.7.11) are the required separated differential equations. Equation (6.7.7) has a solution similar to the solution obtained in Case 2 of Example 6.5; that is, Z 1 z 2 5 c1 cosh lz 1 c2 sinh lz(6.7.12) The solution to eq. (6.7.10) is similar to the solution obtained in Case 3 of Example 6.5; that is, 1 f 2 5 c3 cos mf 1 c4 sin mf(6.7.13) Equation (6.7.11) is known as the Bessel differential equation and its solution is beyond the scope of this text.1 PRACTICE EXERCISE 6.7 Repeat Example 6.7 for V 1 r, u, f 2 . m2 2 Answer: If V 1 r, u, f 2 5 R 1 r 2 F 1 u 2 ϕ 1 f 2 , ϕ s 1 l2ϕ 5 0, Rs 1 Rr 2 2 R 5 0, r r Fs 1 cos u Fr 1 1 m2 sin u 2 l2 csc u 2 F 5 0. 6.5 RESISTANCE AND CAPACITANCE In Section 5.4 the concept of resistance was covered and we derived eq. (5.16) for finding the resistance of a conductor of uniform cross section. If the cross section of the conductor is not uniform, eq. (5.16) becomes invalid and the resistance is obtained from eq. (5.17): R5 V 5 I eL E # dl (6.16) eS sE # dS The problem of finding the resistance of a conductor of nonuniform cross section can be treated as a boundary-value problem. Using eq. (6.16), the resistance R (or conductance G 5 1/R) of a given conducting material can be found by following these steps: 1. Choose a suitable coordinate system. 2. Assume Vo as the potential difference between conductor terminals. 3. Solve Laplace's equation 2V 5 0 to obtain V.

Then determine E from E 5 2=V and find I from I 5 eS sE # dS. 4. Finally, obtain R as Vo /I. In essence, we assume Vo, find I, and determine R 5 Vo /I . Alternatively, it is possible to assume current Io, find the corresponding potential difference V, and determine R from 1 For a complete solution of Laplace's equation in cylindrical or spherical coordinates, see, for e xample, D. T. Paris and F. K. Hurd, Basic Electromagnetic Theory. New York: McGraw-Hill, 1969, pp. 150–159. 06_Sadiku_Ch06.indd 249 23/09/17 1:39 PM 250 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS FIGURE 6.12 A two-conductor capacitor.
R 5 V/Io. As will be discussed shortly, the capacitance of a capacitor is obtained using a similar technique. Generally speaking, to have a capacitor we must have two (or more) conductors carrying equal but opposite charges. This implies that all the flux lines leaving one conductor must necessarily terminate at the surface of the other conductor. The conductors are sometimes referred to as the plates of the capacitor.

The plates may be separated by free space or a dielectric. Consider the two-conductor capacitor of Figure 6.12. The conductors are maintained at a potential difference V given by V 5 V1 2 V2 5 23 E # dl(6.17) 1 2 where E is the electric field existing between the conductors and conductor 1 is assumed to carry a positive charge.
(Note that the E field is always normal to the conducting surfaces.) We define the capacitance C of the capacitor as the ratio of the magnitude of the charge on one of the plates to the potential difference between them; that is, C5 e eS E # dS Q 5 (6.18) V eL E # dl The negative sign before V 5 2 eL E # dl has been dropped because we are interested in the absolute value of V. The capacitance C is a physical property of the capacitor and is measured in farads (F). Most capacitances are practically much smaller than a farad and are specified in microfarads (mF) or picofarads (pF). We can use eq. (6.18) to obtain C for any given two-conductor capacitance by following either of these methods: 1.
Assuming Q and determining V in terms of Q (involving Gauss's law) Q (assume) V (find) 2.

Assuming V and determining Q in terms of V (involving solving Laplace's equation) C5 C5 06_Sadiku_Ch06.indd 250 Q (find) V (assume) 23/09/17 1:39 PM 6.5 Resistance and Capacitance 251 We shall use the former method here, and the latter method will be illustrated in Examples 6.10 and 6.11. The former method involves taking the following steps: 1. Choose a suitable coordinate system. 2. Let the two conducting plates carry charges 1Q and 2Q. 3. Determine E by using Coulomb's or Gauss's law and find V from V 5 2 eL E # dl. The negative sign may be ignored in this case because we are interested in the absolute value of V. 4. Finally, obtain C from C 5 Q/V . We will now apply this mathematically attractive procedure to determine the capacitance of some important two-conductor configurations. A. Parallel-Plate Capacitor Consider the parallel-plate capacitor of Figure 6.13(a). Suppose that each of the plates has an area S and they are separated by a distance d. We assume that plates 1 and 2, respectively, carry charges 1Q and 2Q uniformly distributed on them so that rS 5 Q (6.19) S FIGURE 6.13 (a) Parallel-plate capacitor. (b) Fringing effect due to a parallel-plate capacitor. D 06_Sadiku_Ch06.indd 251 P 23/09/17 1:39 PM 252 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS An ideal parallel-plate capacitor is one in which the plate separation d is very small compared with the dimensions of the plate. Assuming such an ideal case, the fringing field at the edge of the plates, as illustrated in Figure 6.13(b), can be ignored so that the field between them is considered uniform. If the space between the plates is filled with a homogeneous dielectric with permittivity « and we ignore flux fringing at the edges of the plates, from eq. (4.27), D 5 2rSax or E5 rS 1 2ax 2 e 52 Q a (6.20) eS x Hence, d Q Qd # (6.21) V 5 23 E dl 5 23 c2 ax d # dx ax 5 eS eS 2 0 1 and thus for a parallel-plate capacitor C5 Q eS 5 (6.22) V d This formula offers a means of measuring the dielectric constant «r of a given dielectric. By measuring the capacitance C of a parallel-plate capacitor with the space between the plates filled with the dielectric and the capacitance Co with air between the plates, we find «r from er 5 C (6.23) Co Using eq. (4.96), it can be shown that the energy stored in a capacitor is given by WE 5 Q2 1 1 (6.24) CV2 5 QV 5 2 2 2C To verify this for a parallel-plate capacitor, we substitute eq. (6.20) into eq. (4.96) and obtain WE 5 5 Q2 eQ2Sd 1 3 e 2 2 dv 5 2 v eS 2e2S2 Q2 d Q2 1 5 QV a b5 2 eS 2C 2 as expected. 06_Sadiku_Ch06.indd 252 23/09/17 1:39 PM 6.5 Resistance and Capacitance 253 FIGURE 6.14 A coaxial capacitor. D B. Coaxial Capacitor A coaxial capacitor is essentially a coaxial cable or coaxial cylindrical capacitor. Consider length L of two coaxial conductors of inner radius a and outer radius b 1 b . a 2 as shown in Figure 6.14. Let the space between the conductors be filled with a homogeneous dielectric with permittivity «. We assume that conductors 1 and 2, respectively, carry 1Q and 2Q uniformly distributed on them. By applying Gauss's law to an arbitrary Gaussian cylindrical surface of radius r 1 a , r , b 2 , we obtain Q 5 e C E # dS 5 eEr2prL(6.25) S Hence, E5 Q a (6.26) 2perL r Neglecting flux fringing at the cylinder ends, V 5 23 E # dl 5 23 c a 1 b 2 5 Q a d # dr ar(6.27a) 2perL r Q b ln (6.27b) 2peL a Thus the capacitance of a coaxial cylinder is given by C5 Q 2peL (6.28) 5 V b ln a C. Spherical Capacitor A spherical capacitor is the case of two concentric spherical conductors. Consider the inner sphere of radius a and outer sphere of radius b 1 b . a 2 separated by a dielectric medium with permittivity « as shown in Figure 6.15. We assume charges 1Q and 2Q on the inner 06_Sadiku_Ch06.indd 253 23/09/17 1:39 PM 254 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS FIGURE 6.15 A spherical capacitor. D and outer spheres, respectively. By applying Gauss's law to an arbitrary Gaussian spherical surface of radius r 1 a , r , b 2 , we have Q 5 e C E # dS 5 eEr 4pr2(6.29) S that is, E5 Q ar(6.30) 4per2 The potential difference between the conductors is Q V 5 23 E # dl 5 23 c ar d # dr ar 4per2 2 b a 1 5 Q 1 1 c 2 d 4pe a b (6.31) Thus the capacitance of the spherical capacitor is C5 Q 4pe (6.32) 5 V 1 1 2 a b By letting b S ` , C 5 4pea, which is the capacitance of a spherical capacitor whose outer plate is infinitely large. Such is the case of a spherical conductor at a large distance from other conducting bodies—the isolated sphere. Even an irregularly shaped object of about the same size as the sphere will have nearly the same capacitance. This fact is useful in estimating the stray capacitance of an isolated body or piece of equipment. Recall from network theory that if two capacitors with capacitance C1 and C2 are in series (i.e., they have the same charge on them) as shown in Figure 6.16(a), the total capacitance is 1 1 1 5 1 C C1 C2 06_Sadiku_Ch06.indd 254 23/09/17 1:39 PM 6.5 Resistance and Capacitance 255 FIGURE 6.16 Capacitors (a) in series and (b) in parallel. or C5 C1 C2 (6.33) C1 1 C2 If the capacitors are in parallel (i.e., if they have the same voltage across their plates) as shown in Figure 6.16(b), the total capacitance is C 5 C1 1 C2(6.34) Let us reconsider the expressions for finding the resistance R and the capacitance C of an electrical system. The expressions were given in eqs. (6.16) and (6.18): R 5 C 5 V 5 I eLE # dl (6.16) eS sE # dS e AS E # dS Q 5 (6.18) V eLE # dl The product of these expressions yields RC 5 e (6.35) s which is the relaxation time Tr of the medium separating the conductors. It should be remarked that eq. (6.35) is valid only when the medium is homogeneous; this is easily inferred from eqs. (6.16) and (6.18). Assuming homogeneous media, the resistance of various capacitors mentioned earlier can be readily obtained using eq. (6.35). The following examples are provided to illustrate this idea. For a parallel-plate capacitor, 06_Sadiku_Ch06.indd 255 C5 eS , d R5 d (6.36) sS 23/09/17 1:39 PM 256 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS For a cylindrical capacitor, 2 peL C5 , b ln a b a R5 (6.37) 2 psL ln For a spherical capacitor, 4pe C5 , 1 1 2 a b 1 1 2 a b R5 (6.38) 4ps And finally for an isolated spherical conductor, C 5 4pea, R5 1 4psa (6.39) It should be noted that the resistance R in each of eqs. (6.35) to (6.39) is not the resistance of the capacitor plate but the leakage resistance between the plates; therefore, in those equations is the conductivity of the dielectric medium separating the plates. EXAMPLE 6.8 A metal bar of conductivity is bent to form a flat 90 sector of inner radius a, outer radius b, and thickness t as shown in Figure 6.17. Show that (a) the resistance of the bar between the vertical curved surfaces at r 5 a and r 5 b is b a R5 spt 2 ln and (b) the resistance between the two horizontal surfaces at z 5 0 and z 5 t is Rr 5 06_Sadiku_Ch06.indd 256 4t sp 1 b2 2 a2 2 FIGURE 6.17 Bent metal bar for Example 6.8. 23/09/17 1:39 PM 6.5 Resistance and Capacitance 257 Solution: (a) Between the vertical curved ends located at r 5 a and r 5 b, the bar has a nonuniform cross section and hence eq. (5.16) does not apply. We have to use eq.
(6.16). Let a potential difference Vo be maintained between the curved surfaces at r 5 a and r 5 b so that V 1 r 5 a 2 5 0 and V 1 r 5 b 2 5 Vo. We solve for V in Laplace's equation =2V 5 0 in cylindrical coordinates. Since V 5 V 1 r 2 , =2V 5 1 d dV ar b50 r dr dr As r 5 0 is excluded, upon multiplying by r and integrating once, this becomes dV 5A dr r or dV A 5 r dr Integrating once again yields V 5 A ln r 1 B where A and B are constants of integration to be determined from the boundary conditions.

V 1 r 5 a 2 5 0 S 0 5 A ln a 1 B or B 5 2A ln a b V V 1 r 5 b 2 5 Vo S Vo 5 A ln b 1 B 5 A ln b 2 A ln a 5 A ln or A 5 o a b ln a Hence, r r V 5 o ln a b a ln a dV A V E 5 2=V 5 2 a 5 2 ar 5 2 o ar r dr r b r ln a J 5 sE, dS 5 2r df dz ar V 5 A ln r 2 A ln a 5 A ln I 5 3 J # dS 5 3 S 06_Sadiku_Ch06.indd 257 p/2 f50 3 t Vos p tVos dz r df 5 b 2 b z50 r ln ln a a 23/09/17 1:40 PM 258 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS Thus b 2 ln a Vo R5 5 I spt as required. (b) Let V o be the potential difference between the two horizontal surfaces so that V 1 z 5 0 2 5 0 and V 1 z 5 t 2 5 Vo. V 5 V 1 z 2 , so Laplace's equation =2V 5 0 becomes d2V 50 dz2 Integrating twice gives V 5 Az 1 B We apply the boundary conditions to determine A and B: V1 z 50 2 50 S 0 50 1B or Hence, V 1 z 5 t 2 5 Vo S Vo 5 At V5 or B 50 A5 Vo t Vo z t Vo dV az 5 2 a z dz t sVo J 5 sE 5 2 a , dS 5 2r df dr az t z E 5 2=V 5 2 I 5 3 J # dS 5 3 S Thus b r50 3 V0s r df dr f50 t p/2 Vos # p r2 b Vo s p 1 b2 2 a2 2 5 ` 5 t 2 2 a 4t Rr 5 Vo 4t 5 I sp 1 b2 2 a2 2 Alternatively, for this case, the cross section of the bar is uniform between the horizontal surfaces at z 5 0 and z 5 t and eq. (5.16) holds. Hence, 06_Sadiku_Ch06.indd 258 23/09/17 1:40 PM 6.5 Resistance and Capacitance 259 Rr 5 t , 5 sS p s 1 b2 2 a2 2 4 4t 5 sp 1 b2 2 a2 2 as required. PRACTICE EXERCISE 6.8 A disk of thickness t has radius b and a central hole of radius a. Taking the conductivity of the disk as , find the resistance between (a) The hole and the rim of the disk (b) The two flat sides of the disk b a t Answer: (a) , (b) . 2 2pts sp 1 b2 2 a2 2 ln EXAMPLE 6.9 A coaxial cable contains an insulating material of conductivity .
If the radius of the central wire is a and that of the sheath is b, show that the conductance of the cable per unit length is [see eq. (6.37)] G5 2ps b ln a Solution: Consider length L of the coaxial cable as shown in Figure 6.14. Let Vo be the potential difference between the inner and outer conductors so that V 1 r 5 a 2 5 0 and V 1 r 5 b 2 5 Vo, which allows V and E to be found just as in part (a) of Example 6.8. Hence, J 5 sE 5 2sVo a, b r r ln a I 5 3 J # dS 5 3 S 5 06_Sadiku_Ch06.indd 259 2p f50 dS 5 2r df dz ar 3 L Vos r dz df b z50 r ln a 2pLsVo b ln a 23/09/17 1:40 PM 260 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS The resistance of the cable of length is given by R5 lnba Vo # 5 I L ps and the conductance per unit length is G5 as required. 1 2ps 5 R ln 1 ab 2 PRACTICE EXERCISE 6.9 A coaxial cable contains an insulating material of conductivity 1 in its upper half and another material of conductivity 2 in its lower half (similar to the situation shown later in Figure 6.19b). If the radius of the central wire is a and that of the sheath is b, show that the leakage resistance of length of the cable is R5 Answer: Proof. EXAMPLE 6.10 b 1 ln p, 1 s1 1 s2 2 a Conducting spherical shells with radii a 5 10 cm and b 5 30 cm are maintained at a potential difference of 100 V such that V 1 r 5 b 2 5 0 and V 1 r 5 a 2 5 100 V. Determine V and E in the region between the shells. If er 5 2.5 in the region, determine the total charge induced on the shells and the capacitance of the capacitor. Solution: Consider the spherical shells shown in Figure 6.18 and assume that V depends only on r. Hence Laplace's equation becomes FIGURE 6.18 Potential V(r) due to conducting spherical shells. 06_Sadiku_Ch06.indd 260 23/09/17 1:40 PM 6.5 Resistance and Capacitance 261 =2V 5 1 d 2 dV cr d 50 dr r2 dr Since r 2 0 in the region of interest, we multiply through by r2 to obtain d 2 dV cr d 50 dr dr Integrating once gives r2 dV 5A dr or dV A 5 2 dr r Integrating again gives V52 A 1B r As usual, constants A and B are determined from the boundary conditions. When r 5 b, V 5 0 S 0 5 2 A A 1 B or B 5 b b Hence, 1 1 V 5 Ac 2 d r b 1 1 Also when r 5 a, V 5 Vo S Vo 5 A c 2 d a b or A5 Vo 1 1 2 a b Thus 1 c 2 r V 5 Vo 1 2 a 06_Sadiku_Ch06.indd 261 1 d b 1 b 23/09/17 1:40 PM 262 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS E 5 2=V 5 2 dV A ar 5 2 2 ar dr r Vo a 1 1 r r2 c 2 d a b Q 5 3 eE # dS 5 3 S 5 p u 50 4peoerVo 1 1 2 a b 3 2p eoerVo r2 sin u df du 1 f50 2 1 r c 2 d a b Alternatively, rs 5 Dn 5 eEr, Q 5 3 rs dS 5 The capacitance is easily determined as C5 Q 4pe 5 Vo 1 1 2 a b which is the same as we obtained in eq. (6.32); there in Section 6.5, we assumed Q and found the corresponding Vo, but here we assumed Vo and found the corresponding Q to determine C. Substituting a 5 0.1 m, b 5 0.3 m, Vo 5 100 V yields 1 10 c 2 d r 3 1 10 V 5 100 5 15 c 2 d V r 10 2 10/3 3 Check: =2V 5 0, V 1 r 5 0.3 m 2 5 0, V 1 r 5 0.1 m 2 5 100. E5 100 15 ar 5 2 ar V/m r r 3 10 2 10/3 4 2 Q 5 64p # 1029 # 1 2.5 2 # 1 100 2 36p 10 2 10/3 5 64.167 nC The positive charge is induced on the inner shell; the negative charge is induced on the outer shell. Also C5 06_Sadiku_Ch06.indd 262 0Q0 Vo 5 4.167 3 1029 5 41.67 pF 100 23/09/17 1:40 PM 6.5 Resistance and Capacitance 263 FIGURE 6.19 For Practice Exercises 6.9, 6.10, and 6.12. PRACTICE EXERCISE 6.10 If Figure 6.19 represents the cross sections of two spherical capacitors, determine their capacitances. Let a 5 1 mm, b 5 3 mm, c 5 2 mm, er1 5 2.5, and er2 5 3.5. Answer: (a) 0.53 pF, (b) 0.5 pF. EXAMPLE 6.11 In Section 6.5, it was mentioned that the capacitance C 5 Q/V of a capacitor can be found by either assuming Q and finding V, as in Section 6.5, or by assuming V and finding Q, as in Example 6.10. Use the latter method to derive eq. (6.22). Solution: Assume that the parallel plates in Figure 6.13 are maintained at a potential difference Vo so that V 1 x 5 0 2 and V 1 x 5 d 2 5 Vo. This necessitates solving a one-dimensional boundary-value problem; that is, we solve Laplace's equation =2V 5 d2V 50 dx2 Integrating twice gives V 5 Ax 1 B where A and B are integration constants to be determined from the boundary conditions. At x 5 0, V 5 0 S 0 5 0 1 B, or B 5 0, and at x 5 d, V 5 Vo S Vo 5 Ad 1 0 or A 5 Vo /d. Hence, V5 Vo x d Notice that this solution satisfies Laplace's equation and the boundary conditions. We have assumed the potential difference between the plates to be Vo. Our goal is to find the charge Q on either plate so that we can eventually find the capacitance C 5 Q/Vo. The charge on either plate is 06_Sadiku_Ch06.indd 263 23/09/17 1:40 PM 264 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS Q 5 3 rS dS S But rS 5 D # an 5 eE # an, where E 5 2=V 5 2 Vo dV a 5 2Aax 5 2 ax dx x d On the lower plate, an 5 ax, so rS 5 2 eVo d and Q52 eVo d and Q5 eVoS d On the upper plate, an 5 2ax, so rS 5 eVoS d As expected, Q is equal but opposite on each plate.

Thus C5 0Q0 Vo 5 eS d which is in agreement with eq. (6.22). PRACTICE EXERCISE 6.11 Derive the formula for the capacitance C 5 Q/Vo of a cylindrical capacitor in eq. (6.28) by assuming Vo and finding Q. EXAMPLE 6.12 Determine the capacitance of each of the capacitors in Figure 6.20. Take er1 5 4, er2 5 6, d 5 5 mm, S 5 30 cm2. Solution: (a) Since D and E are normal to the dielectric interface, the capacitor in Figure 6.20(a) can be treated as consisting of two capacitors C1 and C2 in series as in Figure 6.16(a). C1 5 eoer1S 2eoer1S 5 , d/2 d C2 5 2eoer2S d The total capacitor C is given by C5 06_Sadiku_Ch06.indd 264 2eoS 1 er1er2 2 C1C2 5 C1 1 C2 d er1 1 er2 23/09/17 1:40 PM 6.5 Resistance and Capacitance 265 FIGURE 6.20 For Example 6.12. 1029 # 30 3 1024 # 4 3 6 (6.12.1) 52# 36p 5 3 1023 10 C 5 25.46 pF (b) In this case, D and E are parallel to the dielectric interface. We may treat the capacitor as consisting of two capacitors C1 and C2 in parallel (the same voltage across C1 and C2) as in Figure 6.16(b). C1 5 eoer1S/2 eoer1S 5 , d 2d C2 5 eoer2S 2d The total capacitance is eoS 1 e 1 er2 2 2d r1 1029 # 30 3 1024 # 5 10 (6.12.2) 36p 2 # 1 5 3 1023 2 C 5 26.53 pF C 5 C1 1 C2 5 Notice that when er1 5 er2 5 er, eqs. (6.12.1) and (6.12.2) agree with eq. (6.22) as e xpected. PRACTICE EXERCISE 6.12 Determine the capacitance of 10 m length of the cylindrical capacitors shown in Figure 6.19. Take a 5 1 mm, b 5 3 mm, c 5 2 mm, er1 5 2.5, and er2 5 3.5. Answer: (a) 1.54 F, (b) 1.52 nF.

EXAMPLE 6.13 06_Sadiku_Ch06.indd 265 A cylindrical capacitor has radii a 5 1 cm and b 5 2.5 cm. If the space between the plates is filled with an inhomogeneous dielectric with er 5 1 10 1 r 2 /r, where r is in centimeters, find the capacitance per meter of the capacitor. 23/09/17 1:40 PM 266 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS Solution: The procedure is the same as that taken in Section 6.5 except that eq. (6.27a) now becomes V 5 23 a b 5 5 Q Q dr 5 2 2peoerrL 2peoL 3b a dr 10 1 r ra b r a a dr 2Q 2Q 5 ln 1 10 1 r 2 ` 3 2peoL b 10 1 r 2peoL b Q 10 1 b ln 2peoL 10 1 a Thus the capacitance per meter is 1 L 5 1 m 2 Q 5 V 2peo 1029 # 1 5 2p # 10 1 b 36p 12.5 ln ln 10 1 a 11.0 C 5 434.6 pF/m C5 PRACTICE EXERCISE 6.13 A spherical capacitor with a 5 1.5 cm, b 5 4 cm has an inhomogeneous dielectric of e 5 10eo /r . Calculate the capacitance of the capacitor. Answer: 1.13 nF. 6.6 METHOD OF IMAGES The method of images, introduced by Lord Kelvin in 1848, is commonly used to determine V, E, D, and rS due to charges in the presence of conductors.

By this method, we avoid solving Poisson's or Laplace's equation but rather utilize the fact that a conducting surface is an equipotential. Although the method does not apply to all electrostatic problems, it can reduce a formidable problem to a simple one. The image theory states that a given charge configuration above an infinite grounded perfect conducting plane may be replaced by the charge configuration itself, its image, and an equipotential surface in place of the conducting plane. Typical examples of point, line, and volume charge configurations are portrayed in Figure 6.21(a), and their corresponding image configurations are in Figure 6.21(b). 06_Sadiku_Ch06.indd 266 23/09/17 1:40 PM 6.6 Method of Images 267 FIGURE 6.21 Image system: (a) charge configurations above a perfectly conducting plane, (b) image configuration with the conducting plane replaced by equipotential surface. In applying the image method, two conditions must always be satisfied: 1. The image charge(s) must be located in the conducting region. 2. The image charge(s) must be located such that on the conducting surface(s) the potential is zero or constant. The first condition is necessary to satisfy Poisson's equation, and the second condition ensures that the boundary conditions are satisfied. Let us now apply the image theory to two specific problems. A. A Point Charge above a Grounded Conducting Plane Consider a point charge Q placed at a distance h from a perfect conducting plane of infinite extent as in Figure 6.22(a). The image configuration is in Figure 6.22(b). The electric field in the region above the plane at point P 1 x, y, z 2 is given by FIGURE 6.22 (a) Point charge and grounded conducting plane. (b) Image configuration and field lines. 06_Sadiku_Ch06.indd 267 23/09/17 1:40 PM 268 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS E 5 E1 1 E2(6.40) 5 Q r1 2Q r2 (6.41) 3 1 4peor1 4peor32 The distance vectors r1 and r2 are given by so eq. (6.41) becomes E5 r1 5 1 x, y, z 2 2 1 0, 0, h 2 5 1 x, y, z 2 h 2 (6.42) r2 5 1 x, y, z 2 2 1 0, 0, 2h 2 5 1 x, y, z 1 h 2 (6.43) xax 1 yay 1 1 z 2 h 2 az xax 1 yay 1 1 z 1 h 2 az Q 2 2 c 2 d (6.44) 2 2 3/2 3 x 1 y2 1 1 z 1 h 2 2 4 3/2 4peo 3 x 1 y 1 1 z 2 h 2 4 It should be noted that when z 5 0, E has only the z-component, confirming that E is normal to the conducting surface. The potential at P is easily obtained from eq. (6.41) or (6.44) using V 5 2 eL E # dl. Thus V 5 V1 1 V2 5 V5 Q 2Q (6.45) 1 4peor1 4peor2 Q 1 1 2 2 e f 3 x 1 y2 1 1 z 1 h 2 2 4 1/2 4peo 3 x2 1 y2 1 1 z 2 h 2 2 4 1/2 for z $ 0 and V 5 0 for z # 0. Note that V 1 z 5 0 2 5 0. The surface charge density of the induced charge can also be obtained from eq. (6.44) as rS 5 Dn 5 eoEn ` z50 2Qh 5 2 2p 3 x 1 y2 1 h2 4 3/2 (6.46) The total induced charge on the conducting plane is Qi 5 3 rS dS 5 3 ` 2` 3 2Qh dx dy (6.47) 2 2 2 3/2 3 2` 2p x 1 y 1 h 4 ` By changing variables, r2 5 x2 1 y2, dx dy 5 r dr df, and we have 06_Sadiku_Ch06.indd 268 Qi 5 2 2p ` r dr df Qh (6.48) 2p 30 30 3 r2 1 h2 4 3/2 23/09/17 1:40 PM 6.6 Method of Images 269 1 Integrating over f gives 2p, and letting r dr 5 d 1 r2 2 , we obtain 2 Qh 1 2p 3 3 r2 1 h2 4 23/2 d 1 r2 2 2p 2 0 ` Qi 5 2 5 ` Qh (6.49) ` 3 r2 1 h2 4 1/2 0 5 2Q as expected, because all flux lines terminating on the conductor would have terminated on the image charge if the conductor were absent. B. A Line Charge above a Grounded Conducting Plane Consider an infinite line charge with density rL C/m located at a distance h from the grounded conducting plane at z 5 0. This may be regarded as a problem of a long conductor over the earth. The image system of Figure 6.22(b) applies to the line charge except that Q is replaced by rL. The infinite line charge rL may be assumed to be at x 5 0, z 5 h, and the image 2rL at x 5 0, z 5 2h so that the two are parallel to the y-axis. The electric field at point P is given (from eq. 4.21) by E 5 E1 1 E2(6.50) rL 2rL 5 a 1 a (6.51) 2peor1 r1 2peor2 r2 The distance vectors 1 and 2 are given by r1 5 1 x, y, z 2 2 1 0, y, h 2 5 1 x, 0, z 2 h 2 (6.52) so eq. (6.51) becomes r2 5 1 x, y, z 2 2 1 0, y, 2h 2 5 1 x, 0, z 1 h 2 (6.53) E5 xax 1 1 z 1 h 2 az rL xax 1 1 z 2 h 2 az 2 2 c d (6.54) 2peo x2 1 1 z 2 h 2 2 x 1 1z 1 h22 Again, notice that when z 5 0, E has only the z-component, confirming that E is normal to the conducting surface. The potential at P is obtained from eq. (6.51) or (6.54) using V 5 2 eL E # dl. Thus V 5 V1 1 V2 52 06_Sadiku_Ch06.indd 269 rL 2rL ln r1 2 ln r2 2peo 2peo 52 rL r1 ln (6.55) 2peo r2 23/09/17 1:40 PM 270 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS Substituting r1 5 0 r1 0 and r2 5 0 r2 0 in eqs. (6.52) and (6.53) into eq.

(6.55) gives V52 rL x2 1 1 z 2 h 2 2 1/2 ln c 2 d (6.56) 2peo x 1 1z 1 h22 for z $ 0 and V 5 0 for z # 0. Note that V 1 z 5 0 2 5 0. The surface charge induced on the conducting plane is given by rS 5 Dn 5 eoEz ` 5 z50 2rLh (6.57) p 1 x2 1 h 2 2 The induced charge per length on the conducting plane is ri 5 3 rS dx 5 2 L rLh ` dx (6.58) p 32` x2 1 h2 By letting x 5 h tan a, eq. (6.58) becomes rLh p/2 da ri 5 2 p 32p/2 h (6.59) 5 2rL as expected. EXAMPLE 6.14 A point charge Q is located at point 1 a, 0, b 2 between two semi-infinite conducting planes intersecting at right angles as in Figure 6.23. Determine the potential at point P 1 x, y, z 2 in region z $ 0 and x $ 0 and the force on Q. Solution: The image configuration is shown in Figure 6.24. Three image charges are necessary to satisfy the two conditions listed at the beginning of this section. From Figure 6.24(a), the potential at point P 1 x, y, z 2 is the superposition of the potentials at P due to the four point charges; that is, V5 where Q 1 1 1 1 c 2 1 2 d r2 r3 r4 4peo r1 r1 5 3 1 x 2 a 2 2 1 y2 1 1 z 2 b 2 2 4 1/2 r2 5 3 1 x 1 a 2 2 1 y2 1 1 z 2 b 2 2 4 1/2 r3 5 3 1 x 1 a 2 2 1 y2 1 1 z 1 b 2 2 4 1/2 r4 5 3 1 x 2 a 2 2 1 y2 1 1 z 1 b 2 2 4 1/2 06_Sadiku_Ch06.indd 270 23/09/17 1:40 PM 6.6 Method of Images 271 FIGURE 6.23 Point charge between two semi-infinite conducting planes. From Figure 6.24(b), the net force on Q is F 5 F1 1 F2 1 F3 Q2 1 2aax 1 2baz 2 Q2 Q2 52 2 az 2 2 ax 1 4peo 1 2b 2 4peo 1 2a 2 4peo 3 1 2a 2 2 1 1 2b 2 2 4 3/2 5 Q2 a 1 b 1 2 2 d ax 1 c 2 2 2 d az f ec 2 2 3/2 2 3/2 1a 1 b 2 1a 1 b 2 16peo a b The electric field due to this system can be determined similarly, and the charge induced on the planes can also be found. In general, when the method of images is used for a system consisting of a point charge between two semi-infinite conducting planes inclined at an angle f (in degrees), the number of images is given by N5a 360° 2 b f FIGURE 6.24 Determining (a) the potential at P and (b) the force on charge Q. 06_Sadiku_Ch06.indd 271 23/09/17 1:40 PM 272 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS FIGURE 6.25 Point charge between two semiinfinite conducting walls inclined at f 5 60 to each other. because the charge and its images all lie on a circle. For example, when f 5 180, N 5 1 as in the case of Figure 6.22; for f 5 90, N 5 3 as in the case of Figure 6.23; and for f 5 60, we expect N 5 5 as shown in Figure 6.25. PRACTICE EXERCISE 6.14 If the point charge Q 5 10 nC in Figure 6.25 is 10 cm away from point O and along the line bisecting f 5 60, find the magnitude of the force on Q due to the charge induced on the conducting walls. Answer: 60.54 mN. † 6.7 APPLICATION NOTE—CAPACITANCE OF MICROSTRIP LINES The increasing application of integrated circuits at microwave frequencies has generated interest in the use of rectangular and circular microstrip disk capacitors as lumped-element circuits. The fringing field effects of such capacitors were first observed in 1877 by Kirchhoff, who used conformal mapping to account for the fringing. But his analysis was limited by the assumption that the capacitor is air filled.

In microstrip applications, the capacitor plates are separated by a dielectric material instead of free space. Lately, others have come up with better approximate closed-form solutions to the problem taking into account the presence of the dielectric material and fringing. We consider only the circular disk c apacitor. The geometry of the circular microstrip capacitor, with radius r and separation distance d, is shown in Figure 6.26. Again, if disk area S 1 S 5 pr2 2 is very large compared with the separation distance (i.e., !S W d), then fringing is minimal and the capacitance is given by 06_Sadiku_Ch06.indd 272 C5 eoerpr2 (6.60) d 23/09/17 1:40 PM 6.7 Application Note—Capacitance of Microstrip Lines 273 FIGURE 6.26 Circular microstrip capacitor. r d Several researchers have attempted to account for the effect of fringing and to obtain a closed-form solution. We consider the following cases. CASE 1. According to Kirchhoff,2 the fringing capacitance is so that the total capacitance is DC 5 eoerr alog CT 5 16pr 2 1b(6.61) d eoerpr2 16pr 1 eoerr alog 2 1b(6.62) d d It should be noted that Kirchhoff 's approximation is valid only for er 5 1. CASE 2. According to Chew and Kong,3 the total capacitance including fringing is CT 5 eoerpr2 2d r d e1 1 clna b 1 1 1.41er 1 1.77 2 1 1 0.268er 1 1.65 2 d f (6.63) pe r r d 2d r CASE 3. Wheeler used interpolation to match the three cases of small, medium, and large disk sizes. According to Wheeler,4 we first define the following 2 Cks 5 eor c4 1 1 1 er 2 1 erpr d (6.64) d L. D. Landau and E. M.
Lifshitz, Electrodynamics of Continuous Media. Oxford: Pergamon Press, 1960, p. 20. 3 W. C. Chew and J. A. Kong, "Effects of fringing fields on the capacitance of circular microstrip disk," IEEE Transactions on Microwave Theory and Techniques, vol. 28, no. 2, Feb. 1980, pp. 98–103. 4 H. A.
Wheeler, "A simple formula for the capacitance of a disc on dielectric on a plane," IEEE Transactions on Microwave Theory and Techniques, vol.
30, no. 11, Nov. 1982, pp. 2050–2054. 06_Sadiku_Ch06.indd 273 23/09/17 1:40 PM 274 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS 250 Chew and Kong Wheeler Kirchhoff, without fringing Capacitance (pF) 200 150 100 50 0 0 20 40 60 80 100 120 Radius (mil) 140 160 180 200 FIGURE 6.27 Capacitance of the circular microstrip capacitor. where k 5 er. When k 5 1, eq. (6.64) becomes C1s 5 eor c8 1 The total capacitance is CT 5 where pr d (6.65) d Cks 1 C1 1 a1 2 bC2Cks(6.66) kcC1s kc 1 1 0.8 1 r/d 2 2 1 1 0.31r/d 2 4 pr 2 1 ln a b d (6.67) d 3 1 1 0.9 1 r/d 2 1 C2 5 1 2 (6.68) r d 4 1 2.6 1 2.9 r d C1 5 eor c 8 1 kc 5 0.37 1 0.63er(6.69) A MATLAB program was developed by using eqs. (6.62) to (6.69). With specific values of d 5 10 mil and er 5 74.04, the values of C and CT for 10 , r , 200 mil are plotted in Figure 6.27 for the three cases. The curve for Kirchhoff 's approximation coincides with the case without fringing. 06_Sadiku_Ch06.indd 274 23/09/17 1:40 PM 6.8 Application Note—RF MEMS 275 6.8 APPLICATION NOTE—RF MEMS Linear spring Driving voltage Air gap Floating electrode Driven electrode Radio-frequency microelectromechanical systems (RF MEMS) are electronic devices with a submillimeter-sized moving part in the form of a beam, comb, disk, or ring, capable of providing RF functionality. Figure 6.28 shows one example. Another class consists of bulk or surface micromachined devices, such as thin-film bulk acoustic resonators (FBARs), which rely on energy transduction from the electrical energy domain to the acoustic energy domain and vice versa to provide RF functionality. A myriad of devices can be made using RF MEMS. Examples include RF MEMS switches, switched capacitors, varactors, and vibrating RF MEMS resonators. Several national laboratories and universities are actively engaged in developing these devices. Most notably, IBM Research Laboratory, Hughes Research Laboratories, Northeastern University in cooperation with Analog Devices, Raytheon, Rockwell Science, Westinghouse Research Laboratories, and the University of Michigan, Ann Arbor, and a few others are known to pioneer in this area of research.

Modeling RF MEMS devices is a challenge because they exhibit nonlinear behavior and hysteresis. The electrostatic force in MEMS, which depends on the square of the drive voltage, causes electrostatic force strengthening. This results in pull-in of the electrodes and limits the linearity of these devices. Also, residual charge at the interfaces influences the switching times. The Verilog-A hardware description language (HDL) is well suited for RF MEMS device modeling by means of nonlinear-physics-based equations. In addition, simplified empirical equations or model-order-reduced equations can be used to trade accuracy for feasibility. Device models that use empirical equations in addition to physicsbased equations are referred to as compact models. + − FIGURE 6.28 RF MEMS device. 06_Sadiku_Ch06.indd 275 23/09/17 1:40 PM 276 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS RF MEMS have a deflecting cantilever or fixed-fixed beam and can be classified further by actuation method (electrostatic, electrothermal, magnetostatic, or piezoelectric), axis of deflection (lateral or vertical), circuit configuration (series or shunt), suspension (cantilever or fixed-fixed), or contact interface (capacitive or ohmic). Electrostatically actuated RF MEMS switches offer low insertion loss and high isolation, good linearity, greater power handling, and a higher electrical quality factor Qe. A capacitive fixed-fixed beam RF MEMS switch is a micromachined capacitor with a moving top electrode called the beam. The beam, which is sometimes perforated, is suspended by springs over a lower electrode, separated by an air gap and a dielectric. It is generally connected in shunt with the transmission line. This type of switch is generally used for the X- to W-band, which lies in the range of 8 to 110 GHz. An ohmic cantilever RF MEMS switch is capacitive in the up-state but makes an ohmic contact in the down-state. It is an asymmetrical device with the clamp designated as source, the bias electrode designated as gate, and the contact electrode designated as drain, analogous to the field effect transistor.

An ohmic cantilever switch is generally connected in series with the transmission line and is used from dc to about 40 GHz into the Ka-band. A vibrating RF MEMS resonator has a vibrating beam, comb, disk, or ring (wine glass), which is sufficiently isolated from the surroundings to obtain a high mechanical quality factor Qm. Vacuum encapsulation with an ambient pressure P , 1025 mbar results in negligible air damping. The vibrating RF MEMS resonator is often driven into a weakly nonlinear regime to increase the energy storage, taking advantage of the reduction in stored energy that occurs as the size is reduced. Vibrating RF MEMS resonators have a precise self-referenced oscillation frequency. Reference oscillators are used in local oscillators, which are implemented as voltage-controlled oscillators (VCOs). RF MEMS are extensively used in broadband wireless radio communications. Also, RF MEMS switches, switched capacitors, and varactors are applied in electronically scanned arrays, software-defined radios, reconfigurable antennas, and tunable bandpass filters. The opening vignette of Chapter 12 provides an additional discussion of RF MEMS. † 6.9 APPLICATION NOTE—SUPERCAPACITORS Historically, capacitors came in the range of picofarads and microfarads. Electrolytic capacitors had higher values of capacitance. But today, we find capacitors even of few kilofarads capacitance. They are called supercapacitors (SCs), ultracapacitors, or electric double-layer capacitors and are becoming more popular due to their growing use in electric vehicles. The internal structure of a typical SC is shown in Figure 6.29. SCs have a lower voltage rating. They are supposed to bridge the gap between electrolytic capacitors and batteries. As they store more energy per unit volume or mass than electrolytic capacitors, they seem to gain more prominence than batteries.

The charging and discharging times of SCs are very, very short compared to rechargeable batteries. For applications requiring long-term compact energy storage and regenerative braking in electric vehicles, SCs prove to be a good choice. Even in cranes and elevators, with short-term energy storage or burst-mode power delivery, these units can 06_Sadiku_Ch06.indd 276 23/09/17 1:40 PM 6.9 Application Note—Supercapacitors 277 be used. The higher value of capacitance of SCs is contributed from one or a combination of more than one source such as faradic electron charge-transfer or formation of electrostatic double layer. The inner surface of each electrode in these devices is not smooth, but rather padded with activated porous carbon resulting in a surface area that is about 100,000 times as large as the surface area of an ordinary capacitor. The large surface area is not the only key feature. Charges in the form of ions stick to the inner surfaces of the electrodes. Thereby, the effective distance between the positive and the negative charges at each electrode is of the order of nanometers. Large surface area and small distance of separation are among the prime reasons for high nominal values of capacitance of SCs. A third source of increasing the value of capacitance is by choosing high value of permittivity for the film that separates the electrodes. Barium titanate and its composites, such as barium strontium titanate, have recently been fabricated in nanometer-sized crystals and thin films. If such nanometer-sized crystals are deposited on the internal surface of the activated carbon electrode, this will result in a very substantial increase in the overall dielectric constant of the capacitor because the spacing between the electrolyte and the surface of the electrode will essentially remain a few nanometers thick, while the relative permittivity is increased substantially.

The dielectric constant of powdered BaSrTiO3, for example, is typically 12,000 to 15,000 at room temperature, whereas, many electrolytes have it in the range of 37–65. Although research in the field of the electrochemical double-layer capacitors (EDLCs) began in early 1950s, it was only after the late 1970s that the technology of SCs appeared on the market. Compared to the rechargeable batteries, EDLCs have the additional advantage that they have a relatively low internal resistance and can store and deliver energy at higher power rating. As for their constructional details, the double-layer capacitors usually are made of two carbon electrodes isolated from one another through a porous membrane. The entire assembly is immersed in an electrolyte, which allows the ionic flow between the electrodes. The membrane under ideal conditions precludes the possibility of any electrical short circuit between the electrodes. A current-collecting plate is connected with each of the electrodes with the intent of minimizing the internal resistance. Most of the energy is stored in a polarized liquid, which is formed when a voltage is applied to the capacitor terminals. Thus, a double layer of negative and positive charges is generated on the contact surface between the electrode and the electrolyte, whence comes the name, EDLC. The separation of charge is of the order of a few angstrom units, much smaller than in a conventional capacitor. Pseudocapacitance is established by faradic electron charge transfer with redox reactions, intercalation, or electrosorption. Electrochemical pseudocapacitors use metal oxide or conducting polymer electrodes with a high amount of electrochemical pseudocapacitance in addition to the double-layer capacitance. Another category is the hybrid capacitor, such as the lithium-ion capacitor, that uses electrodes with differing characteristics: one exhibiting mostly electrostatic capacitance and the other mostly electrochemical capacitance. Between the two electrodes, the electrolyte such as manganese dioxide or a conducting polymer serves as an ionic conductive pathway. This is part of the second electrode and is a feature that distinguishes them from the conventional electrolytic capacitors. 06_Sadiku_Ch06.indd 277 23/09/17 1:40 PM 278 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS Supercapacitors are polarized by design with asymmetric electrodes or, for symmetric electrodes, by a potential applied during manufacture. + Electrolyte Separator Activated Carbon Electrode – FIGURE 6.29 Cross-sectional view of a supercapacitor. MATLAB 6.1 Consider a trace of thickness t 5 15 mm, length l 5 8 mm, and with exponentially increasing width from w1 5 0.2 mm to w2 5 2.5 mm connected to a 10 V source by perfect electric conductor (PEC) slabs at the ends. Such a shape is commonly encountered around via pads or near impedance transformers. The trace has conductivity s 5 6.5 3 105 S/m. Determine the total resistance of the trace by integrating the incremental cross-sectional resistance. V0 _ + x = aebz w1 t w2 y l z x 06_Sadiku_Ch06.indd 278 23/09/17 1:40 PM 6.9 Application Note—Supercapacitors 279 To simplify, we can place the origin between the PEC–trace junction on the wide side of the trace, at the center of the bottom side. The incremental resistance is a slab of length dz in the xy-plane given by dR 5 dz stw 1 z 2 We must determine the function w(z).
Since the trace tapers according to a known expression, we can determine the coefficients a, b: w1z2 5 a 5 x 1 z 5 2 5 aebz 2 We know that x1z 5 02 5 So, w2 w1 , x1z 5 l2 5 2 2 x1z 5 02 5 a 5 And x 1 z 5 l 2 5 aebl 5 w2 2 w2 bl w1 e 5 2 2 w2ebl 5 w1 b5 1 ln , w1 w2 This integral can be solved as dz dz 1 dz R 5 3 dR 5 3 53 5 3 1 2 1 2 1 2 stw z st2x z 2st x z 0 0 0 0 l l l 2 1 5 e w2st 30 l l 1 ln w1 z w2 l dz Plugging in numbers for the geometric parameters, R 5 1.49 . % This script computes the integral of an exponential trace % a function using two different methods: % 1. the built-in matlab 'quad' function % 2.
user-defined summation % % The user must first create a separate file for the function 06_Sadiku_Ch06.indd 279 23/09/17 1:40 PM 280 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS % y = (-1/20)*x^3+(3/5)*x.^2-(21/10)*x+4; % The file should be named fun.m and stored in the same directory % as this file, and it should contain the following two lines: % function y = fun(x) % y = (-1/20)*x.^3+(3/5)*x.^2-2.1*x+4; % % We will determine the integral of this function from % x = 0 to x = 8 clear % the parameters of the trace w1 = 0.2e-3; w2 = 2.5e-3; cond = 6.5e5; % conductivity t = 15e-6; % thickness l = 8e-3; % length sum=0; % set initial total sum to zero dz=l/1000; for z=0:dz:l, sum=sum+1/(w2*cond*t)*exp(-1/l*log(w1/w2)*z)*dz; % add the partial sums to the total sum end disp('') disp(sprintf('The total resistance of the trace is %f', sum)) SUMMARY 1. Boundary-value problems are those in which the potentials or their derivatives at the boundaries of a region are specified and we are to determine the potential field within the region. They are solved by using Poisson's equation if rv 2 0 or Laplace's equation if rv 5 0.

2.

In a nonhomogeneous region, Poisson's equation is = # e =V 5 2rv For a homogeneous region, « is independent of space variables. Poisson's equation becomes rv =2V 5 2 e In a charge-free region 1 rv 5 0 2 , Poisson's equation becomes Laplace's equation; that is, =2V 5 0 3. We solve the differential equation resulting from Poisson's or Laplace's equation by integrating twice if V depends on one variable or by the method of separation of variables if V is a function of more than one variable. We then apply the prescribed boundary conditions to obtain a unique solution. 06_Sadiku_Ch06.indd 280 23/09/17 1:40 PM Review Questions 281 4. The uniqueness theorem states that if V satisfies Poisson's or Laplace's equation and the prescribed boundary condition, V is the only possible solution for that problem.

This enables us to find the solution to a given problem via any expedient means because we are assured of one, and only one, solution. 5. The problem of finding the resistance R of an object or the capacitance C of a capacitor may be treated as a boundary-value problem. To determine R, we assume a potential difference Vo between the ends of the object, solve Laplace's equation, find I 5 eS sE # dS, and obtain R 5 Vo /I . Similarly, to determine C, we assume a potential difference of Vo between the plates of the capacitor, solve Laplace's equation, find Q 5 eS eE # dS, and obtain C 5 Q/Vo. 6. A boundary-value problem involving an infinite conducting plane or wedge may be solved by using the method of images. This basically entails replacing the charge configuration by itself, its image, and an equipotential surface in place of the conducting plane. Thus the original problem is replaced by "an image problem," which is solved by using techniques covered in Chapters 4 and 5. 7. The computation of the capacitance of microstrip lines has become important because such lines are used in microwave devices. Three formulas for finding the capacitance of a circular microstrip line have been presented. REVIEW QUESTIONS 6.1 Equation = # 1 2e=V 2 5 rv may be regarded as Poisson's equation for an inhomogeneous medium. (a) True (b) False 6.2 In cylindrical coordinates, the equation '2c 1 'c '2c 1 2 1 10 5 0 2 1 r 'r 'r 'z is called (a) Maxwell's equation (b) Laplace's equation (c) Poisson's equation (d) Helmholtz's equation (e) Lorentz's equation 6.3 Two potential functions V1 and V2 satisfy Laplace's equation within a closed region and assume the same values on its surface. V1 must be equal to V2. (a) True (b) False (c) Not necessarily 6.4 Which of the following potentials does not satisfy Laplace's equation? (a) V 5 2x 1 5 (b) V 5 10 xy (c) V 5 r cos f 06_Sadiku_Ch06.indd 281 10 r (e) V 5 r cos f 1 10 (d) V 5 23/09/17 1:40 PM 282 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS 6.5 Which of the following is not true? (a) 25 cos 3x is a solution to fs 1 x 2 1 9f 1 x 2 5 0 (b) 10 sin 2x is a solution to fs 1 x 2 2 4f 1 x 2 5 0 (c) 24 cosh 3y is a solution to Rs 1 y 2 2 9R 1 y 2 5 0 (d) sinh 2y is a solution to Rs 1 y 2 2 4R 1 y 2 5 0 hs 1 y 2 gs 1 x 2 (e) 52 5 f 1 z 2 5 21 where g 1 x 2 5 sin x and h 1 y 2 5 sinh y g1x2 h1y2 6.6 If V1 5 X1Y1 is a product solution of Laplace's equation, which of these are not solutions of Laplace's equation? (a) 210X1Y1 (b) X1Y1 1 2xy (c) X1Y1 2 x 1 y (d) X1 1 Y1 (e) 1 X1 2 2 2 1 Y1 1 3 2 6.7 The capacitance of a capacitor filled by a linear dielectric is independent of the charge on the plates and the potential difference between the plates. (a) True (b) False 6.8 A parallel-plate capacitor connected to a battery stores twice as much charge with a given dielectric as it does with air as dielectric.

The susceptibility of the dielectric is (a) 0 (b) 1 (c) 2 (d) 3 (e) 4 6.9 A potential difference Vo is applied to a mercury column in a cylindrical container. The mercury is now poured into another cylindrical container of half the radius and the same potential difference Vo applied across the ends. As a result of this change of space, the resistance will be increased (a) 2 times (b) 4 times (c) 8 times (d) 16 times 6.10 Two conducting plates are inclined at an angle 30 to each other with a point charge between them. The number of image charges is (a) 12 (b) 11 (c) 6 (d) 5 (e) 3 Answers: 6.1a, 6.2c, 6.3a, 6.4c, 6.5b, 6.6d,e, 6.7a, 6.8b, 6.9d, 6.10b. PROBLEMS Section 6.2—Poisson's and Laplace's Equations 6.1 Given V 5 5x3y2z and e 5 2.25eo, find (a) E at point P 1 23, 1, 2 2 , (b) rv at P. 6.2 Let V 5 06_Sadiku_Ch06.indd 282 10 cos u sin f and « 5 «o. (a) Find E at point P(1, 60, 30).

(b) Determine rv at P. r2 23/09/17 1:40 PM Problems 283 6.3 Conducting sheets are located at y 5 1 and y 5 3 planes. The space between them is y filled with a nonuniform charge distribution rv 5 nC/m3 and e 5 4eo. Assuming that 4p V 1 y 5 1 2 5 0 and V 1 y 5 3 2 5 50 V, find V 1 y 5 2 2 . 6.4 In free space, V 5 10r0.8 V. Find E and the volume charge density at r = 0.6 m. 6.5 A certain material occupies the space between two conducting slabs located at y 562 cm.

When heated, the material emits electrons such that rv 5 50 1 1 2 y2 2 mC/m3. If the slabs are both held at 30 kV, find the potential distribution within the slabs. Take e 5 3eo. 6.6 Two large flat metal sheets are located at z 5 0 and z 5 d and are maintained at 0 and Vo, respectively. The charge density between the sheets is rv(z) 5 roz/d, where ro is a constant. Determine the potential at all points between the plates. 6.7 In cylindrical coordinates, V 5 0 at r 5 2 m and V 5 60 V at r 5 5 m due to charge 10 distribution rv 5 pC/m3. If «r 5 3.6, find E. r 6.8 The region between two cylinders r 5 a and r 5 b has charge density ro. Determine the potential distribution V. 6.9 The dielectric region (e 5 6eo) between a pair of concentric spheres r 5 1 and r 5 4 has 10 charge distribution rv 5 nC/m3. If V(r 5 1 2 5 0 and V(r 5 4 2 5 50 V, determine r V(r 5 2). 6.10 Determine whether each of the following potentials satisfies Laplace's equation.
(a) V1 5 3xyz 1 y 2 z2 10sinf r 5sinf (c) V3 5 r (b) V2 5 6.11 Given V 5 x3y 1 yz 1 cz2, find c such that V satisfies Laplace's equation. 6.12 The potential field V 5 2x2yz 2 y3z exists in a dielectric medium having e 5 2eo. (a) Does V satisfy Laplace's equation?
(b) Calculate the total charge within the unit cube 0 , x , 1 m, 0 , y , 1 m, 0 , z , 1 m.

6.13 Two conducting coaxial cylinders are located at r 5 1 cm and r 5 1.5 cm. The inner conductor is maintained at 50 V while the outer one is grounded. If the cylinders are separated by a dielectric material with e 5 4eo, find the surface charge density on the inner conductor. 6.14 Consider the conducting plates shown in Figure 6.30. If V 1 z 5 0 2 5 0 and V 1 z 5 2 mm 2 5 50 V, determine V, E, and D in the dielectric region 1 er 5 1.5 2 between the plates and rS on the plates. 6.15 The cylindrical structure whose cross section is in Figure 6.31 has inner and outer radii of 5 mm and 15 mm, respectively. If V 1 r 5 5 mm 2 5 100 V and V 1 r 5 15 mm 2 5 0 V, calculate V, E, and D at r 5 10 mm and rS on each plate. Take er 5 2.0. 06_Sadiku_Ch06.indd 283 02/11/17 2:43 PM 284 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS o FIGURE 6.30 For Problem 6.14. FIGURE 6.31 Cylindrical capacitor of Problem 6.15. FIGURE 6.32 Conducting cones of Problem 6.20. 6.16 In cylindrical coordinates, V 5 50 V on plane 5 /2 and V 5 0 on plane 5 0. Assuming that the planes are insulated along the z-axis, determine E between the planes.

*6.17 (a) Show that V 5 Vo(12a2/r2) r sin f (where Vo is constant) satisfies Laplace's equation. (b) Determine E for r2 .. a2. 6.18 Two conducting planes are located at x 5 0 and x 5 50 mm. The zero voltage reference is at x 5 20 mm. Given that E 5 2110ax V/m, calculate the conductor voltages. 6.19 The region between concentric spherical conducting shells r 5 0.5 m and r 5 1 m is charge free. If V 1 r 5 0.5 2 5 250 V and V 1 r 5 1 2 5 50 V, determine the potential distribution and the electric field strength in the region between the shells. 6.20 Find V and E at 1 3, 0, 4 2 due to the two conducting cones of infinite extent shown in Figure 6.32. *6.21 The inner and outer electrodes of a diode are coaxial cylinders of radii a 5 0.6 mm and b 5 30 mm, respectively. The inner electrode is maintained at 70 V, while the outer electrode is grounded. (a) Assuming that the length of the electrodes , W a, b and ignoring the effects of space charge, calculate the potential at r 5 15 mm.
(b) If an electron is injected radially through a small hole in the inner electrode with velocity 107 m/s, find its velocity at r 5 15 mm. 06_Sadiku_Ch06.indd 284 02/11/17 2:43 PM Problems 285 FIGURE 6.33 For Problem 6.22. FIGURE 6.34 For Problem 6.23. 6.22 An electrode with a hyperbolic shape 1 xy 5 4 2 is placed above a grounded right-angle corner as in Figure 6.33. Calculate V and E at point 1 1, 2, 0 2 when the electrode is connected to a 20 V source. *6.23 Solve Laplace's equation for the two-dimensional electrostatic systems of Figure 6.34 and find the potential V 1 x, y 2 . *6.24 Find the potential V 1 x, y 2 due to the two-dimensional systems of Figure 6.35.
6.25 A conducting strip is defined as shown in Figure 6.35(b). The potential distribution is V 1 x,y 2 5 4Vo a p n5odd ` sina npy b a exp 1 2 npx/a 2 n Find the electric field E. 6.26 Figure 6.36 shows the cross-sectional view of an infinitely long rectangular slot.
Find the potential distribution in the slot. 06_Sadiku_Ch06.indd 285 02/11/17 2:43 PM 286 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS V = V2 V=0 V=0 V = V1 FIGURE 6.35 For Problems 6.24 and 6.25. x=a V = Vo sin y=0 V=0 x πy b Slot y=b a V=0 y+x=0 V=0 b FIGURE 6.36 For Problem 6.26. 6.27 By letting V 1 r, f 2 5 R 1 r 2 ϕ 1 f 2 be the solution of Laplace's equation in a region where r 2 0, show that the separated differential equations for R and ϕ are Rs 1 Rr l 2 2R50 r r and ϕs 1 lϕ 5 0 where is the separation constant. 06_Sadiku_Ch06.indd 286 02/11/17 2:43 PM Problems 287 FIGURE 6.37 For Problem 6.30. 6.28 A potential in spherical coordinates is a function of r and but not f. Assuming that V 1 r, u 2 5 R 1 r 2 F 1 u 2 , obtain the separated differential equations for R and F in a region for which rv 5 0. Section 6.5—Resistance and Capacitance 6.29 Show that the resistance of the bar of Figure 6.17 between the vertical ends located at f 5 0 and f 5 p/2 is R5 p 2st ln b a *6.30 Show that the resistance of the sector of a spherical shell of conductivity , with cross section shown in Figure 6.37 1 where 0 # f , 2p 2 , between its base (i.e., from r 5 a to r 5 b) is R5 1 1 1 c 2 d b 2ps 1 1 2 cos a 2 a 6.31 A spherical shell has inner and outer radii a and b, respectively. Assume that the shell has a uniform conductivity and that it has copper electrodes plated on the inner and outer surfaces. Show that R5 1 1 1 a 2 b 4ps a b *6.32 A hollow conducting hemisphere of radius a is buried with its flat face lying flush with the earth's surface, thereby serving as an earthing electrode. If the conductivity of earth is , show that the leakage conductance between the electrode and earth is 2pa. 06_Sadiku_Ch06.indd 287 02/11/17 2:43 PM 288 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS 6.33 Another method of finding the capacitance of a capacitor is by using energy considerations, that is, C5 2WE 1 2 3 e 0 E 0 dv 2 5 Vo V2o Using this approach, derive eqs. (6.22), (6.28), and (6.32). 6.34 A cylindrical capacitor has inner radius a and outer radius b. The region between the cylinders has conductivity s. Determine the conductance per unit length of the capacitor. 6.35 A coaxial cable with inner radius a and outer radius b has a steady-state voltage V across it. Determine the power loss per unit length. Assume that the conductivity of the region between the cylinders is s. 6.36 In an integrated circuit, a capacitor is formed by growing a silicon dioxide layer 1 er 5 4 2 of thickness 1 mm over the conducting silicon substrate and covering it with a metal electrode of area S. Determine S if a capacitance of 2 nF is desired. 6.37 Calculate the capacitance of the parallel-plate capacitor shown in Figure 6.38.
Depth = 15 cm εr1 = 3 εr2 = 5 εr3 = 8 20 cm 20 cm 20 cm 2 mm FIGURE 6.38 For Problem 6.37. FIGURE 6.39 For Problem 6.38. ε0 0 ε0 ε0εr a 2a 3a x FIGURE 6.40 For Problem 6.39. 06_Sadiku_Ch06.indd 288 06/11/17 6:31 PM Problems 289 FIGURE 6.41 For Problem 6.41. 6.38 A capacitor consists of two infinitely large plates of area A and 3a apart as shown in Figure 6.39. If a dielectric slab of thickness a is located midway between the plates, determine the capacitance. 6.39 The parallel-plate capacitor of Figure 6.40 is quarter-filled with mica 1 er 5 6 2 . Find the capacitance of the capacitor.
6.40 To appreciate the physical size of 1 F capacitor, consider a parallel-plate capacitor filled with air and with separation distance of 1 mm.

Find the area of the plates to provide a capacitance of 1 F. *6.41 An air-filled parallel plate capacitor of length L, width a, and plate separation d has its plates maintained at constant potential difference Vo. If a dielectric slab of dielectric constant «r is slid between the plates and is withdrawn until only a length x remains between the plates as in Figure 6.41, show that the force tending to restore the slab to its original position is F5 eo 1 er 2 1 2 a V2o 2d 6.42 A parallel-plate capacitor has plate area 200 cm2 and plate separation of 3 mm.

The charge density is 1 mC/m2 and air is the dielectric. Find (a) The capacitance of the capacitor (b) The voltage between the plates (c) The force with which the plates attract each other 6.43 The capacitance of a parallel-plate capacitor is 56 F when the dielectric material is in place. The capacitance drops to 32 F when the dielectric material is removed. Calculate the dielectric constant «r of the material. 6.44 A parallel-plate capacitor has a 4 mm plate separation, 0.5 m2 surface area per plate, and a dielectric with «r 5 6.8. If the plates are maintained at 9 V potential difference, calculate (a) the capacitance, (b) the charge density on each plate. 6.45 A parallel-plate capacitor remains connected to a voltage source while the separation between the plates changes from d to 3d. Express new values of C, Q, E, and W in terms of the old values C0, Q0, E0, and W0. 6.46 A parallel-plate capacitor has plate area 40 cm2. The dielectric has two layers with permittivity e1 5 4eo and e2 5 6eo, and each layer is 2 mm thick. If the capacitor is connected to a voltage 12 V, calculate: (a) the capacitance of the capacitor, (b) the total charge on each plate, (c) the values of E, D, and P.
6.47 The space between spherical conducting shells r 5 5 cm and r 5 10 cm is filled with a dielectric material for which e 5 2.25eo. The two shells are maintained at a potential difference of 80 V. (a) Find the capacitance of the system. (b) Calculate the charge density on shell r 5 5 cm. 06_Sadiku_Ch06.indd 289 02/11/17 2:43 PM 290 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS y a c b π/4 0 a FIGURE 6.42 For Problem 6.49. b x FIGURE 6.43 For Problem 6.52. 6.48 A spherical capacitor has inner radius d and outer radius a. Concentric with the spherical conductors and lying between them is a spherical shell of outer radius c and inner radius b. If the regions d , r , c, c , r , b, and b , r , a are filled with materials with permittivities «1, «2, and «3, respectively, determine the capacitance of the system. 6.49 Determine the capacitance of a conducting sphere surrounded by a thick spherical shell as shown in Figure 6.42. 6.50 The coaxial cable in Figure 6.14 has two dielectrics with «r1 for a , r , c and «r2 for c , r , b where a , c , b. Determine the capacitance of the system. 6.51 A coaxial cable has inner radius of 5 mm and outer radius of 8 mm. If the cable is 3 km long, calculate its capacitance. Assume « 5 2.5«o. 6.52 A capacitor consists of two plates with equal width (b 2 a), and a length L in the z-direction. The plates are separated by /4, as shown in Figure 6.43.
Assume that the plates are separated by a dielectric material (« 5 «o«r) and ignore fringing. Determine the capacitance. 6.53 A segment of the cylindrical capacitor is defined by r1 , r , r2, 0 , f , a. If V(f 5 0 2 5 0 and V(f 5 a 2 5 Vo , show that the capacitance of the segment is r eL C 5 lna 2 b, where L is the length and « is the permittivity of the dielectric. a r1 *6.54 In an ink-jet printer the drops are charged by surrounding the jet of radius 20 mm with a concentric cylinder of radius 600 mm as in Figure 6.44.
Calculate the minimum voltage required to generate a charge 50 fC on the drop if the length of the jet inside the cylinder is 100 mm. Take e 5 eo. 6.55 The cross section of a cable is shown in Figure 6.45. Determine the capacitance per unit length. 06_Sadiku_Ch06.indd 290 02/11/17 2:43 PM Problems 291 ε2 ε1 a D c oir b L FIGURE 6.44 Simplified geometry of an ink-jet printer; for Problem 6.54. FIGURE 6.45 For Problem 6.55. FIGURE 6.46 For Problem 6.57.
ε1 a b ε2 6.56 A spherical capacitor has inner radius of a = 2 cm and outer radius of b = 4 cm. The interior is a dielectric material with «o «r .
The outer conductor is grounded while the inner one is maintained at 100 V. (a) Determine «r if the surface charge density on the inner conductor is 400 nC/m2. (b) Find the capacitance of the structure. 6.57 One half of the dielectric region of a spherical capacitor has permittivity «1 while the other half has «2 as shown in Figure 6.46. Show that the capacitance of the system is given by C5 2p(e1 1 e2)ab b2a *6.58 A spherical capacitor has an inner conductor of radius a carrying charge Q and is maintained at zero potential. If the outer conductor contracts from a radius b to c under internal forces, prove that the work performed by the electric field as a result of the contraction is W5 06_Sadiku_Ch06.indd 291 Q2 1 b 2 c 2 8pebc 06/11/17 6:31 PM 292 CHAPTER 6 ELECTROSTATIC BOUNDARY-VALUE PROBLEMS *6.59 A parallel-plate capacitor has its plates at x 5 0, d and the space between the plates is x filled with an inhomogeneous material with permittivity e 5 eo a1 1 b.
If the plate at d x 5 d is maintained at Vo while the plate at x 5 0 is grounded, find: (a) V and E (b) P (c) rPs at x 5 0, d (d) the capacitance, assuming that each plate has area S 6.60 Two parallel conducting plates are located at x d and x 2d. The plate at x d is held at Vo, while the plate at x 2d is grounded. If the space between the plates is filled with an inhomogeneous dielectric medium with e5 2eo x 2 11a b d find the capacitance.
Assume that each plate has an area S. 6.61 A spherical capacitor has inner radius a and outer radius b and is filled with an inhomogeneous dielectric with e 5 eok/r2. Show that the capacitance of the capacitor is C5 4peok b2a 6.62 If the earth is regarded as a spherical capacitor, what is its capacitance? Assume the radius of the earth to be approximately 6370 km. 6.63 A capacitor is formed by two coaxial metal cylinders of radii a 5 1 mm and b 5 5 mm. If the space between the cylinders is filled with a dielectric having er 5 3 1 1 1 r 2 , a r b, and r is in millimeters, determine the capacitance per meter. 6.64 A two-wire transmission line is formed with two identical wires which are widely separated. If the radius of each wire is a and the center-to-center spacing is D, the approximate formula for the capacitance per unit length is C5 pe D2a lna b a where D .. a. For « = 4«o and D/a = 12, calculate C. 06_Sadiku_Ch06.indd 292 02/11/17 2:43 PM Problems 293 Section 6.6—Method of Images 6.65 A 10-nC point charge is located at point (0, 0, 10 m) above a grounded conducting plane z 5 0. (a) Find the surface charge density at point (2, –4, 0). (b) Calculate the total charge on the plate. 6.66 Two point charges of 3 nC and 24 nC are placed, respectively, at 1 0, 0, 1 m 2 and 1 0, 0, 2 m 2 while an infinite conducting plane is at z 5 0. Determine (a) The total charge induced on the plane (b) The magnitude of the force of attraction between the charges and the plane *6.67 A point charge of 10 mC is located at 1 1, 1, 1 2 and the positive portions of the coordinate planes are occupied by three mutually perpendicular plane conductors maintained at zero potential. Find the force on the charge due to the conductors. 6.68 A point charge Q is placed between two earthed intersecting conducting planes that are inclined at 45 to each other. Determine the number of image charges and their locations.
6.69 Infinite line x 5 3, z 5 4 carries 16 nC/m and is located in free space above the conducting plane z 5 0. (a) Find E at 1 2, 22, 3 2 . (b) Calculate the induced surface charge density on the conducting plane at 1 5, 26, 0 2 . 6.70 In free space, infinite planes y 5 4 and y 5 8 carry charges 20 nC/m2 and 30 nC/m2, respectively. If plane y 5 2 is grounded, calculate E at P 1 0, 0, 2 and Q 1 24, 6, 2 2 . 06_Sadiku_Ch06.indd 293 02/11/17 2:43 PM 06_Sadiku_Ch06.indd 294 02/11/17 2:43 PM PA R T 3 MAGNETOSTATICS 07_Sadiku_Ch07.indd 295 23/09/17 2:38 PM Jean-Baptiste Biot (1774–1862), a French physicist and mathematician, made advances in geometry, astronomy, elasticity, electricity, magnetism, heat, and optics.

Born in Paris, he studied at the École Polytechnique in Paris, where he realized his potential. Biot studied a wide range of mathematical topics, mostly on the applied mathematics. Biot, together with the French physicist Felix Savart, discovered that the magnetic field intensity of a current flowing through a wire varies inversely with the distance from the wire.

This relation, now known as Biot–Savart's law, will be covered in this chapter. Biot discovered that when light passes through some substances, including sugar solutions, the plane of polarization of the light is rotated by an amount that depends on the color of the light. In addition to his scientific pursuits, Biot was a prolific writer. He completed over 250 works of various types, the most renowned of which is his Elementary Treatise on Physical Astronomy (1805). André-Marie Ampère (1775–1836), a French physicist, mathematician, and natural philosopher, is best known for defining a way to measure the flow of current. He has been called the Newton of electricity. Born at Polemieux, near Lyons, Ampère took a passionate delight in the pursuit of knowledge from his very infancy. Although André never attended school, he received an excellent education.

His father taught him Latin, which enabled him to master the works of Euler and Bernouilli. André poured over his studies of mathematics and soon began to create his own theories and ideas. His reading embraced a wide range of knowledge—history, travels, poetry, philosophy, metaphysics, and the natural sciences.

As an adult, Ampère was notoriously absent-minded. He became a professor of mathematics at the École Polytechnique, and later at the Collège de France. He developed Oersted's discovery of the link between electric and magnetic fields and introduced the concepts of current element and the force between current elements. Ampère made several contributions to electromagnetism, including the formulation of the law that bears his name, which will be discussed in this chapter. The ampere unit of electric current is named after him. 07_Sadiku_Ch07.indd 296 23/09/17 2:38 PM CHAPTER 7 MAGNETOSTATIC FIELDS The highest happiness on earth is in marriage. Every man who is happily married is a successful man even if he has failed in everything else. —WILLIAM L. PHELPS 7.1 INTRODUCTION In Chapters 4 to 6, we limited our discussions to static electric fields characterized by E or D. We now focus our attention on static magnetic fields, which are characterized by H or B. There are similarities and dissimilarities between electric and magnetic fields. As E and D are related according to D 5 eE for linear, isotropic material space, H and B are related according to B 5 mH. Table 7.1 further shows the analogy between electric and magnetic field quantities. Some of the magnetic field quantities will be introduced later in this chapter, and others will be presented in the next. The analogy is presented here to show that most of the equations we have derived for the electric fields may be r eadily used to obtain corresponding equations for magnetic fields if the equivalent analogous quantities are substituted. This way it does not appear as if we are learning new concepts. A definite link between electric and magnetic fields was established by Oersted1 in 1820. As we have noticed, an electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced. A magnetostatic field is produced by a constant current flow (or direct current). This current flow may be due to magnetization currents as in permanent magnets, electron-beam currents as in vacuum tubes, or conduction currents as in current-carrying wires. In this chapter, we consider magnetic fields in free space due to direct current. Magnetostatic fields in material space are covered in Chapter 8. Our study of magnetostatics is not a dispensable luxury but an indispensable necessity. Motors, transformers, microphones, compasses, telephone bell ringers, television focusing controls, advertising displays, magnetically levitated high-speed vehicles, memory stores, magnetic separators, and so on, which play an important role in our everyday life,2 could not have been developed without an understanding of magnetic phenomena. 1 Hans Christian Oersted (1777–1851), a Danish professor of physics, after 13 years of frustrating efforts discovered that electricity could produce magnetism. 2 Various applications of magnetism can be found in J. K. Watson, Applications of Magnetism. New York: John Wiley & Sons, 1980. 297 07_Sadiku_Ch07.indd 297 23/09/17 2:38 PM 298 CHAPTER 7 MAGNETOSTATIC FIELDS TABLE 7.1 Analogy between Electric and Magnetic Fields* Term Basic laws Electric F5 Q1Q2 4peR2 Magnetic aR dB 5 moI dl 3 aR 4pR2 # C D dS 5 Qenc # C H d l 5 Ienc Force law F 5 QE F 5 Qu 3 B Source element dQ dQu 5 Id l Field intensity Relationship between fields V E 5 1 V/m 2 , c D 5 1 C/m2 2 S D 5 eE Potentials E 5 2=V Flux density rLdl V53 L 4peR Flux c5 e D # dS Poisson's equation H 5 2=Vm 1 J 5 0 2 A53 L c5 m I d, 4pR e B # dS S c 5 Q 5 CV c 5 L LI dV dt 1 # wE 5 D E 2 rv 2 =V52 e dI dt 1 wm 5 B # H 2 I5C Energy density I 1 A/m 2 , c B 5 1 Wb/m2 2 S B 5 mH H5 V5L =2A 5 2mJ *A similar analogy can be found in R. S. Elliot, "Electromagnetic theory: a simplified representation," IEEE Transactions on Education, vol. E-24, no. 4, Nov. 1981, pp. 294–296. There are two major laws governing magnetostatic fields: (1) Biot–Savart's law,3 and (2) Ampère's circuit law. Like Coulomb's law, Biot–Savart's law is the general law of magnetostatics. Just as Gauss's law is a special case of Coulomb's law, Ampère's law is a special case of Biot–Savart's law and is easily applied in problems involving symmetrical current distribution. The two laws of magnetostatics are stated and applied first, with their derivations provided later in the chapter.
7.2 BIOT–SAVART'S LAW Biot–Savart's law states that the differential magnetic field intensity dH produced at a point P, as shown in Figure 7.1, by the differential current element I dl is proportional to the product I dl and the sine of the angle a between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element. The experiments and analyses of the effect of a current element were carried out by Ampère and by Jean-Baptiste Biot and Felix Savart around 1820. 3 07_Sadiku_Ch07.indd 298 04/11/17 1:29 PM 7.2 Biot–Savart's Law 299 FIGURE 7.1 Magnetic field dH at P due to current element I d l. That is, dH ~ I dl sin a (7.1) R2 dH 5 kI dl sin a (7.2) R2 or where k is the constant of proportionality. In SI units, k 5 1/4p, so eq. (7.2) becomes dH 5 I dl sin a (7.3) 4pR2 From the definition of cross product in eq. (1.21), it is easy to notice that eq. (7.3) is better put in vector form as dH 5 I dl 3 aR I dl 3 R (7.4) 5 2 4pR 4pR3 where R 5 0 R 0 and aR 5 R/R; R and dl are illustrated in Figure 7.1. Thus the direction of dH can be determined by the right-hand rule with the right-hand thumb pointing in the d irection of the current and the right-hand fingers encircling the wire in the direction of dH as shown in Figure 7.2(a). Alternatively, we can use the right-handed-screw rule to determine the direction of dH: with the screw placed along the wire and pointed in the direction of current flow, the direction of rotation of the screw is the direction of d H as in Figure 7.2(b). It is customary to represent the direction of the magnetic field intensity H (or current I) by a small circle with a dot or cross sign depending on whether H (or I) is out of the page, or into it respectively, as illustrated in Figure 7.3. Just as we can have different charge configurations (see Figure 4.5), we can have different current distributions: line current, surface current, and volume current as shown in Figure 7.4. If we define K as the surface current density in amperes per meter and J as the volume current density in amperes per meter squared, the source elements are related as 07_Sadiku_Ch07.indd 299 I dl ; K dS ; J dv(7.5) 23/09/17 2:38 PM 300 CHAPTER 7 MAGNETOSTATIC FIELDS FIGURE 7.3 Conventional representation of H (or I) (a) out of the page and (b) into the page. FIGURE 7.2 Determining the direction of dH using (a) the right-hand rule or (b) the right-handed-screw rule. Thus in terms of the distributed current sources, the Biot–Savart's law as in eq. (7.4) becomes H53 H53 H53 v I dl 3 aR 4pR2 L K dS 3 aR 4pR2 S 1 line current 2 (7.6) J dv 3 aR 4pR2 1 surface current 2 (7.7) 1 volume current 2 (7.8) where aR is a unit vector pointing from the differential element of current to the point of interest.
As an example, let us apply eq. (7.6) to determine the field due to a straight currentcarrying filamentary conductor of finite length AB as in Figure 7.5. We assume that the FIGURE 7.4 Current distributions: (a) line current, (b) surface current, (c) volume current. 07_Sadiku_Ch07.indd 300 23/09/17 2:38 PM 7.2 Biot–Savart's Law 301 FIGURE 7.5 Field at point P due to a straight filamentary conductor. c onductor is along the z-axis with its upper and lower ends, respectively, subtending angles a2 and a1 at P, the point at which H is to be determined. Particular note should be taken of this assumption, as the formula to be derived will have to be applied accordingly. Note that current flows from point A, where a 5 a1, to point B, where a 5 a2. If we consider the contribution dH at P due to an element dl at 1 0, 0, z 2 , dH 5 I dl 3 R (7.9) 4pR3 But dl 5 dz az and R 5 rar 2 zaz, so dl 3 R 5 r dz af(7.10) Hence, H53 Ir dz af(7.11) 4p 3 r2 1 z2 4 3/2 Letting z 5 r cot a, dz 5 2r csc2 a da, 3 r2 1 z2 4 3/2 5 r3 csc a3, and eq.
(7.11) becomes a2 2 r csc2 a da 1 a 4p 3a1 r3 csc3 a f a2 I 52 af 3 sin a da 4pr a1 H52 or 07_Sadiku_Ch07.indd 301 H5 I 1 cos a2 2 cos a1 2 af(7.12) 4pr 23/09/17 2:38 PM 302 CHAPTER 7 MAGNETOSTATIC FIELDS This expression is generally applicable for any straight filamentary conductor. The conductor need not lie on the z-axis, but it must be straight. Notice from eq. (7.12) that H is always along the unit vector af (i.e., along concentric circular paths) irrespective of the length of the wire or the point of interest P. As a special case, when the conductor is semi-infinite (with respect to P) so that point A is now at O 1 0, 0, 0 2 while B is at 1 0, 0, ` 2 , a1 5 90°, a2 5 0°, and eq. (7.12) becomes H5 I a (7.13) 4pr f Another special case is found when the conductor is infinite in length. For this case, point A is at 1 0, 0, 2` 2 while B is at 1 0, 0, ` 2 ; a1 5 180°, a2 5 0°, and eq. (7.12) reduces to H5 I a (7.14) 2pr f To find unit vector af in eqs. (7.12) to (7.14) is not always easy. A simple approach is to determine af from af 5 a, 3 ar(7.15) where a is a unit vector along the line current and ar is a unit vector along the perpendicular line from the line current to the field point. EXAMPLE 7.1 The conducting triangular loop in Figure 7.6(a) carries a current of 10 A. Find H at 1 0, 0, 5 2 due to side 1 of the loop. Solution: This example illustrates how eq. (7.12) is applied to any straight, thin, current-carrying conductor. The key point to keep in mind in applying eq. (7.12) is figuring out a1, a2, r, and af. To find H at 1 0, 0, 5 2 due to side 1 of the loop in Figure 7.6(a), consider Figure 7.6(b), where side 1 is treated as a straight conductor. Notice that we join the point of interest 1 0, 0, 5 2 to the beginning and end of the line current. Observe that a1, a2, and r are assigned in the same manner as in Figure 7.5 on which eq. (7.12) is based: cos a1 5 cos 90° 5 0, cos a2 5 2 , r55 "29 To determine af is often the hardest part of applying eq. (7.12). According to eq. (7.15), a, 5 ax and ar 5 az, so af 5 ax 3 az 5 2ay 07_Sadiku_Ch07.indd 302 23/09/17 2:38 PM 7.2 Biot–Savart's Law 303 z 5 ρ y αρ 1 3 2 10 A a 1 α1 x 0 1 α2 0 2 10 A x 2 (b) (a) FIGURE 7.6 For Example 7.1: (a) conducting triangular loop, (b) side 1 of the loop. Hence, H1 5 I 10 2 1 cos a2 2 cos a1 2 af 5 2 0b 1 2ay 2 a 4pr 4p 1 5 2 "29 5 259.1ay mA/m PRACTICE EXERCISE 7.1 Find H at 1 0, 0, 5 2 due to side 3 of the triangular loop in Figure 7.6(a). Answer: 230.63ax 1 30.63ay mA/m. EXAMPLE 7.2 Find H at 1 23, 4, 0 2 due to the current filament shown in Figure 7.7(a).

Solution: Let H 5 H1 1 H2, where H1 and H2 are the contributions to the magnetic field intensity at P 1 23, 4, 0 2 due to the portions of the filament along x and z, respectively. H2 5 I 1 cos a2 2 cos a1 2 af 4pr At PA23, 4, 0B, r 5 A9 1 16B 1/2 5 5, a1 5 90°, a2 5 0°, and af is obtained as a unit vector along the circular path through P on plane z 5 0 as in Figure 7.7(b).

The direction of af 07_Sadiku_Ch07.indd 303 23/09/17 2:38 PM 304 CHAPTER 7 MAGNETOSTATIC FIELDS P 3A 3A FIGURE 7.7 For Example 7.2: (a) current filament along semi-infinite x- and z-axes, a and ar for H2 only; (b) determining ar for H2. is determined using the right-handed-screw rule or the right-hand rule. From the geometry in Figure 7.7(b), 4 3 af 5 sin u ax 1 cos u ay 5 ax 1 ay 5 5 Alternatively, we can determine af from eq.
(7.15). At point P, a and ar are as illustrated in Figure 7.7(a) for H2. Hence, 3 4 4 3 af 5 2az 3 a2 ax 1 ay b 5 ax 1 ay 5 5 5 5 as obtained before.
Thus H2 5 1 4ax 1 3ay 2 3 11 2 02 5 4p 1 5 2 5 38.2ax 1 28.65ay mA/m It should be noted that in this case af happens to be the negative of the regular af of cylindrical coordinates. H2 could have also been obtained in cylindrical coordinates as H2 5 3 1 1 2 0 2 1 2af 2 4p 1 5 2 5 247.75af mA/m Similarly, for H1 at P, r 5 4, a2 5 0°, cos a1 5 3/5, and af 5 az or af 5 a, 3 ar 5 ax 3 ay 5 az. Hence, H1 5 3 3 a1 2 b az 5 4p 1 4 2 5 23.87az mA/m 07_Sadiku_Ch07.indd 304 23/09/17 2:38 PM 7.2 Biot–Savart's Law 305 Thus H 5 H1 1 H2 5 38.2ax 1 28.65ay 1 23.87az mA/m or H 5 247.75af 1 23.87az mA/m Notice that although the current filaments appear to be semi-infinite (they occupy the positive z- and x-axes), it is only the filament along the z-axis that is semi-infinite with respect to point P. Thus H2 could have been found by using eq. (7.13), but the equation could not have been used to find H1 because the filament along the x-axis is not semi- infinite with respect to P. PRACTICE EXERCISE 7.2 The positive y-axis (semi-infinite line with respect to the origin) carries a filamentary current of 2 A in the 2ay direction. Assume it is part of a large circuit. Find H at (a) A 1 2, 3, 0 2 (b) B 1 3, 12, 24 2 Answer: (a) 145.8az mA/m, (b) 48.97ax 1 36.73az mA/m. EXAMPLE 7.3 A circular loop located on x2 1 y2 5 9, z 5 0 carries a direct current of 10 A along af. Determine H at 1 0, 0, 4 2 and 1 0, 0, 24 2 .
Solution: Consider the circular loop shown in Figure 7.8(a). The magnetic field intensity dH at point P 1 0, 0, h 2 contributed by current element I dl is given by Biot–Savart's law: dH 5 I dl 3 R 4pR3 where dl 5 r df af, R 5 1 0, 0, h 2 2 1 x, y, 0 2 5 2rar 1 haz, and ar af dl 3 R 5 † 0 r df 2r 0 az 0 † 5 rh df ar 1 r2 df az h Hence, dH 5 07_Sadiku_Ch07.indd 305 I 1 rh df ar 1 r2 df az 2 5 dHr ar 1 dHz az 4p 3 r2 1 h2 4 3/2 23/09/17 2:38 PM 306 CHAPTER 7 MAGNETOSTATIC FIELDS FIGURE 7.8 For Example 7.3: (a) circular current loop, (b) flux lines due to the current loop. By symmetry, the contributions along ar add up to zero because the radial components produced by current element pairs 180° apart cancel. This may also be shown mathematically by writing ar in rectangular coordinate systems (i.e., ar 5 cos f ax 1 sin f ay). Integrating cos f or sin f over 0 # f # 2p gives zero, thereby showing that Hr 0. Thus H 5 3 dHz az 5 3 2p 0 or Ir2 df az Ir22paz 5 2 2 3/2 4p 3 r 1 h 4 4p 3 r2 1 h2 4 3/2 H5 Ir2az 2 3 r2 1 h2 4 3/2 (a) Substituting I 5 10 A, r 5 3, h 5 4 gives H 1 0, 0, 4 2 5 10 1 3 2 2az 5 0.36az A/m 2 3 9 1 16 4 3/2 (b) Notice from dl 3 R in the Biot–Savart law that if h is replaced by 2h, the z-component of dH remains the same while the r-component still adds up to zero due to the axial symmetry of the loop. Hence H 1 0, 0, 24 2 5 H 1 0, 0, 4 2 5 0.36az A/m The flux lines due to the circular current loop are sketched in Figure 7.8(b). 07_Sadiku_Ch07.indd 306 23/09/17 2:38 PM 7.2 Biot–Savart's Law 307 PRACTICE EXERCISE 7.3 A thin ring of radius 5 cm is placed on plane z 5 1 cm so that its center is at 1 0, 0, 1 cm 2 . If the ring carries 50 mA along af, find H at (a) 1 0, 0, 21 cm 2 (b) 1 0, 0, 10 cm 2 Answer: (a) 400az mA/m, (b) 57.3az mA/m. EXAMPLE 7.4 A solenoid of length and radius a consists of N turns of wire carrying current I.

Show that at point P along its axis, H5 nI 1 cos u 2 2 cos u 1 2 az 2 where n 5 N/,, u1 and u2 are the angles subtended at P by the end turns as illustrated in Figure 7.9. Also show that if , W a, at the center of the solenoid, H 5 nIaz Solution: Consider the cross section of the solenoid as shown in Figure 7.9. Since the solenoid consists of circular loops, we apply the result of Example 7.3. The contribution to the magnetic field H at P by an element of the solenoid of length dz is dHz 5 I dl a2 Ia2n dz 5 2 3 a2 1 z2 4 3/2 2 3 a2 1 z2 4 3/2 where dl 5 n dz 5 1 N/, 2 dz. From Figure 7.9, tan u 5 a/z; that is, dz 5 2a csc2 u du 5 2 3 z2 1 a2 4 3/2 sin u du a2 FIGURE 7.9 For Example 7.4; cross section of a solenoid. 07_Sadiku_Ch07.indd 307 23/09/17 2:38 PM 308 CHAPTER 7 MAGNETOSTATIC FIELDS Hence, dHz 5 2 nI sin u du 2 or nI 2 sin u du 2 3u1 u Hz 5 2 Thus H5 nI 1 cos u 2 2 cos u 1 2 az 2 as required. Substituting n 5 N/, gives H5 At the center of the solenoid, cos u 2 5 and NI 1 cos u 2 2 cos u 1 2 az 2, ,/2 5 2cos u 1 3 a 1 ,2 /4 4 1/2 H5 If , W a or u 2 . 0°, u 1 . 180°, 2 In, az 2 3 a 1 ,2 /4 4 1/2 2 H 5 nI az 5 NI a , z PRACTICE EXERCISE 7.4 The solenoid of Figure 7.9 has 2000 turns, a length of 75 cm, and a radius of 5 cm. If it carries a current of 50 mA along af, find H at (a) 1 0, 0, 0 2 (b) 1 0, 0, 75 cm 2 (c) 1 0, 0, 50 cm 2 Answer: (a) 66.52az A/m, (b) 66.52az A/m, (c) 131.7az A/m. 07_Sadiku_Ch07.indd 308 23/09/17 2:38 PM 7.4 Applications of Ampère's Law 309 7.3 AMPÈRE'S CIRCUIT LAW—MAXWELL'S EQUATION Ampère's circuit law states that the line integral of H around a closed path is the same as the net current Ienc enclosed by the path. In other words, the circulation of H equals Ienc; that is, # C H dl 5 Ienc(7.16) L Ampère's law is similar to Gauss's law, since Ampère's law is easily applied to determine H when the current distribution is symmetrical. It should be noted that eq. (7.16) always holds regardless of whether the current distribution is symmetrical or not, but we can use the equation to determine H only when a symmetrical current distribution exists. Ampère's law is a special case of Biot–Savart's law; the former may be derived from the latter. By applying Stokes's theorem to the left-hand side of eq. (7.16), we obtain But Ienc 5 C H # dl 5 3 1 = 3 H 2 # dS(7.17) L S Ienc 5 3 J # dS(7.18) S Comparing the surface integrals in eqs. (7.17) and (7.18) clearly reveals that = 3 H 5 J(7.19) This is the third Maxwell's equation to be derived; it is essentially Ampère's law in differential (or point) form, whereas eq. (7.16) is the integral form. From eq.
(7.19), we should observe that = 3 H 5 J 2 0; that is, a magnetostatic field is not conservative. 7.4 APPLICATIONS OF AMPÈRE'S LAW We now apply Ampère's circuit law to determine H for some symmetrical current distri butions as we did for Gauss's law. We will consider an infinite line current, an infinite sheet of current, and an infinitely long coaxial transmission line. In each case, we apply # A LH dl 5 Ienc. For symmetrical current distribution, H is either parallel or perpendicular to dl. When H is parallel to dl, 0 H 0 5 constant.
07_Sadiku_Ch07.indd 309 23/09/17 2:38 PM 310 CHAPTER 7 MAGNETOSTATIC FIELDS A. Infinite Line Current Consider an infinitely long filamentary current I along the z-axis as in Figure 7.10. To determine H at an observation point P, we allow a closed path to pass through P. This path, on which Ampère's law is to be applied, is known as an Amperian path (analogous to the term "Gaussian surface"). We choose a concentric circle as the Amperian path in view of eq. (7.14), which shows that H is constant provided r is constant. Since this path encloses the whole current I, according to Ampère's law, I 5 3 Hfaf # r df af 5 Hf 3 r df 5 Hf # 2pr L L or H5 I a (7.20) 2pr f as expected from eq.
(7.14). B. Infinite Sheet of Current Consider an infinite current sheet in the z 5 0 plane.

If the sheet has a uniform current density K 5 Kyay A/m as shown in Figure 7.11, applying Ampère's law to the rectangular closed path 1-2-3-4-1 (Amperian path) gives # C H dl 5 Ienc 5 Kyb(7.21a) To evaluate the integral, we first need to have an idea of what H is like.
To achieve this, we regard the infinite sheet as comprising filaments; dH above or below the sheet due to a pair of filamentary currents can be found by using eqs. (7.14) and (7.15). As evident in Figure 7.11(b), the resultant dH has only an x-component. Also, H on one side of the sheet is the negative of that on the other side. Owing to the infinite extent of the sheet, the sheet can be FIGURE 7.10 Ampère's law applied to an infinite filamentary line current. 07_Sadiku_Ch07.indd 310 23/09/17 2:38 PM 7.4 Applications of Ampère's Law 311 z z dH1 dH Amperian path dH2 4 h a 1 y 3 K = Ky a y 2 x 1 2 h b dH1 x dH dH2 (b) (a) FIGURE 7.11 Application of Ampère's law to an infinite sheet: (a) closed path 1-2-3-4-1, (b) symmetrical pair of current filaments with current along ay. regarded as consisting of such filamentary pairs so that the characteristics of H for a pair are the same for the infinite current sheet, that is, H5 e Ho a x 2Hoax z.0 (7.21b) z,0 where Ho is yet to be determined. Evaluating the line integral of H in eq. (7.21a) along the closed path in Figure 7.11(a) gives # # C H dl 5 a 3 1 3 1 3 1 3 b H dl 2 1 3 2 4 1 3 4 5 0 1 2a 2 1 1 2Ho 2 1 2b 2 1 0 1 a 2 1 Ho 1 b 2 (7.21c) 5 2Hob From eqs. (7.21a) and (7.21c), we obtain Ho 5 1 K . Substituting Ho in eq. (7.21b) gives 2 y 1 Ka, 2 y x H5 μ 1 2 Kyax, 2 z.0 (7.22) z,0 In general, for an infinite sheet of current density K A/m, H5 1 K 3 an(7.23) 2 where an is a unit normal vector directed from the current sheet to the point of interest. 07_Sadiku_Ch07.indd 311 23/09/17 2:38 PM 312 CHAPTER 7 MAGNETOSTATIC FIELDS FIGURE 7.12 Cross section of the transmission line; the positive z-direction is out of the page. C. Infinitely Long Coaxial Transmission Line Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the z-axis. The cross section of the line is shown in Figure 7.12, where the z-axis is out of the page. The inner conductor has radius a and carries current I, while the outer conductor has inner radius b and thickness t and carries return current I. We want to determine H everywhere, assuming that current is uniformly distributed in both conductors. Since the current distribution is symmetrical, we apply Ampère's law along the Amperian path for each of the four possible regions: 0 # r # a, a # r # b, b # r # b 1 t , and r $ b 1 t . For region 0 # r # a, we apply Ampère's law to path L1, giving # # C H dl 5 Ienc 5 3 J dS(7.24) L1 S Since the current is uniformly distributed over the cross section, J5 Ienc 5 3 J # dS 5 S I az, pa2 dS 5 r df dr az 2p r Ir2 I I 2 r df dr 5 pr 5 3 3 pa2 f50 r50 pa2 a2 Hence eq. (7.24) becomes Hf 3 dl 5 Hf 2pr 5 L1 Ir2 a2 or 07_Sadiku_Ch07.indd 312 Hf 5 Ir (7.25) 2pa2 23/09/17 2:38 PM 7.4 Applications of Ampère's Law 313 For region a # r # b, we use path L2 as the Amperian path, # C H dl 5 Ienc 5 I L2 Hf2pr 5 I or Hf 5 I (7.26) 2pr since the whole current I is enclosed by L2. Notice that eq. (7.26) is the same as eq. (7.14), and it is independent of a. For region b # r # b 1 t , we use path L3, getting # # C H dl 5 Hf 2pr 5 Ienc(7.27a) L3 where Ienc 5 I 1 3 J # dS and J in this case is the current density (current per unit area) of the outer conductor and is along 2az, that is, J52 Thus I az 3 1 p b 1 t 2 2 2 b2 4 I 3 3 r dr df p 3 1 b 1 t 2 2 2 b2 4 f50 r5b 2p Ienc 5 I 2 5 I c1 2 r r2 2 b2 d t2 1 2bt Substituting this in eq.
(7.27a), we have Hf 5 r2 2 b2 I c1 2 2 d (7.27b) 2pr t 1 2bt For region r $ b 1 t , we use path L4, getting # C H dI 5 I 2 I 5 0 L4 07_Sadiku_Ch07.indd 313 23/09/17 2:38 PM 314 CHAPTER 7 MAGNETOSTATIC FIELDS FIGURE 7.13 Plot of Hf against r. b or Hf 5 0(7.28) Putting eqs. (7.25) to (7.28) together gives Ir af, 2pa2 I a , H 5 g 2pr f 0#r#a a#r#b r2 2 b2 I d af, c1 2 2 2pr t 1 2bt 0, (7.29) b#r#b1t r$b1t The magnitude of H is sketched in Figure 7.13. From these examples, it can be observed that the ability to take H from under the integral sign is the key to using Ampère's law to determine H. In other words, Ampère's law can be used to find H only due to symmetric current distributions for which it is possible to find a closed path over which H is constant in magnitude. EXAMPLE 7.5 Planes z 5 0 and z 5 4 carry current K 5 210ax A/m and K 5 10ax A/m, respectively. Determine H at (a) 1 1, 1, 1 2 (b) 1 0, 23, 10 2 Solution: The parallel current sheets are shown in Figure 7.14. Let H 5 Ho 1 H4 07_Sadiku_Ch07.indd 314 23/09/17 2:38 PM 7.4 Applications of Ampère's Law 315 FIGURE 7.14 For Example 7.5: parallel infinite current sheets. where Ho and H4 are the contributions due to the current sheets z 5 0 and z 5 4, respectively. We make use of eq. (7.23). (a) At 1 1, 1, 1 2 , which is between the plates 1 0 , z 5 1 , 4 2 , Ho 5 1/2 K 3 an 5 1/2 1 210ax 2 3 az 5 5ay A/m Hence, H4 5 1/2 K 3 an 5 1/2 1 10ax 2 3 1 2az 2 5 5ay A/m H 5 10ay A/m (b) At 1 0, 23, 10 2 , which is above the two sheets 1 z 5 10 . 4 . 0 2 , Ho 5 1/2 1 210ax 2 3 az 5 25ay A/m Hence, H4 5 1/2 1 10ax 2 3 az 5 25ay A/m H 5 0 A/m PRACTICE EXERCISE 7.5 Plane y 5 1 carries current K 5 50az mA/m. Find H at (a) 1 0, 0, 0 2 (b) 1 1, 5, 23 2 Answer: (a) 25ax mA/m, (b) 225ax mA/m. EXAMPLE 7.6 A toroid whose dimensions are shown in Figure 7.15 has N turns and carries current I. Determine H inside and outside the toroid. Solution: We apply Ampère's circuit law to the Amperian path, which is a circle of radius r shown dashed in Figure 7.15. Since N wires cut through this path each carrying current I, the net current enclosed by the Amperian path is NI. Hence, 07_Sadiku_Ch07.indd 315 23/09/17 2:38 PM 316 CHAPTER 7 MAGNETOSTATIC FIELDS FIGURE 7.15 For Example 7.6: a toroid with a circular cross section. # S H # 2pr 5 NI C H dl 5 Ienc or H5 NI , 2pr for ro 2 a , r , ro 1 a where ro is the mean radius of the toroid as shown in Figure 7.15. An approximate value of H is Happrox 5 NI NI 5 2pro , Notice that this is the same as the formula obtained for H for points well inside a very long solenoid 1 , W a 2 . Thus a straight solenoid may be regarded as a special toroidal coil for which ro S ` . Outside the toroid, the current enclosed by an Amperian path is NI 2 NI 5 0 and hence H 5 0. PRACTICE EXERCISE 7.6 A toroid of circular cross section whose center is at the origin and axis the same as the z-axis has 1000 turns with ro 5 10 cm, a 5 1 cm. If the toroid carries a 100 mA current, find 0 H 0 at (a) 1 3 cm, 24 cm, 0 2 (b) 1 6 cm, 9 cm, 0 2 Answer: (a) 0, (b) 147.1 A/m. 07_Sadiku_Ch07.indd 316 23/09/17 2:38 PM 7.5 Magnetic Flux Density—Maxwell's Equation 317 7.5 MAGNETIC FLUX DENSITY—MAXWELL'S EQUATION The magnetic flux density B is similar to the electric flux density D. As D 5 eoE in free space, the magnetic flux density B is related to the magnetic field intensity H according to B 5 moH (7.30) where mo is a constant known as the permeability of free space. The constant is in Henrys per meter (H/m) and has the value of mo 5 4p 3 1027 H/m (7.31) The precise definition of the magnetic flux density B, in terms of the magnetic force, will be given in the next chapter. The magnetic flux through a surface S is given by 5 3 B # dS (7.32) S where the magnetic flux Ψ is in webers (Wb) and the magnetic flux density is in webers per square meter (Wb/m2) or teslas (T). A magnetic flux line is a path to which B is tangential at every point on the line. It is a line along which the needle of a magnetic compass will orient itself if placed in the presence of a magnetic field. For example, the magnetic flux lines due to a straight long wire are shown in Figure 7.16. The flux lines are determined by using the same principle followed in Section 4.10 for the electric flux lines. The direction of B is taken as that indicated as "north" by the needle of the magnetic compass. Notice that each flux line is closed and has no beginning or end. Though Figure 7.16 is for a straight, current-carrying conductor, it is generally true that magnetic flux lines are closed and do not cross each other regardless of the current distribution. FIGURE 7.16 Magnetic flux lines due to a straight wire with current coming out of the page. 07_Sadiku_Ch07.indd 317 23/09/17 2:38 PM 318 CHAPTER 7 MAGNETOSTATIC FIELDS C C FIGURE 7.17 Flux leaving a closed surface due to (a) isolated electric charge c 5 AS D # dS 5 Q, (b) magnetic charge, c 5 AS B # dS 5 0.

In an electrostatic field, the flux passing through a closed surface is the same as the charge enclosed; that is, 5 AS D # dS 5 Q . Thus it is possible to have an isolated electric charge as shown in Figure 7.17(a), which also reveals that electric flux lines are not necessarily closed. Unlike electric flux lines, magnetic flux lines always close upon themselves as in Figure 7.17(b). This is because it is not possible to have isolated magnetic poles (or magnetic charges). For example, if we desire to have an isolated magnetic pole by dividing a magnetic bar successively into two, we end up with pieces each having north and south poles as illustrated in Figure 7.18. We find it impossible to separate the north pole from the south pole. An isolated magnetic charge does not exist. Thus the total flux through a closed surface in a magnetic field must be zero; that is, # C B dS 5 0 (7.33) S FIGURE 7.18 Successive division of a bar magnet results in pieces with north and south poles, showing that magnetic poles cannot be isolated.
07_Sadiku_Ch07.indd 318 23/09/17 2:38 PM 7.6 Maxwell's Equations for Static Fields 319 This equation is referred to as the law of conservation of magnetic flux or Gauss's law for magnetostatic fields, just as AS D # dS 5 Q is Gauss's law for electrostatic fields. Although the magnetostatic field is not conservative, magnetic flux is conserved. By applying the divergence theorem to eq. (7.33), we obtain # # C B dS 5 3 = B dv 5 0 S v or = # B 5 0(7.34) This equation is the fourth Maxwell's equation to be derived. Equation (7.33) or (7.34) shows that magnetostatic fields have no sources or sinks. Equation (7.34) suggests that magnetic field lines are always continuous. 7.6 MAXWELL'S EQUATIONS FOR STATIC FIELDS Having derived Maxwell's four equations for static fields, we may take a moment to put them together as in Table 7.2. From the table, we notice that the order in which the equations are presented differs from the order in which they were derived. This was done for the sake of clarity. The choice between differential and integral forms of the equations depends on a given problem. It is evident from Table 7.2 that a vector field is defined completely by specifying its curl and its divergence. A field can be electric or magnetic only if it satisfies the corresponding Maxwell equations (see Problems 7. 40 and 7.41). It should be noted that Maxwell's equations as in Table 7.2 are only for static electric and magnetic fields. As will be discussed in Chapter 9, the divergence equations will remain the same for time-varying EM fields, but the curl equations will have to be modified. TABLE 7.2 Maxwell's Equations for Static Electric and Magnetic Fields Differential (or Point) Form Integral Form Remarks = # D 5 rv # C D dS 5 3 rv dv Gauss's law =#B50 # C B dS 5 0 =3E50 S v Nonexistence of magnetic monopole S # C E dl 5 0 Conservative nature of electrostatic field L =3H5J 07_Sadiku_Ch07.indd 319 # # C H d l 5 3 J dS L S Ampère's law 23/09/17 2:38 PM 320 CHAPTER 7 MAGNETOSTATIC FIELDS 7.7 MAGNETIC SCALAR AND VECTOR POTENTIALS We recall that some electrostatic field problems were simplified by relating the electric potential V to the electric field intensity E 1 E 5 2=V 2 . Similarly, we can define a potential associated with magnetostatic field B. In fact, the magnetic potential could be scalar Vm or vector A. To define Vm and A involves recalling two important identities (see Example 3.10 and Practice Exercise 3.10): = 3 1 =V 2 5 0(7.35a) = # 1 = 3 A 2 5 0(7.35b) which must always hold for any scalar field V and vector field A. Just as E 5 2=V , we define the magnetic scalar potential Vm (in amperes) as related to H according to H 5 2=Vm if J 5 0(7.36) The condition attached to this equation is important and will be explained. Combining eq. (7.36) and eq. (7.19) gives J 5 = 3 H 5 = 3 1 2=Vm 2 5 0(7.37) since Vm must satisfy the condition in eq. (7.35a). Thus the magnetic scalar potential Vm is only defined in a region where J 5 0 as in eq. (7.36). We should also note that Vm satisfies Laplace's equation just as V does for electrostatic fields; hence, =2Vm 5 0, 1 J 5 0 2 (7.38) We know that for a magnetostatic field, = # B 5 0 as stated in eq. (7.34).
To satisfy eqs.
(7.34) and (7.35b) simultaneously, we can define the magnetic vector potential A (in Wb/m) such that B 5 = 3 A(7.39) Just as we defined V53 dQ (7.40) 4peoR we can define 07_Sadiku_Ch07.indd 320 A53 moI dl for line current(7.41) L 4pR 23/09/17 2:38 PM 7.7 Magnetic Scalar and Vector Potentials 321 A53 A53 moK dS for surface current (7.42) S 4pR moJ dv for volume current (7.43) v 4pR Rather than obtaining eqs. (7.41) to (7.43) from eq. (7.40), an alternative approach would be to obtain eqs. (7.41) to (7.43) from eqs. (7.6) to (7.8). For example, we can derive eq. (7.41) from eq. (7.6) in conjunction with eq. (7.39). To do this, we write eq. (7.6) as B5 mo I dlr 3 R (7.44) 4p 3L R3 where R is the distance vector from the line element dl at the source point 1 xr, yr, zr 2 to the field point 1 x, y, z 2 as shown in Figure 7.19 and R 5 0 R 0 , that is, Hence, or R 5 0 r 2 r9 0 5 3 1 x 2 xr 2 2 1 1 y 2 yr 2 2 1 1 z 2 zr 2 2 4 1/2 (7.45) 1 x 2 xr 2 ax 1 1 y 2 yr 2 ay 1 1 z 2 zr 2 az 1 R =a b 5 2 52 3 2 2 2 3/2 3 1 x 2 xr 2 1 1 y 2 yr 2 1 1 z 2 zr 2 4 R R 1 R 5 2=a b R3 R a5 aR b(7.46) R2 where the differentiation is with respect to x, y, and z. Substituting this into eq. (7.44), we obtain B52 mo 1 3 I dlr 3 =a b(7.47) 4p L R FIGURE 7.19 Illustration of the source point (x, y, z) and the field point (x, y, z).
07_Sadiku_Ch07.indd 321 23/09/17 2:38 PM 322 CHAPTER 7 MAGNETOSTATIC FIELDS We apply the vector identity = 3 1 f F 2 5 f = 3 F 1 1 =f 2 3 F(7.48) where f is a scalar field and F is a vector field. Taking f 5 1/R and F 5 dlr, we have 1 1 dlr dlr 3 =a b 5 = 3 dlr 2 = 3 a b R R R Since operates with respect to 1 x, y, z 2 while dl is a function of 1 xr, yr, zr 2 , = 3 dlr 5 0.
Hence, 1 dlr dlr 3 =a b 5 2= 3 (7.49) R R With this equation, eq. (7.47) reduces to B5=33 moI dlr (7.50) L 4pR Comparing eq. (7.50) with eq.
(7.39) shows that A53 moI dlr L 4pR verifying eq.
(7.41). By substituting eq. (7.39) into eq. (7.32) and applying Stokes's theorem, we obtain 5 3 B # dS 5 3 1 = 3 A 2 # dS 5 C A # dl S S L or 5 C A # dl (7.51) L Thus the magnetic flux through a given area can be found by using either eq. (7.32) or (7.51). Also, the magnetic field can be determined by using either Vm or A; the choice is dictated by the nature of the given problem except that Vm can be used only in a source-free region. The use of the magnetic vector potential provides a powerful, elegant approach to solving EM problems, particularly those relating to antennas. As we shall notice in Chapter 13, it is more convenient to find B by first finding A in antenna problems.
07_Sadiku_Ch07.indd 322 23/09/17 2:38 PM 7.7 Magnetic Scalar and Vector Potentials 323 EXAMPLE 7.7 Given the magnetic vector potential A 5 2r2 /4 az Wb/m, calculate the total magnetic flux crossing the surface f 5 p/2, 1 # r # 2 m, 0 # z # 5 m. Solution: We can solve this problem in two different ways: using eq. (7.32) or eq. (7.51). Method 1: B5=A52 r 'Az af 5 af, 'r 2 dS 5 dr dz af Hence, 5 2 1 1 1 15 # 5 3 B dS 5 3 3 r dr dz 5 r2 ` 1 5 2 5 2 4 4 S 2 z50 r51 5 3.75 Wb Method 2: We use 5 C A # dl 5 1 1 2 1 3 1 4 L where L is the path bounding surface S; Ψ1, Ψ2, Ψ3, and Ψ4 are, respectively, the evaluations of eLA # dl along the segments of L labeled 1 to 4 in Figure 7.20. Since A has only a z-component, 1 5 0 5 3 FIGURE 7.20 For Example 7.7. 07_Sadiku_Ch07.indd 323 23/09/17 2:38 PM 324 CHAPTER 7 MAGNETOSTATIC FIELDS That is, 5 0 1 5 2 1 4 5 2 c 1 1 2 2 3 dz 1 1 2 2 2 3 dzd 4 0 5 1 15 5 2 11 2 42 152 5 4 4 5 3.75 Wb as obtained by Method 1. Note that the direction of the path L must agree with that of dS. PRACTICE EXERCISE 7.7 A current distribution gives rise to the vector magnetic potential A 5 x2yax 1 y2x ay 2 4xyz az Wb/m. Calculate the following: (a) B at 1 21, 2, 5 2 (b) The flux through the surface defined by z 5 1, 0 # x # 1, 21 # y # 4 Answer: (a) 20ax 1 40ay 1 3az Wb/m2, (b) 20 Wb. EXAMPLE 7.8 If plane z 5 0 carries uniform current K 5 Kyay, H5 e 1/2 Kyax, 21/2 Kyax, z.0 z,0 This was obtained in Section 7.4 by using Ampère's law. Obtain this by using the concept of vector magnetic potential. Solution: Consider the current sheet as in Figure 7.21. From eq. (7.42), dA 5 moK dS 4pR In this problem, K 5 Kyay, dS 5 dxr dyr, and for z . 0, R 5 0 R 0 5 0 1 0, 0, z 2 2 1 xr, yr, 0 2 0 5 3 1 xr 2 2 1 1 yr 2 2 1 z2 4 1/2 (7.8.1) where the primed coordinates are for the source point while the unprimed coordinates are for the field point. It is necessary (and customary) to distinguish between the two points to avoid confusion (see Figure 7.19). Hence 07_Sadiku_Ch07.indd 324 23/09/17 2:38 PM 7.7 Magnetic Scalar and Vector Potentials 325 FIGURE 7.21 For Example 7.8: infinite current sheet. moKy dxr dyr ay dA 5 4p 3 1 xr 2 2 1 1 yr 2 2 1 z2 4 1/2 ' dB 5 = 3 dA 5 2 d Ay ax 'z moKyz dxr dyr ax 5 4p 3 1 xr 2 2 1 1 yr 2 2 1 z2 4 3/2 B5 moKyzax 4p 3 ` 2` 3 ` 2` dxr dyr (7.8.2) 3 1 xr 2 1 1 yr 2 2 1 z2 4 3/2 2 In the integrand, we may change coordinates from Cartesian to cylindrical for convenience so that B5 5 5 5 moKyzax 4p 3 2p rr50 fr50 moKyzax 4p rr dfr drr 3 1 rr 2 2 1 z2 4 3/2 2p 3 3 1 rr 2 2 1 z2 4 23/2 1/2 d 3 1 rr 2 2 4 ` 0 moKyzax 2 moKyax 2 3 ` ` 21 ` 3 1 rr 2 2 1 z2 4 1/2 rr50 Hence H5 Ky B 5 a, mo 2 x for z . 0 By simply replacing z by 2z in eq. (7.8.2) and following the same procedure, we obtain H52 07_Sadiku_Ch07.indd 325 Ky 2 ax, for z , 0 23/09/17 2:38 PM 326 CHAPTER 7 MAGNETOSTATIC FIELDS PRACTICE EXERCISE 7.8 Repeat Example 7.8 by using Biot–Savart's law to determine H at points 1 0, 0, h 2 and 1 0, 0, 2h 2 . † 7.8 DERIVATION OF BIOT–SAVART'S LAW AND AMPÈRE'S LAW Both Biot–Savart's law and Ampère's law may be derived by using the concept of magnetic vector potential. The derivation will involve the use of the vector identities in eq. (7.48) and = 3 = 3 A 5 = 1 = # A 2 2 =2A(7.52) Since Biot–Savart's law as given in eq. (7.4) is defined in terms of line current, we begin our derivation with eqs. (7.39) and (7.41); that is, moI dlr moI 1 B5=3 C 5 = 3 dlr(7.53) C 4p L R L 4pR where R is as defined in eq.
(7.45). If the vector identity in eq. (7.48) is applied by letting F 5 dlr and f 5 1/R, eq. (7.53) becomes B5 moI 1 1 c = 3 dlr 1 a= b 3 dlr d (7.54) C 4p L R R Since operates with respect to 1 x, y, z 2 and d l is a function of 1 xr, yr, zr 2 , = 3 dlr 5 0. Also 1 5 3 1 x 2 xr 2 2 1 1 y 2 yr 2 2 1 1 z 2 zr 2 2 4 21/2(7.55) R 1 x 2 xr 2 ax 1 1 y 2 yr 2 ay 1 1 z 2 zr 2 az aR 1 =c d 5 2 5 2 2 2 (7.56) 2 2 2 3/2 3 1 x 2 xr 2 1 1 y 2 yr 2 1 1 z 2 zr 2 4 R R where aR is a unit vector from the source point to the field point.
Thus eq. (7.54) (upon dropping the prime in dl) becomes B5 moI dl 3 aR (7.57) 4p CL R2 which is Biot–Savart's law. 07_Sadiku_Ch07.indd 326 23/09/17 2:38 PM 7.8 Derivation of Biot–Savart's Law and Ampère's Law 327 Using the identity in eq. (7.52) with eq. (7.39), we obtain = 3 B 5 = 1 = # A 2 2 =2A(7.58) For reasons that will be obvious in Chapter 9, we choose = # A 5 0(7.59) which is called Coulomb's gauge. Upon replacing B with moH and using eq. (7.19), eq. (7.58) becomes =2A 5 2mo= 3 H or =2A 5 2moJ(7.60) which is called the vector Poisson equation. It is similar to Poisson's equation 1 =2V 5 2rv /e 2 in electrostatics. In Cartesian coordinates, eq. (7.60) may be decomposed into three scalar equations: =2Ax 5 2moJx =2Ay 5 2moJy(7.61) =2Az 5 2moJz which may be regarded as the scalar Poisson equations. It can also be shown that Ampère's circuit law is consistent with our definition of the magnetic vector potential. From Stokes's theorem and eq. (7.39), # # C H dl 5 3 = 3 H dS L S 5 1 = 3 1 = 3 A 2 # dS mo 3S (7.62) From eqs. (7.52), (7.59), and (7.60), = 3 = 3 A 5 2=2 A 5 moJ Substituting this into eq. (7.62) yields # # C H dl 5 3 J dS 5 I L S which is Ampère's circuit law. 07_Sadiku_Ch07.indd 327 23/09/17 2:38 PM 328 CHAPTER 7 MAGNETOSTATIC FIELDS † 7.9 APPLICATION NOTE—LIGHTNING Lightning is the discharge of static electricity generated in clouds by natural processes. Lightning may also be regarded as a transient, high-current electric discharge. It is a major natural source of electromagnetic radiation that interferes with modern electronics and communication systems. Lightning strikes somewhere on the surface of the earth about 100 times every second. Lightning, the thunderbolt from mythology, has long been feared as an atmospheric flash of supernatural origins: the great weapon of the gods. Today, scientific rather than mystical techniques are used to explain lightning, with experimental procedures replacing intuitive concepts. Yet, we remain in awe of lightning, which still shines with its mystery, and rightly so. Deaths and injuries to livestock and other animals, thousands of forest and brush fires, and millions of dollars in damage to buildings, communications systems, power lines, and electrical systems are among the results of lightning. Since lightning can reach from clouds to the ground or to other clouds, lightning may be classified into two types: (1) cloud-to-cloud and (2) cloud-to-ground. A typical cloud-toground lighting is shown in Figure 7.22.

The cloud-to-cloud discharge is more common and is important for aircraft in flight. However, cloud-to-ground lightning has been studied more extensively because of its practical interest (e.g., as the cause of injuries and death or disturbances in power and communication systems). A typical cloud-to-ground lightning carries about 10 C to 20 C at an average height of 5 km above the ground. The portion of the cloud-to-ground discharge that produces physical damage at ground level by virtue of its high current is called the return stroke. The current in a return stroke is typically 10 kA but can be as high as 200 kA. Under good weather conditions, an electric field of the order 100 V/m exists near the earth's surface. Movements inside a cloud cause the cloud to become an electric dipole, with negative charges in the lower part and positive charges in the upper part. FIGURE 7.22 A cloud-to-ground lightning. 07_Sadiku_Ch07.indd 328 23/09/17 2:38 PM 7.10 Application Note—Polywells 329 The approach of the negatively charged particles to the ground induces more positive charges, especially on tall, sharp structures. A lightning bolt follows the path of least resistance at the moment of initiation; this is rarely a straight line, and it is unique for each strike. However, if we assume that lightning strikes arrive in the vertical direction, we can estimate the striking distance as a function of the amplitude of the current of the return stroke. The base striking distance D in meters, and the current I, in kiloamperes, are related as D 5 10I 0.65(7.63) Humans and animals within the striking distance may be hurt. A common way to protect people, buildings, and other structures from lightning is to use lightning rods. Originally developed by

Benjamin Franklin, a lightning rod is a pointed metal rod attached to the roof of a building. It is connected to a copper or aluminum wire, and the wire is connected to a conductive grid buried in the ground nearby. Lightning rods provide a low-resistance path to ground that can be used to conduct the enormous e lectrical currents when lightning strikes occur. When lightning strikes, the system attempts to carry the harmful electrical current away from the structure and safely to ground. 7.10 APPLICATION NOTE—POLYWELLS A polywell is a polyhedral group of metal rings; inside each ring is a coil, which produces a magnetic field. As schematically illustrated in Figure 7.23, the position of each of the rings and the direction of current flow in each coil are set to create a null magnetic field at FIGURE 7.23 Coils in a polywell. 07_Sadiku_Ch07.indd 329 23/09/17 2:38 PM 330 CHAPTER 7 MAGNETOSTATIC FIELDS the center of the cube. On two opposing sides of the cube a stream of electrons is injected through the rings.

These injected electrons are pushed by the magnetic field toward the (magnetically null) center of the cube, forming a cloud of electrons. When this cloud of electrons is large enough, it will create what is known as an electrostatic potential well. The polywell's name comes from its polyhedral shape and the electrostatic potential well produces. In a nuclear fusion reactor two lighter atomic particles fuse together to form a heavier particle, releasing large amounts of energy. The normal activity inside a nuclear fusion reactor is as follows: Two spherically concentric, gridded electrodes create a radial electric field that acts as an electrostatic potential well. Then the radial electric field accelerates ions to fusion-revelant energies and confines them in the central grid region. The fusion reactor system, however, suffers from substantial energy loss due to collisions between the grid itself and the ions. The polywell overcomes this problem by replacing the physical cathode with a virtual cathode, the electron cloud. In the polywell the ion streams are injected into the polyhedron through the remaining four rings.

These ions are attracted to the electron cloud and are accelerated to the energy at which fusion can occur. All these parts—the polywell and the electron and ion guns—are encapsulated in a collection sphere, with all of this inside a vacuum chamber. This collection sphere captures the energy released from the fusion process in the form of alpha particles, which come from the fusion, inside the electron cloud, of boron and hydrogen ions.

The use of boron and hydrogen in nuclear fusion is becoming more popular than the use of deuterium and tritium as fuel.

Unlike the fusion of deuterium and tritium, the fusion of boron and hydrogen produces little to no radiation and, since the only by-product is helium, there is no radioactive waste. MATLAB 7.1 y y = x2 P(–0.5, 0.5, 0) I x Suppose a 0.5 mA segment of current travels along the parabola y 5 x2 between a 5 1 0, 0, 0 2 and b 5 1 1, 1, 0 2 cm. Using the Biot–Savart law, determine the magnetic field at point P(–0.5, 0.5, 0) due to the segment. 07_Sadiku_Ch07.indd 330 23/09/17 2:38 PM 7.10 Application Note—Polywells 331 We will find the general solution and evaluate at the observation point. We use the Biot–Savart law to obtain the magnetic field at point P: H53 Idl 3 aR 2 L 4pR The unit vector from the incremental current filament at 1 xr, yr, 0 2 to the observation point P (x, y, z) is aR 5 1 x 2 xr 2 ax 1 1 y 2 yr 2 ay 1 zaz R R 5 " 1 x 2 xr 2 2 1 1 y 2 yr 2 2 1 z2 The incremental current element is given by The cross product is IdL 5 I 1 dxrax 1 dyray 2 IdL 3 aR 5 I 1 dxray 1 dyray 2 3 Thus 1 x 2 xr 2 ax 1 1 y 2 yr 2 ay 1 zaz " 1 x 2 xr 2 2 1 1 y 2 yr 2 2 1 1 z 2 zr 2 2 H53 b 3 1 y 2 yr 2 dxr 2 1 x 2 xr 2 dyr 4 a 2 z dxra 1 z dyra IdL 3 aR z y x I 5 3 2 2 2 2 1.5 1 2 3 1 2 4 4p 4pR 1 y 2 yr 1 z x 2 xr L a This integral is numerically evaluated as 0.85I; thus the magnetic field at P is given by: H 5 1.9437az clear I=0.5e-3; mA m % the current value % prompt for observation point disp('Enter the observation point (in the '); p0 = input('format [x y z])... > '); if isempty(p0); p0 = [0 0 0]; end xpstart = 0; xpend = 1e-2; % start and end points for %integration variable x prime dxp=1e-7; % integration variable increment dx H = [0, 0, 0]; % initial field values before integration sum zp = 0; % current lies only in the xy-plane for xp=xpstart:dxp:xpend, % begin integration loop yp=xp^2*1e2; % make substitution for y prime in terms % of x prime 07_Sadiku_Ch07.indd 331 23/09/17 2:38 PM 332 CHAPTER 7 MAGNETOSTATIC FIELDS dyp=2*3*dxp; % % % % the 1e2 is to offset the 1e-2 squared term which relates x prime and y prime in space make substitution for dy prime in terms % of dx prime num = [(p0(3)-zp)*dyp,-(p0(3)-zp)*dxp,((p0(2)-yp)*dxp-... (p0(1)-xp)*dyp)]; % numerator den = ((p0(1)-xp)^2+(p0(2)-yp)^2)^(3/2); % denominator H = H + num/den; % total field including all three coordinates end H=H*I/(4*pi); % display the output disp('') disp('The magnetic field at'); disp(sprintf(' (%f, %f, %f) cm is (%f %f %f) A/m', ... p0(1), p0(2), p0(2), H(1), H(2), H(3))) MATLAB 7.2 % This script allows the user to specify a current % directed out of the page (+z direction) that lies on the origin, % is assumed infinite, and points in the z direction % and plot the vector magnetic field in the xy-plane % % % inputs: I (value of the current), x and y limits of the plot % outputs: the magnetic field vector plot clear % prompt user for input materials disp('Enter the graph limits '); plotlim = input(' [xmin xmax ymin ymax]... > '); if isempty(plotlim); plotlim = [-1 1 -1 1]; end % check if entered correctly I = input('Enter the current in Amperes... > '); if isempty(I); I = 1; end % check if current is entered dx=(plotlim(2)-plotlim(1))/10; dy=(plotlim(4)-plotlim(3))/10; xrange=plotlim(1):dx:plotlim(2); yrange=plotlim(3):dy:plotlim(4); [X,Y]=meshgrid(xrange,yrange); U=zeros(length(xrange), length(yrange)); V=zeros(length(xrange), length(yrange)); for x=1:length(xrange) 07_Sadiku_Ch07.indd 332 23/09/17 2:38 PM Summary 333 for y=1:length(yrange) r=sqrt(xrange(x)^2+yrange(y)^2); % the distance from the current phiuvector=[-yrange(y),xrange(x)]/r; % the unit vector in the phi direction H=I/(2*pi*r)*phiuvector; % Ampere's law for an infinite current % fill matrices which contain the vector % components in x and y direction U(y, x)=H(1); % vector x corresponds to columns V(y, x)=H(2); % vector x corresponds to columns end end % Display results figure quiver(xrange,yrange,U,V) axis square axis(plotlim) xlabel('X location (m)') ylabel('Y location (m)') disp('Value of first vector to the right of') disp(sprintf(' origin = %f A/m',I/(2*pi*dx))) SUMMARY 1. The basic laws (Biot–Savart's and Ampère's) that govern magnetostatic fields are discussed. Biot–Savart's law, which is similar to Coulomb's law, states that the magnetic field intensity dH at r due to current element I dl at r is dH 5 I dl 3 R (in A/m) 4pR3 where R 5 r 2 rr and R 5 0 R 0 . For surface or volume current distribution, we replace I dl with K dS or J dv, respectively; that is, I dl ; K dS ; J dv 2. Ampère's circuit law, which is similar to Gauss's law, states that the circulation of H around a closed path is equal to the current enclosed by the path; that is, # # C H dl 5 Ienc 5 3 J dS L 07_Sadiku_Ch07.indd 333 S 23/09/17 2:38 PM 334 CHAPTER 7 MAGNETOSTATIC FIELDS or = 3 H 5 J (third Maxwell equation to be derived) When current distribution is symmetric so that an Amperian path (on which H 5 Hfaf is constant) can be found, Ampère's law is useful in determining H; that is, Ienc Hf C dl 5 Ienc or Hf 5 , L 3. The magnetic flux through a surface S is given by 5 3 B # dS (in Wb) S where B is the magnetic flux density (in Wb/m2). In free space, B 5 moH where mo 5 4p 3 1027 H/m 5 permeability of free space. 4. Since an isolated or free magnetic monopole does not exist, the net magnetic flux through a closed surface is zero: 5 C B # dS 5 0 S or = # B 5 0 (fourth Maxwell equation to be derived) 5. At this point, all four Maxwell equations for static EM fields have been derived, namely: = # D 5 rv =#B50 =3E50 =3H5J 6. The magnetic scalar potential Vm is defined as H 5 2=Vm, if J 5 0 and the magnetic vector potential A as B5=3A where = # A 5 0. With the definition of A, the magnetic flux through a surface S can be found from 5 C A # dl L 07_Sadiku_Ch07.indd 334 23/09/17 2:38 PM Review Questions 335 where L is the closed path defining surface S (see Figure 3.21). Rather than using Biot– Savart's law, the magnetic field due to a current distribution may be found by using A, a powerful approach that is particularly useful in antenna theory. For a current element I dl at r, the magnetic vector potential at r is A53 moI dl , 4pR R 5 0 r 2 rr 0 7. Elements of similarity between electric and magnetic fields exist. Some of these are listed in Table 7.1. Corresponding to Poisson's equation =2V 5 2rv /e, for example, is =2A 5 2moJ 8. Lightning may be regarded as a transient, high-current electric discharge.

A common way to protect people, buildings, and other structures from lightning is to use lightning rods. REVIEW QUESTIONS 7.1 One of the following is not a source of magnetostatic fields: (a) A dc current in a wire (b) A permanent magnet (c) An accelerated charge (d) An electric field linearly changing with time (e) A charged disk rotating at uniform speed 7.2 Identify the configuration in Figure 7.24 that is not a correct representation of I and H. 7.3 Consider points A, B, C, D, and E on a circle of radius 2 as shown in Figure 7.25. The items in the right-hand list are the values of af at different points on the circle. Match these items with the points in the list on the left. (a) A (i) ax (b) B (ii) 2ax (c) C (iii) ay (d) D (iv) 2ay FIGURE 7.24 For Review Question 7.2. 07_Sadiku_Ch07.indd 335 23/09/17 2:39 PM 336 CHAPTER 7 MAGNETOSTATIC FIELDS FIGURE 7.25 For Review Question 7.3. (e) E (v) (vi) ax 1 ay "2 2ax 2 ay "2 2ax 1 ay (vii) "2 ax 2 ay (viii) "2 7.4 The z-axis carries filamentary current of 10p A along az. Which of these is incorrect? (a) H 5 2ax A/m at 1 0, 5, 0 2 (b) H 5 af A/m at 1 5, p/4, 0 2 (c) H 5 20.8ax 2 0.6ay at 1 23, 4, 0 2 (d) H 5 2af at 1 5, 3p/2, 0 2 7.5 Plane y 5 0 carries a uniform current of 30az mA/m. At 1 1, 10, 22 2 , the magnetic field intensity is (a) 215ax mA/m (b) 15ax mA/m (c) 477.5ay mA/m (d) 18.85ay nA/m (e) None of the above 7.6 For the currents and closed paths of Figure 7.26, calculate the value of AL H # d l. 7.7 Which of these statements is not characteristic of a static magnetic field?

(a) It is solenoidal. (b) It is conservative. (c) It has no sinks or sources. (d) Magnetic flux lines are always closed. (e) The total number of flux lines entering a given region is equal to the total number of flux lines leaving the region. 07_Sadiku_Ch07.indd 336 23/09/17 2:39 PM Review Questions 337 FIGURE 7.26 For Review Question 7.6. 7.8 Two identical coaxial circular coils carry the same current I but in opposite directions. The magnitude of the magnetic field B at a point on the axis midway between the coils is (a) Zero (b) The same as that produced by one coil (c) Twice that produced by one coil (d) Half that produced by one coil. 7.9 Which one of these equations is not Maxwell's equation for a static electromagnetic field in a linear homogeneous medium?

(a) = # B 5 0 (b) = 3 D 5 0 (c) AL B # d l 5 moI (d) AS D # dS 5 Q (e) =2A 5 moJ 7.10 Two bar magnets with their north poles having strength Qm1 5 10 A # m and Qm2 5 10 A # m (magnetic charges) are placed inside a volume as shown in Figure 7.27. The magnetic flux leaving the v olume is (a) 200 Wb (b) 30 Wb (c) 10 Wb (d) 0 Wb (e) 210 Wb Answers: 7.1c, 7.2c, 7.3 (a)-(ii), (b)-(vi), (c)-(i), (d)-(v), (e)-(iii), 7.4d, 7.5a, 7.6 (a) 10 A, (b) 220 A, (c) 0, (d) 210 A, 7.7b, 7.8a, 7.9e, 7.10d. FIGURE 7.27 For Review Question 7.10. 07_Sadiku_Ch07.indd 337 23/09/17 2:39 PM 338 CHAPTER 7 MAGNETOSTATIC FIELDS PROBLEMS Section 7.2—Biot–Savart's Law 7.1 (a) State Biot–Savart's law. (b) The y- and z-axes, respectively, carry filamentary currents 10 A along ay and 20 A along 2az. Find H at 1 23, 4, 5 2 . 7.2 A long, straight wire carries current 2A. Calculate the distance from the wire when the magnetic field strength is 10 mA/m. 7.3 Two infinitely long wires, placed parallel to the z-axis, carry currents 10 A in opposite directions as shown in Figure 7.28. Find H at point P. 7.4 Two current elements I1dl1 5 4 3 10 25 ax A.m at (0, 0, 0) and I2dl2 5 6 3 10 25 ay A.m at (0, 0, 1) are in free space. Find H at (3, 1, –2). 7.5 A conducting filament carries current I from point A 1 0, 0, a 2 to point B 1 0, 0, b 2 . Show that at point P 1 x, y, 0 2 , H5 I b a 2 c d af 4p"x2 1 y3 "x2 1 y2 1 b2 "x2 1 y2 1 a2 7.6 Consider AB in Figure 7.29 as part of an electric circuit. Find H at the origin due to AB. 7.7 Line x 5 0, y 5 0, 0 # z # 10 m carries current 2 A along az. Calculate H at points (a) 1 5, 0, 0 2 (b) 1 5, 5, 0 2 (c) 1 5, 15, 0 2 (d) 1 5, 215, 0 2 *7.8 (a) Find H at 1 0, 0, 5 2 due to side 2 of the triangular loop in Figure 7.6(a). (b) Find H at 1 0, 0, 5 2 due to the entire loop.

y 5 X 10 A 4 y 3 1 2 6A 1 • 10 A P 1 2 3 4 FIGURE 7.28 For Problem 7.3. 07_Sadiku_Ch07.indd 338 A 5 B x 0 x 1 FIGURE 7.29 For Problem 7.6. 04/11/17 1:30 PM Problems 339 FIGURE 7.30 Current filament for Problem 7.9. y 5A x 5A 7.9 An infinitely long conductor is bent into an L shape as shown in Figure 7.30. If a direct current of 5 A flows in the conductor, find the magnetic field intensity at (a) 1 2, 2, 0 2 , (b) 1 0, 22, 0 2 , and (c) 1 0, 0, 2 2 . 7.10 Find H at the center C of an equilateral triangular loop of side 4 m carrying 5 A of current as in Figure 7.31. 7.11 A rectangular loop carrying 10 A of current is placed on z 5 0 plane as shown in Figure 7.32. Evaluate H at (a) 1 2, 2, 0 2 (c) 1 4, 8, 0 2 (b) 1 4, 2, 0 2 (d) 1 0, 0, 2 2 7.12 A square conducting loop of side 4 cm lies on the z 5 0 plane and is centered at the origin. If it carries a current 5 mA in the counterclockwise direction, find H at the center of the loop. *7.13 (a) A filamentary loop carrying current I is bent to assume the shape of a regular p olygon of n sides. Show that at the center of the polygon nI p sin n 2pr where r is the radius of the circle circumscribed by the polygon. H5 (b) Apply this for the cases of n 5 3 and n 5 4 and see if your results agree with those for the triangular loop of Problem 7.10. FIGURE 7.31 Equilateral triangular loop for Problem 7.10. 07_Sadiku_Ch07.indd 339 FIGURE 7.32 Rectangular loop of Problem 7.11. 04/11/17 1:30 PM 340 CHAPTER 7 MAGNETOSTATIC FIELDS y 10 A 4 cm O 100 cm x 10 A FIGURE 7.33 Filamentary loop of Problem 7.14 (not drawn to scale). y I 0 φo ρ1 ρ2 x FIGURE 7.34 Problem 7.15. (c) As n becomes large, show that the result of part (a) becomes that of the circular loop of Example 7.3. 7.14 For the filamentary loop shown in Figure 7.33, find the magnetic field strength at O. 7.15 Figure 7.34 shows a portion of a circular loop. Find H at the origin.

7.16 Two identical loops are parallel and separated by distance d as shown in Figure 7.35. Calculate H at (0, 0, d) assuming that a 5 3 cm, d 5 4 cm, and I 5 10 A. 7.17 A solenoid of radius 4 mm and length 2 cm has 150 turns/m and carries a current of 500 mA. Find (a) 0 H 0 at the center, (b) 0 H 0 at the ends of the solenoid. 7.18 Plane x 5 10 carries a current of 100 mA/m along az, while line x 5 1, y 5 22 carries a filamentary current of 20p mA along az. Determine H at 1 4, 3, 2 2 . 07_Sadiku_Ch07.indd 340 04/11/17 1:30 PM Problems 341 FIGURE 7.35 Problem 7.16. z I a Loop 1 d I y a Loop 2 x Section 7.3—Ampère's Circuit Law 7.19 (a) State Ampère's circuit law. (b) A hollow conducting cylinder has inner radius a and outer radius b and carries current I along the positive z-direction. Find H everywhere. 7.20 Current sheets of 20ax A/m and 220ax A/m are located at y 5 1 and y 5 21, respectively. Find H in region 21 , y , 1. 7.21 The z 5 0 plane carries current K 5 10ax A/m, while current filament situated at y 5 0, z 5 6 carries current I along ax. Find I such that H 1 0, 0, 3 2 5 0. 7.22 A conducting cylinder of radius a carries current I along 1az. (a) Use Ampère's law to find H for a and .

a. (b) Find J. 7.23 An infinitely long cylindrical conductor of radius a is placed along the z-axis. If the Jo current density is J 5 az, where Jo is constant, find H everywhere. r 7.24 Let H 5 y2ax 1 x2ay A/m. (a) Find J. (b) Determine the current through the strip z 51, 0 x 2, 1 y < 5. r 7.25 Let H 5 ko a baf, r , a, where ko is a constant. (a) Find J for r , a. (b) Find H for r . a. a 7.26 Let H 5 y2ax 1 x2ay A/m. Find J at (1, 24, 7). 7.27 Assume a conductor, H 5 103ρ2af A/m. (a) Find J. (b) Calculate the current through the surface 0 ρ 2, 0 2, z 5 0.

07_Sadiku_Ch07.indd 341 06/11/17 6:23 PM 342 CHAPTER 7 MAGNETOSTATIC FIELDS FIGURE 7.36 Two-wire line of Problem 7.30. 7.28 A cylindrical conductor of radius a 5 1 cm carries current I which produces H 5 4ρaf A/m. Find I. 7.29 An infinitely long filamentary wire carries a current of 2 A along the z-axis in the 1z-direction. Calculate the following: (a) B at 1 23, 4, 7 2 (b) The flux through the square loop described by 2 # r # 6, 0 # z # 4, f 5 90°. 7.30 Consider the two-wire transmission line whose cross section is illustrated in Figure 7.36. Each wire is of radius 2 cm, and the wires are separated 10 cm. The wire centered at 1 0, 0 2 carries a current of 5 A, while the other centered at 1 10 cm, 0 2 carries the return current. Find H at (a) 1 5 cm, 0 2 (b) 1 10 cm, 5 cm 2 7.31 An electron beam forms a current of density J5 e Jo 1 12r2@ 2 2 az , a 0, r.a r,a (a) Determine the total current. (b) Find the magnetic field intensity everywhere. Section 7.5—Magnetic Flux Density 7.32 Determine the magnetic flux through a rectangular loop 1 a 3 b 2 due to an infinitely long conductor carrying current I as shown in Figure 7.37. The loop and the straight conductors are separated by distance d. 7.33 A semicircular loop of radius a in free space carries a current I.

Determine the magnetic flux density at the center of the loop. 07_Sadiku_Ch07.indd 342 16/11/17 1:15 PM Problems 343 FIGURE 7.37 For Problem 7.32. z I b d a FIGURE 7.38 Cross section of a brass ring enclosing a long straight wire; for Problem 7.35.

7.34 In free space, the magnetic flux density is B 5 y2ax 1 z2ay 1 x2az Wb/m2 (a) Show that B is a magnetic field (b) Find the magnetic flux through x 5 1, 0 , y , 1, 1 , z , 4. (c) Calculate J.

(d) Determine the total magnetic flux through the surface of a cube defined by 0 < x < 2, 0 y 2, 0 z 2. *7.35 A brass ring with triangular cross section encircles a very long straight wire concentrically as in Figure 7.38. If the wire carries a current I, show that the total number of magnetic flux lines in the ring is C 5 moIh a1b c b 2 a ln d 2pb b Calculate C if a 5 30 cm, b 5 10 cm, h 5 5 cm, and I 5 10 A. 07_Sadiku_Ch07.indd 343 16/11/17 1:15 PM 344 CHAPTER 7 MAGNETOSTATIC FIELDS P A FIGURE 7.39 Electric motor pole of Problem 7.36. 7.36 The electric motor shown in Figure 7.39 has field H5 106 sin 2f ar A/m r Calculate the flux per pole passing through the air gap if the axial length of the pole is 20 cm. 20 2 sin faz Wb/m2. Determine the magnetic flux crossing the strip r z = 0, 1 r 2 m, 0 f /4. 7.37 In free space, B 5 2 1 2 3 cos uar 1 3 sin uau Wb/m , find the magnetic flux through the spherical cap r r r 5 1, /3.

7.38 If B 5 7.39 In a hydrogen atom, an electron revolves at velocity 2.2 3 106 m/s. Calculate the magnetic flux density at the center of the electron's orbit. Assume that the radius of the orbit is R 5 5.3 3 10211 m. Section 7.6—Maxwell's Equations 7.40 Consider the following arbitrary fields. Find out which of them can possibly represent an electrostatic or magnetostatic field in free space. (a) A 5 y cos axax 1 1 y 1 e2x 2 az (b) B 5 20 a r r (c) C 5 r2 sin u af 07_Sadiku_Ch07.indd 344 16/11/17 1:15 PM Problems 345 7.41 Reconsider Problem 7.40 for the following fields.

(a) D 5 y2zax 1 2 1 x 1 1 2 yzay 2 1 x 1 1 2 z2az 1z 1 12 sin f cos f ar 1 az r r 1 (c) F 5 2 1 2 cos u ar 1 sin u au 2 r (b) E 5 Section 7.7—Magnetic Scalar and Vector Potentials 7.42 A current element of length L carries current I in the z direction. Show that at a very distant point, A5 Find B. moIL a 4pr z 7.43 In free space, A 5 10 sin yax 1 (4 1 cos x)az Wb/m. Find H and J. 7.44 Given that A 5 2cosu sinu ar 1 3 au Wb/m exists in free space. r3 r (a) show that = ? A 5 0 (b) Find B at point T(1, 30°, 60°) 7.45 For a current distribution in free space, A 5 1 2x2y 1 yz 2 ax 1 1 xy2 2 xz3 2 ay 2 1 6xyz 2 2x2y2 2 az Wb/m (a) Calculate B. (b) Find the magnetic flux through a loop described by x 5 1, 0 , y , 2, 0 , z , 2. (c) Show that = # A 5 0 and = # B 5 0. 7.46 In free space, a small circular loop of current produces A5 k sin uaf r2 where k is a constant. Find B. 7.47 The magnetic vector potential of a current distribution in free space is given by A 5 15e2r sin f az Wb/m Find H at 1 3, p/4, 210 2 . Calculate the flux through r 5 5, 0 # f # p/2, 0 # z # 10. 07_Sadiku_Ch07.indd 345 16/11/17 1:15 PM 346 CHAPTER 7 MAGNETOSTATIC FIELDS 10 sin uaf Wb/m, find H at point (4, 60o, 30o). r 7.49 An infinitely long conductor of radius a carries a uniform current with J 5 Jo az. Show that the magnetic vector potential for r , a is 7.48 Given that A 5 1 A 5 2 moJor2az 4 7.50 Find the B field corresponding to the magnetic vector potential A 5 sin py px cos a 2 2 z 7.51 The magnetic vector potential at a distant point from a small circular loop is given by A5 Ao sin u af Wb/m r2 where Ao is a constant. Determine the magnetic flux density B. 7.52 The magnetic field intensity in a certain conducting medium is H 5 xy2ax 1 x2zay 2 y2zaz A/m (a) Calculate the current density at point P 12, 21, 3 2. 'rv (b) What is at P? 't 7.53 Let A 5 10r2az mWb/m. (a) Find H and J. (b) Determine the total current crossing the surface z 5 1, 0 r 2, 0 2.

7.54 Prove that the magnetic scalar potential at 1 0, 0, z 2 due to a circular loop of radius a shown in Figure 7.8(a) is Vm 5 I z c1 2 2 d 3 2 z 1 a2 4 1/2 7.55 The z-axis carries a filamentary current 12 A along az. Calculate Vm at (4, 30°, 22) if Vm 5 0 at (10, 60°, 7). 7.56 Plane z 5 22 carries a current of 50ay A/m. If Vm 5 0 at the origin, find Vm at (a) 1 22, 0, 5 2 (b) 1 10, 3, 1 2 07_Sadiku_Ch07.indd 346 16/11/17 1:16 PM Problems 347 7.57 Prove in cylindrical coordinates that (a) = 3 1 =V 2 5 0 (b) = # 1 = 3 A 2 5 0 *7.58 If R 5 r 2 rr and R 5 0 R 0 , show that 1 R 1 = 5 2=r 5 2 3 R R R where and are del operators with respect to 1 x, y, z 2 and 1 xr, yr, zr 2 , respectively.

07_Sadiku_Ch07.indd 347 16/11/17 1:16 PM MAGNETIC RESONANCE IMAGING (MRI) Magnetic resonance imaging (MRI), an exciting technique for probing the human body, was introduced into clinical practice by the early 1980s. In less than 10 years, it became a primary diagnostic tool in several clinical areas such as neurology and orthopedics. Improvements in MRI technology and in computer technology have led to continued growth in the clinical capabilities of the technique. No other technique has proven to be so uniquely flexible and dynamic. What is MRI? When placed in a static magnetic field, certain atomic nuclei assume one of two states: one has a higher energy level and the other has a lower energy level. The energy difference between the two states is linearly proportional to the strength of the applied magnetic field. (This is called the Zeeman effect.) Thus, the MRI signals received by a probe can be analyzed to study the properties of the nuclei and their environment. MRI stems from the application of nuclear magnetic resonance (NMR) to radiological imaging. Unlike other imaging techniques, such as X-ray computed tomography, MRI does not require exposure of the subject to ionizing radiation and hence is considered safe. It provides more information than other imaging techniques because MRI signals are sensitive to several tissue parameters.

An MRI machine consists of a magnet and a giant cube 7 feet tall by 7 feet wide by 10 feet long (2 m 2 m 3 m), although new models are rapidly shrinking. There is a horizontal tube running through the magnet from front to back.

The magnets in use today in MRI machines are in the range of 0.5 T to 2 T. (There is no scientific evidence that fields in that range produce harmful effects in humans.) The patient, lying on his or her back, slides into the tube on a special table. Once the body part to be scanned is in the exact center of the magnetic field, the scan can begin. MRI has changed from a curiosity to the technique of choice for a wide variety of diseases in various regions of the human body. It has been lauded as a technique that represents a breakthrough in medical diagnosis. Today, an estimated 60 million MRI scans are performed annually to visualize patients' internal structures and diagnose a number of conditions including tumors, stroke damage, heart and brain diseases, and back problems. 08_Sadiku_Ch08.indd 348 06/11/17 8:38 PM CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES Always be kind to your A and B students.

Someday one of them will return to your campus as a good professor. And also be kind to your C students. Someday one of them will return and build you a two-million dollar science laboratory. —YALE UNIVERSITY PRESIDENT 8.1 INTRODUC TION Having considered the basic laws and techniques commonly used in calculating magnetic field B due to current-carrying elements, we are prepared to study the force a magnetic field exerts on charged particles, current elements, and loops. Such a study is important to problems on electrical devices such as ammeters, voltmeters, galvanometers, cyclotrons, plasmas, motors, and magnetohydrodynamic generators.

The precise definition of the magnetic field, deliberately sidestepped in the preceding chapter, will be given here. The concepts of magnetic moments and dipole will also be considered. Furthermore, we will consider magnetic fields in material media, as opposed to the magnetic fields in vacuum or free space examined in the preceding chapter. The results of Chapter 7 need only some modification to account for the presence of materials in a magnetic field. Further discussions will cover inductors, inductances, magnetic energy, and magnetic circuits. 8.2 FORCES DUE TO MAGNETIC FIELDS There are at least three ways in which force due to magnetic fields can be experienced. The force can be (a) due to a moving charged particle in a B field, (b) on a current element in an external B field, or (c) between two current elements. A. Force on a Charged Particle According to our discussion in Chapter 4, the electric force Fe on a stationary or moving electric charge Q in an electric field is given by Coulomb's experimental law and is related to the electric field intensity E as Fe 5 QE (8.1) This shows that if Q is positive, Fe and E have the same direction.

349 08_Sadiku_Ch08.indd 349 06/11/17 8:38 PM 350 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES A magnetic field can exert force only on a moving charge. From experiments, it is found that the magnetic force Fm experienced by a charge Q moving with a velocity u in a magnetic field B is Fm 5 Qu 3 B (8.2) This clearly shows that Fm is perpendicular to both u and B. From eqs. (8.1) and (8.2), a comparison between the electric force Fe and the magnetic force Fm can be made. We see that Fe is independent of the velocity of the charge and can perform work on the charge and change its kinetic energy. Unlike Fe, Fm depends on the charge velocity and is normal to it. However, Fm cannot perform work because it is at right angles to the direction of motion of the charge 1 Fm # dl 5 0 2 ; it does not cause an increase in kinetic energy of the charge.

The magnitude of Fm is generally small in comparison to Fe except at high velocities. For a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by, F 5 Fe 1 Fm or F 5 Q 1 E 1 u 3 B 2 (8.3) This is known as the Lorentz force equation.1 It relates mechanical force to electrical force. If the mass of the charged particle moving in E and B fields is m, by Newton's second law of motion. F5m du 5 Q 1 E 1 u 3 B 2 (8.4) dt The solution to this equation is important in determining the motion of charged particles in E and B fields. We should bear in mind that in such fields, energy can be transferred only by means of the electric field. A summary on the force exerted on a charged particle is given in Table 8.1. Since eq. (8.2) is closely parallel to eq. (8.1), which defines the electric field, some authors and instructors prefer to begin their discussions on magnetostatics from eq. (8.2), just as discussions on electrostatics usually begin with Coulomb's force law. B. Force on a Current Element To determine the force on a current element I dl of a current-carrying conductor due to the magnetic field B, we modify eq.

(8.2) using the fact that for convection current [see eq. (5.7)]: 1 08_Sadiku_Ch08.indd 350 J 5 rnu (8.5) After Hendrik Lorentz (1853–1928), who first applied the equation of motion in electric fields. 25/09/17 10:32 AM 8.2 Forces due to Magnetic Fields 351 TABLE 8.1 Force on a Charged Particle State of Particle E Field B Field Combined E and B Fields Stationary QE — QE Moving QE Qu 3 B Q1E 1 u 3 B2 From eq. (7.5), we recall the relationship between current elements: I dl 5 K dS 5 J dv (8.6) Combining eqs. (8.5) and (8.6) yields I dl 5 rnu dv 5 dQ u Alternatively, I dl 5 dQ dl dl 5 dQ 5 dQ u dt dt Hence, I dl 5 dQ u (8.7) This shows that an elemental charge dQ moving with velocity u (thereby producing convection current element dQ u) is equivalent to a conduction current element I d l. Thus the force on a current element I d l in a magnetic field B is found from eq. (8.2) by merely replacing Qu by I dl; that is, dF 5 I dl 3 B (8.8) If the current I is through a closed path L or circuit, the force on the circuit is given by F 5 C I dl 3 B (8.9) L In using eq. (8.8) or (8.9), we should keep in mind that the magnetic field produced by the current element I dl does not exert force on the element itself, just as a point charge does not exert force on itself. The B field that exerts force on I dl must be due to another element. In other words, the B field in eq. (8.8) or (8.9) is external to the current element I dl. If instead of the line current element I dl, we have surface current elements K dS or a volume current element J dv, we simply make use of eq. (8.6) so that eq. (8.8) becomes dF 5 K dS 3 B or dF 5 J dv 3 B (8.8) F 5 3 K dS 3 B or F 5 3 J dv 3 B (8.9) while eq. (8.9) becomes 08_Sadiku_Ch08.indd 351 S v 25/09/17 10:32 AM 352 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES From eq. (8.8) The magnetic field B is defined as the force per unit current element. Alternatively, B may be defined from eq. (8.2) as the vector that satisfies Fm /q 5 u 3 B, just as we defined electric field E as the force per unit charge, Fe/q. Both these definitions of B show that B describes the force properties of a magnetic field. C. Force between Two Current Elements Let us now consider the force between two elements I1 dl1 and I2 d l2. According to Biot– Savart's law, both current elements produce magnetic fields. So we may find the force d(dF1) on element I1 dl1 due to the field dB2 produced by element I2 dl2 as shown in Figure 8.1.

From eq.

(8.8), But from Biot–Savart's law, d 1 dF1 2 5 I1 dl1 3 dB2 (8.10) dB2 5 moI2 dl2 3 aR21 4pR221 (8.11) Hence, d 1 dF1 2 5 moI1 dl1 3 1 I2 dl2 3 aR21 2 4pR221 (8.12) This equation is essentially the law of force between two current elements and is analogous to Coulomb's law, which expresses the force between two stationary charges. From eq. (8.12), we obtain the total force F1 on current loop 1 due to current loop 2 shown in Figure 8.1 as F1 5 dl1 3 1 dl2 3 aR21 2 moI1I2 (8.13) C C 4p L1 L2 R221 FIGURE 8.1 Force between two current loops. 08_Sadiku_Ch08.indd 352 25/09/17 10:32 AM 8.2 Forces due to Magnetic Fields 353 Although this equation appears complicated, we should remember that it is based on eq. (8.10). It is eq. (8.9) or (8.10) that is of fundamental importance. The force F2 on loop 2 due to the magnetic field B1 from loop 1 is obtained from eq.

(8.13) by interchanging subscripts 1 and 2.

It can be shown that F2 5 2F1; thus F1 and F2 obey Newton's third law that action and reaction are equal and opposite. It is worthwhile to mention that eq. (8.13) was experimentally established by Oersted and Ampère; Biot and Savart (Ampère's colleagues) actually based their law on it. EXAMPLE 8.1 A charged particle of mass 2 kg and charge 3 C starts at point 1 1, 22, 0 2 with velocity 4ax 1 3az m/s in an electric field 12ax 1 10ay V/m. At time t 5 1 s, determine (a) The acceleration of the particle (b) Its velocity (c) Its kinetic energy (d) Its position Solution: (a) This is an initial-value problem because initial values are given.

According to Newton's second law of motion, F 5 ma 5 QE where a is the acceleration of the particle. Hence, QE 3 5 1 12ax 1 10ay 2 5 18ax 1 15ay m/s2 m 2 du d a5 5 1 ux, uy, uz 2 5 18ax 1 15ay dt at a5 (b) Equating components and then integrating, we obtain dux 5 18 S ux 5 18t 1 A (8.1.1) dt duy 5 15 S uy 5 15t 1 B (8.1.2) dt duz 5 0 S uz 5 C (8.1.3) dt where A, B, and C are integration constants. But at t 5 0, u 5 4ax 1 3az. Hence, ux 1 t 5 0 2 5 4 S 4 5 0 1 A or A 5 4 uy 1 t 5 0 2 5 0 S 0 5 0 1 B or B 5 0 uz 1 t 5 0 2 5 3 S 3 5 C 08_Sadiku_Ch08.indd 353 25/09/17 10:33 AM 354 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES Substituting the values of A, B, and C into eqs. (8.1.1) to (8.1.3) gives u 1 t 5 1 2 5 1 ux, uy, uz 2 5 1 18t 1 4, 15t, 3 2 Hence u 1 t 5 1 s 2 5 22ax 1 15ay 1 3az m/s 1 1 (c) Kinetic energy 1 K.E. 2 5 m 0 u 0 2 5 1 2 2 1 222 1 152 1 32 2 2 2 5 718 J dl d (d) u 5 5 1 x, y, z 2 5 1 18t 1 4, 15t, 3 2 dt dt Equating components yields dx 5 ux 5 18t 1 4 S x 5 9t2 1 4t 1 A1 (8.1.4) dt dy dt 5 uy 5 15t S y 5 7.5t2 1 B1 (8.1.5) dz 5 uz 5 3 S z 5 3t 1 C1 (8.1.6) dt At t 5 0, 1 x, y, z 2 5 1 1, 22, 0 2 ; hence, x1t 5 02 5 1 S 1 5 0 1 A1 z1t 5 02 5 0 S 0 5 0 1 C1 or A1 5 1 y 1 t 5 0 2 5 22 S 22 5 0 1 B1 or B1 5 22 or C1 5 0 Substituting the values of A1, B1, and C1 into eqs. (8.1.4) to (8.1.6), we obtain 1 x, y, z 2 5 1 9t2 1 4t 1 1, 7.5t2 2 2, 3t 2 (8.1.7) Hence, at t 5 1, 1 x, y, z 2 5 1 14, 5.5, 3 2 . By eliminating t in eq. (8.1.7), the motion of the particle may be described in terms of x, y, and z. PRACTICE EXERCISE 8.1 A charged particle of mass 1 kg and charge 2 C starts at the origin with zero initial velocity in a region where E 3az V/m. Find the following: (a) The force on the particle (b) The time it takes to reach point P(0, 0, 12 m) 08_Sadiku_Ch08.indd 354 25/09/17 10:33 AM 8.2 Forces due to Magnetic Fields 355 (c) Its velocity and acceleration at P (d) Its K.E. at P Answer: (a) 6az N, (b) 2 s, (c) 12az m/s, 6az m/s2, (d) 72 J. EXAMPLE 8.2 A charged particle of mass 2 kg and 1 C starts at the origin with velocity 3ay m/s and travels in a region of uniform magnetic field B 5 10az Wb/m2. At t 5 4 s, do the following. (a) Calculate the velocity and acceleration of the particle. (b) Calculate the magnetic force on it. (c) Determine its K.E. and location. (d) Find the particle's trajectory by eliminating t. (e) Show that its K.E. remains constant. Solution: (a) F 5 m du 5 Qu 3 B dt a5 Q du 5 u3B m dt Hence d 1 ax a y 1 uxax 1 uyay 1 uzaz 2 5 † ux uy dt 2 0 0 az uz † 5 5 1 uyax 2 uxay 2 10 By equating components, we get dux 5 5uy (8.2.1) dt duy 5 25ux (8.2.2) dt duz 5 0 S uz 5 Co (8.2.3) dt We can eliminate ux or uy in eqs. (8.2.1) and (8.2.2) by taking second derivatives of one equation and making use of the other. Thus duy d2ux 5 225ux 2 5 5 dt dt or d2ux 1 25ux 5 0 dt2 08_Sadiku_Ch08.indd 355 25/09/17 10:33 AM 356 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES which is a linear differential equation with solution (see Case 3 of Example 6.5) ux 5 C1 cos 5t 1 C2 sin 5t (8.2.4) From eqs. (8.2.1) and (8.2.4), 5uy 5 dux 5 25C1 sin 5t 1 5C2 cos 5t (8.2.5) dt or uy 5 2C1 sin 5t 1 C2 cos 5t We now determine constants Co, C1, and C2 using the initial conditions. At t 5 0, u 5 3ay. Hence, ux 5 0 S 0 5 C1 # 1 1 C2 # 0 S C1 5 0 uy 5 3 S 3 5 2C1 # 0 1 C2 # 1 S C2 5 3 uz 5 0 S 0 5 Co Substituting the values of Co, C1, and C2 into eqs. (8.2.3) to (8.2.5) gives Hence, u 5 1 ux, uy, uz 2 5 1 3 sin 5t, 3 cos 5t, 0 2 (8.2.6) u 1 t 5 4 2 5 1 3 sin 20, 3 cos 20, 0 2 5 2.739ax 1 1.224ay m/s a5 and (b) du 5 1 15 cos 5t, 215 sin 5t, 0 2 dt a 1 t 5 4 2 5 6.121ax 2 13.694ay m/s2 F 5 ma 5 12.2ax 2 27.4ay N or F 5 Qu 3 B 5 1 1 2 1 2.739ax 1 1.224ay 2 3 10az 5 12.2ax 2 27.4ay N (c) K.E. 5 12 m 0 u 0 2 5 12 1 2 2 1 2.7392 1 1.2242 2 5 9 J 08_Sadiku_Ch08.indd 356 ux 5 dx 3 5 3 sin 5t S x 5 2 cos 5t 1 b1(8.2.7) dt 5 25/09/17 10:33 AM 8.2 Forces due to Magnetic Fields 357 dy 3 5 3 cos 5t S y 5 sin 5t 1 b2 (8.2.8) dt 5 dz uz 5 5 0 S z 5 b3 (8.2.9) dt uy 5 where b1, b2, and b3 are integration constants. At t 5 0, 1 x, y, z 2 5 1 0, 0, 0 2 and hence, 3 x 1 t 5 0 2 5 0 S 0 5 2 # 1 1 b1 S b1 5 0.6 5 3# y1t 5 02 5 0 S 0 5 0 1 b 2 S b2 5 0 5 z 1 t 5 0 2 5 0 S 0 5 b3 Substituting the values of b1, b2, and b3 into eqs. (8.2.7) to (8.2.9), we obtain At t 5 4 s, 1 x, y, z 2 5 1 0.6 2 0.6 cos 5t, 0.6 sin 5t, 0 2 (8.2.10) 1 x, y, z 2 5 1 0.3552, 0.5478, 0 2 (d) From eq. (8.2.10), we eliminate t by noting that or 1 x 2 0.6 2 2 1 y2 5 1 0.6 2 2 1 cos2 5t 1 sin2 5t 2 , 1 x 2 0.6 2 2 1 y2 5 1 0.6 2 2, z 50 z50 which is a circle on plane z 5 0, centered at 1 0.6, 0, 0 2 and of radius 0.6 m. Thus the particle gyrates in an orbit about a magnetic field line. (e) K.E. 5 1 1 m 0 u 0 2 5 1 2 2 1 9 cos2 5t 1 9 sin2 5t 2 5 9 J 2 2 which is the same as the K.E. at t 5 0 and t 5 4 s. Thus the uniform magnetic field has no effect on the K.E. of the particle. Note that the angular velocity v 5 QB/m and the radius of the orbit r 5 uo /v, where uo is the initial speed. An interesting application of the idea in this example is found in a common method of focusing a beam of electrons. The method employs a uniform magnetic field directed parallel to the desired beam as shown in Figure 8.2. Each electron emerging from the electron gun follows a helical path and returns to the axis at the same focal point with other electrons.

If the screen of a cathode-ray tube were at this point, a single spot would appear on the screen. 08_Sadiku_Ch08.indd 357 25/09/17 10:33 AM 358 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES S FIGURE 8.2 For Example 8.2: magnetic focusing of a beam of electrons: (a) helical paths of electrons, (b) end view of paths. F PRACTICE EXERCISE 8.2 A proton of mass m is projected into a uniform field B Boaz with an initial velocity aax baz.

(a) Find the differential equations that the position vector r xax yay zaz must satisfy. (b) Show that a solution to these equations is x5 a sin vt, v y5 a cos vt, v z 5 bt where v eBo/m and e is the charge on the proton. (c) Show that this solution describes a circular helix in space. Answer: (a) EXAMPLE 8.3 dy dx dz 5 a cos vt, 5 2a sin vt, 5 b, (b) and (c) Proof. dt dt dt A charged particle moves with a uniform velocity 4ax m/s in a region where E 5 20 ay V/m and B 5 Boaz Wb/m2. Determine Bo such that the velocity of the particle remains constant.

Solution: If the particle moves with a constant velocity, it is implied that its acceleration is zero. In other words, the particle experiences no net force. Hence, 0 5 F 5 ma 5 Q 1 E 1 u 3 B 2 or 0 5 Q 1 20ay 1 4ax 3 Boaz 2 220ay 5 24Boay Thus Bo 5 5. This example illustrates an important principle employed in a velocity filter shown in Figure 8.3. In this application, E, B, and u are mutually perpendicular so that Qu 3 B is 08_Sadiku_Ch08.indd 358 25/09/17 10:33 AM 8.2 Forces due to Magnetic Fields 359 C FIGURE 8.3 For Example 8.3: a velocity filter for charged particles. directed opposite to QE, regardless of the sign of the charge. When the magnitudes of the two vectors are equal, QuB 5 QE or u5 E B This is the required (critical) speed to balance out the two parts of the Lorentz force. Particles with this speed are undeflected by the fields; they are "filtered" through the aperture. Particles with other speeds are deflected down or up, depending on whether their speeds are greater or less than this critical speed. PRACTICE EXERCISE 8.3 Uniform E and B fields are oriented at right angles to each other. An electron moves with a speed of 8 106 m/s at right angles to both fields and passes undeflected through the field. (a) If the magnitude of B is 0.5 mWb/m2, find the value of E. (b) Will this filter work for positive and negative charges and any value of mass?

Answer: (a) 4 kV/m, (b) yes. EXAMPLE 8.4 A rectangular loop carrying current I2 is placed parallel to an infinitely long filamentary wire carrying current I1 as shown in Figure 8.4(a).

Show that the force experienced by the loop is given by F, 5 2 08_Sadiku_Ch08.indd 359 moI1I2b 1 1 c 2 da N 2p ro ro 1 a r 25/09/17 10:33 AM 360 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES FIGURE 8.4 For Example 8.4: (a) rectangular loop inside the field produced by an infinitely long wire, (b) forces acting on the loop and wire. Solution: Let the force on the loop be F, 5 F1 1 F2 1 F3 1 F4 5 I2 C dl2 3 B1 L where F1, F2, F3, and F4 are, respectively, the forces exerted on sides of the loop labeled 1, 2, 3, and 4 in Figure 8.4(b). Owing to the infinitely long wire B1 5 moI1 a 2pro f Hence, F1 5 I2 3 dl2 3 B1 5 I2 3 L 52 b z50 moI1I2b a 2pro r dz az 3 moI1 a 2pro f 1 attractive 2 F1 is attractive because it is directed toward the long wire; that is, F1 is along 2ar because loop side 1 and the long wire carry currents along the same direction. Similarly, F3 5 I2 3 dl2 3 B1 5 I2 3 L 5 ro1a r5ro 08_Sadiku_Ch08.indd 360 z5b moI1I2b ar 2p 1 ro 1 a 2 F2 5 I2 3 0 dr ar 3 dz az 3 1 repulsive 2 moI1 af 2p 1 ro 1 a 2 moI1 af 2pr 25/09/17 10:33 AM 8.3 Magnetic Torque and Moment 361 5 moI1I2 ro 1 a ln az ro 2p F4 5 I2 3 ro dr ar 3 r5ro1a 52 1 parallel 2 moI1af 2pr moI1I2 ro 1 a ln az ro 2p 1 parallel 2 The total force F on the loop is the sum of F1, F2, F3, and F4; that is, F, 5 moI1I2b 1 1 c 2 d 1 2ar 2 2p ro ro 1 a which is an attractive force trying to draw the loop toward the wire. The force Fw on the wire, by Newton's third law, is 2F,; see Figure 8.4(b). PRACTICE EXERCISE 8.4 In Example 8.4, find the force experienced by the infinitely long wire if I1 10 A, I2 5 A, ro 20 cm, a 10 cm, b 30 cm.

Answer: 5ar mN. 8.3 MAGNETIC TORQUE AND MOMENT Now that we have considered the force on a current loop in a magnetic field, we can determine the torque on it. The concept of a current loop experiencing a torque in a magnetic field is of paramount importance in understanding the behavior of orbiting charged particles, dc motors, and generators. If the loop is placed parallel to a magnetic field, it experiences a force that tends to rotate it. The torque T (or mechanical moment of force) on the loop is the vector product of the moment arm r and the force F. That is, T 5 r 3 F (8.14) and its units are newton-meters 1 N # m 2 . Let us apply this to a rectangular loop of length and width w placed in a uniform magnetic field B as shown in Figure 8.5(a). From Figure 8.5(a), we notice that dl is parallel to B along sides AB and CD of the loop and no force is exerted on those sides. Thus 08_Sadiku_Ch08.indd 361 25/09/17 10:33 AM 362 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES C D A B A FIGURE 8.5 (a) Rectangular planar loop in a uniform magnetic field. (b) Cross-sectional view of part (a). F 5 I3 dl 3 B 1 I3 dl 3 B C A B D 5 I3 dz az 3 B 1 I3 dz az 3 B 0 , 0 , or F 5 Fo 2 Fo 5 0 (8.15) where 0 Fo 0 5 IB, because B is uniform. Thus, no force is exerted on the loop as a whole. However, Fo and 2Fo act at different points on the loop, thereby creating a couple. If the normal to the plane of the loop makes an angle a with B, as shown in the cross-sectional view of Figure 8.5(b), the torque on the loop is 0 T 0 5 0 Fo 0 w sin a or T 5 BI,w sin a (8.16) But ,w 5 S, the area of the loop. Hence, T 5 BIS sin a (8.17) We define the quantity m 5 ISan (8.18) as the magnetic dipole moment (in A # m2) of the loop. In eq. (8.18), an is a unit normal vector to the plane of the loop and its direction is determined by the right-hand rule: fingers in the direction of current and thumb along an. 08_Sadiku_Ch08.indd 362 25/09/17 10:33 AM 8.4 A Magnetic Dipole 363 The magnetic dipole moment is the product of current and area of the loop; its direction is normal to the loop. Introducing eq. (8.18) in eq. (8.17), we obtain T 5 m 3 B (8.19) Although this expression was obtained by using a rectangular loop, it is generally applicable in determining the torque on a planar loop of any arbitrary shape. The only limitation is that the magnetic field must be uniform. It should be noted that the torque is in the direction of the axis of rotation (the z-axis in the case of Figure 8.5(a)). It is directed with the aim of reducing a so that m and B are in the same direction. In an equilibrium position (when m and B are in the same direction), the loop is perpendicular to the magnetic field and the torque will be zero as well as the sum of the forces on the loop. 8.4 A MAGNETIC DIPOLE A magnetic dipole consists of a bar magnet or small current-carrying loop. The reason for this and what we mean by "small" will soon be evident. Let us determine the magnetic field B at an observation point P 1 r, u, f 2 due to a circular loop carrying current I as in Figure 8.6. The magnetic vector potential at P is A5 moI dl (8.20) 4p C r It can be shown that in the far field r W a, so that the loop appears small at the observation point, A has only f-component and it is given by A5 moIpa2 sin u af 4pr2 (8.21a) FIGURE 8.6 Magnetic field at P due to a current loop. 08_Sadiku_Ch08.indd 363 25/09/17 10:33 AM 364 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES or A5 mo m 3 ar (8.21b) 4pr2 where m 5 Ipa2az, the magnetic moment of the loop, and az 3 ar 5 sin u af. We determine the magnetic flux density B from B 5 = 3 A as B5 mom 1 2 cos u ar 1 sin u au 2 (8.22) 4pr3 TABLE 8.2 Comparison between Electric and Magnetic Monopoles and Dipoles 08_Sadiku_Ch08.indd 364 25/09/17 10:33 AM 8.4 A Magnetic Dipole 365 It is interesting to compare eqs. (8.21) and (8.22) with similar expressions in eqs. (4.80) and (4.82) for electrical potential V and electric field intensity E due to an electric dipole. This comparison is done in Table 8.2, in which we notice the striking similarities between B in the far field due to a small current loop and E in the far field due to an electric dipole. It is therefore reasonable to regard a small current loop as a magnetic dipole. The B lines due to a magnetic dipole are similar to the E lines due to an electric dipole. Figure 8.7(a) illustrates the B lines around the magnetic dipole m 5 IS. A short permanent magnetic bar, shown in Figure 8.7(b), may also be regarded as a magnetic dipole. Observe that the B lines due to the bar are similar to those due to a small current loop in Figure 8.7(a). Consider the bar magnet of Figure 8.8. If Qm is an isolated magnetic charge (pole strength) and is the length of the bar, the bar has a dipole moment Qm 0. If B1 5 6ax 2 4.2ay 1 1.8az mWb/m2, find H2 and the angle H2 makes with the interface. 8.31 In medium 1 (z , 0) μ1 5 5μo, while in medium 2 (z . 0) μ2 5 2μo. If B1 5 4ax 2 10ay 1 12az mWb/m2, find B2 and the energy density in medium 2. 8.32 In region x , 0, μ 5 μo, a uniform magnetic field makes angle 42 with the normal to the interface. Calculate the angle the field makes with the normal in region x . 0, μ 5 6.5μo. 8.33 A current sheet with K 5 12ay A/m is placed at x 5 0, which separates region 1, x , 0, μ 5 2μo and region 2, x . 0, μ 5 4μo. If H1 5 10ax 1 6az, A/m, find H2. 8.34 Suppose space is divided into region 1 (y 0, 1 or1) and region 2 (y 0, 2 or2). If H1 ax ay az A/m, find H2. 8.35 If m1 5 2mo for region 1 1 0 , f , p 2 and m2 5 5mo for region 2 1 p , f , 2p 2 and B2 5 10ar 1 15af 2 20az mWb/m2. Calculate (a) B1, (b) the energy densities in the two media. *8.36 Region 1 is defined by x 2 y 1 2z 5 with μ1 5 2μo, while region 2 is defined by x 2 y 1 2z 5 with μ2 5 5μo. If H1 5 40ax 1 20ay 2 30az A/m, find (a) H1n, (b) H2t, (c) B2. 08_Sadiku_Ch08.indd 412 06/11/17 6:31 PM Problems 413 8.37 Inside a right circular cylinder, m1 5 800 mo, while the exterior is free space.

Given that B1 5 mo 1 22ar 1 45af 2 Wb/m2), determine B2 just outside the cylinder. 8.38 The plane z 5 0 separates air 1 z $ 0, m 5 mo 2 from iron 1 z # 0, m 5 200mo 2 . Given that H 5 10ax 1 15ay 2 3az A/m in air, find B in iron and the angle it makes with the interface. 8.39 Region 0 # z # 2 m is filled with an infinite slab of magnetic material 1 m 5 2.5mo 2 . If the surfaces of the slab at z 5 0 and z 5 2, respectively, carry surface currents 30ax A/m and 240ax A/m as in Figure 8.40, calculate H and B for (a) z , 0 (b) 0 , z , 2 (c) z . 2 8.40 Medium 1 is free space and is defined by r a, while medium 2 is a magnetic material with permeability μ2 and defined by r a. The magnetic flux densities in the media are: B1 5 Bo1 c a1 1 Find μ2. 1.6a3 0.8a3 bsin uau d 3 bcos uar 2 a1 2 r r3 B2 5 Bo2 1 cos uar 2 sin uau 2 Section 8.8—Inductors and Inductance *8.41 (a) If the cross section of the toroid of Figure 7.15 is a square of side a, show that the self-inductance of the toroid is L5 moN2a 2ro 1 a ln c d 2p 2ro 2 a (b) If the toroid has a circular cross section as in Figure 7.15, show that L5 moN2a2 2ro where ro W a. FIGURE 8.40 For Problem 8.39. 08_Sadiku_Ch08.indd 413 06/11/17 6:31 PM 414 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES 8.42 An air-filled toroid of square cross section has inner radius 3 cm, outer radius 5 cm, and height 2 cm. How many turns are required to produce an inductance of 45 H? 8.43 A wire of radius 2 mm is 40 m long. Calculate its inductance. Assume μ 5 μo. 8.44 A coaxial cable has an internal inductance that is twice the external inductance.

If the inner radius is 6.5 mm, calculate the outer radius.

8.45 A hollow cylinder of radius a 5 2 cm is 10 m long. Find the inductance of the cylinder.

(See Table 8.3.) 8.46 Show that the mutual inductance between the rectangular loop and the infinite line current of Figure 8.4 is M12 5 a 1 ro mb ln c d ro 2p Calculate M12 when a 5 b 5 ro 5 1 m.

*8.47 Prove that the mutual inductance between the close-wound coaxial solenoids of length 1 and 2 1 ,1 W ,2 2 , turns N1 and N2, and radii r1 and r2 with r1 . r2 is M12 5 mN1N2 2 pr1 ,1 *8.48 A loop resides outside the region between two parallel long wires carrying currents in opposite directions as shown in Figure 8.41.

Find the total flux linking the loop. Section 8.9—Magnetic Energy 8.49 A coaxial cable consists of an inner conductor of radius 1.2 cm and an outer conductor of radius 1.8 cm.

The two conductors are separated by an insulating medium 1 m 5 4mo 2 . If the cable is 3 m long and carries 25 mA current, calculate the energy stored in the medium. w I I h a 08_Sadiku_Ch08.indd 414 b FIGURE 8.41 For Problem 8.48. 06/11/17 6:31 PM Problems 415 0.2 A 500 turns L = 42 cm lg = 0.1 cm FIGURE 8.42 For Problem 8.54. FIGURE 8.43 For Problem 8.55. 8.50 In a certain region for which xm 5 19, H 5 5x2yzax 1 10xy2zay 2 15xyz2az A/m How much energy is stored in 0 , x , 1, 0 , y , 2, 21 , z , 2? 8.51 The magnetic field in a material space (μ 5 15μo) is given by B 5 4ax 1 12ay mWb/m2 Calculate the energy stored in region 0 x 2, 0 y 3, 0 z 4. Section 8.10—Magnetic Circuits 8.52 A cobalt ring 1 mr 5 600 2 has a mean radius of 30 cm. If a coil wound on the ring carries 12 A, calculate the number of turns required to establish an average magnetic flux density of 1.5 Wb/m2 in the ring. 8.53 Refer to Figure 8.27. If the current in the coil is 0.5 A, find the mmf and the magnetic field intensity in the air gap. Assume that m 5 500mo and that all branches have the same cross-sectional area of 10 cm2.

8.54 The magnetic circuit of Figure 8.42 has a current of 10 A in the coil of 2000 turns. Assume that all branches have the same cross section of 2 cm2 and that the material of the core is iron with mr 5 1500. Calculate R, , and for (a) The core (b) The air gap I2 I1 N1 N2 FIGURE 8.44 For Problem 8.56. 08_Sadiku_Ch08.indd 415 06/11/17 6:31 PM 416 CHAPTER 8 MAGNETIC FORCES, MATERIALS, AND DEVICES FIGURE 8.45 For Problem 8.58. FIGURE 8.46 For Problem 8.59.

FIGURE 8.47 For Problem 8.60. FIGURE 8.48 For Problem 8.61. 8.55 Consider the magnetic circuit in Figure 8.42. Assuming that the core 1 m 5 1000mo 2 has a uniform cross section of 4 cm2, determine the flux density in the air gap. 8.56 For the magnetic circuit shown in Figure 8.44, draw the equivalent electric circuit. Assume that all the sections have constant cross-sectional areas.

8.57 An air gap in an electric machine has length 4.4 mm and area 4.82 3 1022 m2. Find the reluctance of the gap.

Section 8.11—Force on Magnetic Materials 8.58 An electromagnetic relay is modeled as shown in Figure 8.45. What force is on the armature (moving part) of the relay if the flux in the air gap is 2 mWb? The area of the gap is 0.3 cm2, and its length 1.5 mm. 8.59 A toroid with air gap, shown in Figure 8.46, has a square cross section. A long conductor carrying current I2 is inserted in the air gap.

If I1 5 200 mA, N 5 750, ro 5 10 cm, a 5 5 mm, and ,a 5 1 mm, calculate (a) The force across the gap when I2 5 0 and the relative permeability of the toroid is 300. (b) The force on the conductor when I2 5 2 mA and the permeability of the toroid is infinite. Neglect fringing in the gap in both cases. 08_Sadiku_Ch08.indd 416 06/11/17 6:31 PM Problems 417 8.60 A section of an electromagnet with a plate below it carrying a load as shown in Figure 8.47. The electromagnet has a contact area of 200 cm2 per pole, and the middle pole has a winding of 1000 turns with I 5 3 A. Calculate the maximum mass that can be lifted. Assume that the reluctance of the electromagnet and the plate is negligible. 8.61 Figure 8.48 shows the cross section of an electromechanical system in which the plunger moves freely between two nonmagnetic sleeves. Assuming that all legs have the same cross-sectional area S, show that F52 08_Sadiku_Ch08.indd 417 2 N2I2moS a 1 a 1 2x 2 2 x 06/11/17 6:31 PM 08_Sadiku_Ch08.indd 418 06/11/17 6:31 PM PA R T 4 WAVES AND APPLICATIONS 09_Sadiku_Ch09.indd 419 25/09/17 1:54 PM Michael Faraday (1791–1867), an English chemist and physicist, is known for his pioneering experiments in electricity and magnetism. Many consider him the greatest experimentalist who ever lived. Born at Newington, near London, to a poor family, he received little more than an elementary education. During a seven-year apprenticeship as a bookbinder, Faraday developed his interest in science and in particular chemistry. As a result, Faraday started a second apprenticeship in chemistry. Following in the footsteps of Benjamin Franklin and other early scientists, Michael Faraday studied the nature of electricity. Later in life, Faraday became professor of chemistry at the Royal Institution.

He discovered benzene and formulated the second law of electrolysis. Faraday's greatest contribution to science was in the field of electricity. Faraday's introduction of the concept of lines of force was initially rejected by most of the mathematical physicists of Europe. He discovered electromagnetic induction (to be covered in this chapter), the battery, the electric arc (plasmas), and the Faraday cage (electrostatics).

His biggest breakthrough was his invention of the electric motor and dynamo (or generator). Despite his achievements, Faraday remained a modest and humble person. In his day, Faraday was deeply religious. The unit of capacitance, the farad, is named after him.

James Clerk Maxwell (1831–1879), Scottish mathematician and physicist, published physical and mathematical theories of the electromagnetic field. Born at Edinburgh, Scotland, Maxwell showed an early understanding and love for the field of mathematics. Dissatisfied with the toys he was given, he made his own scientific toys at the age of 8! Maxwell was a true genius who made several contributions to the scientific community, but his most important achievement was his development of the equations of electromagnetic waves, which we now call Maxwell's equations. In 1931, on the centennial anniversary of Maxwell's birth, Einstein described Maxwell's work as the "most profound and the most fruitful that physics has experienced since the time of Newton." Without Maxwell's work, radio and television could not exist. The 1888 announcement by the German physics professor Heinrich Rudolf Hertz (see Chapter 10) that he had transmitted and received electromagnetic waves was almost universally received as a glorious confirmation of Maxwell's equations. The maxwell (Mx), the unit of measurement of magnetic flux in the centimetergram-second (cgs) system of units, was named in his honor.

09_Sadiku_Ch09.indd 420 25/09/17 1:54 PM CHAPTER 9 MAXWELL'S EQUATIONS Some people make enemies instead of friends because it is less trouble. —E. C. MCKENZIE 9.1 INTRODUCTION In Part 2 (Chapters 4–6) of this text, we mainly concentrated our efforts on electrostatic fields denoted by E 1 x, y, z 2 ; Part 3 (Chapters 7 and 8) was devoted to magnetostatic fields represented by H 1 x, y, z 2 . We have therefore restricted our discussions to static, or timeinvariant, EM fields. Henceforth, we shall examine situations in which electric and magnetic fields are dynamic, or time varying. It should be mentioned first that in static EM fields, electric and magnetic fields are independent of each other, whereas in dynamic EM fields, the two fields are interdependent. In other words, a time-varying electric field necessarily involves a corresponding time-varying magnetic field.

Second, time-varying EM fields, represented by E 1 x, y, z, t 2 and H 1 x, y, z, t 2 , are of more practical value than static EM fields. However, familiarity with static fields provides a good background for understanding dynamic fields. Third, recall that electrostatic fields are usually produced by static electric charges, whereas magnetostatic fields are due to motion of electric charges with uniform velocity (direct current) or static magnetic charges (magnetic poles); timevarying fields or waves are usually due to accelerated charges or time-varying currents such as shown in Figure 9.1. Any pulsating current will produce radiation (time-varying fields). It is worth noting that pulsating current of the type shown in Figure 9.1(b) is the cause of radiated emission in digital logic boards. In summary: stationary charges → electrostatic fields steady currents → magnetostatic fields time-varying currents → electromagnetic fields (or waves) Our aim in this chapter is to lay a firm foundation for our subsequent studies. This will involve introducing two major concepts: (1) electromotive force based on Faraday's experiments and (2) displacement current, which resulted from Maxwell's hypothesis.

As a result of these concepts, Maxwell's equations as presented in Section 7.6 and the boundary conditions for static EM fields will be modified to account for the time variation of the fields. Maxwell's equations, which summarize the laws of electromagnetism, shall be 421 09_Sadiku_Ch09.indd 421 25/09/17 1:54 PM 422 CHAPTER 9 MAXWELL'S EQUATIONS t t t FIGURE 9.1 Examples of time-varying current: (a) sinusoidal, (b) rectangular, (c) triangular. the basis of our discussions in the remaining part of the text. For this reason, Section 9.5 should be regarded as the heart of this text. 9.2 FARADAY'S LAW After Oersted's experimental discovery (upon which Biot–Savart and Ampère based their laws) that a steady current produces a magnetic field, it seemed logical to find out whether magnetism would produce electricity. In 1831, about 11 years after Oersted's discovery, Michael Faraday in London and Joseph Henry in New York discovered that a time-varying magnetic field would produce an electric current.1 According to Faraday's experiments, a static magnetic field produces no current flow; but in a closed circuit, a time-varying field produces an induced voltage (called electromotive force or simply emf) that causes a flow of current. Faraday discovered that the induced emf, Vemf (in volts) in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit. This is called Faraday's law, and it can be expressed as Vemf 5 2 dl d 5 2N dt dt (9.1) where l 5 N is the flux linkage, N is the number of turns in the circuit, and is the flux through each turn. The negative sign shows that the induced voltage acts in such a way as 1 For details on the experiments of Michael Faraday (1791–1867) and Joseph Henry (1797–1878), see W.

F. Magie, A Source Book in Physics. Cambridge, MA: Harvard Univ. Press, 1963, pp. 472–519. 09_Sadiku_Ch09.indd 422 25/09/17 1:54 PM 9.2 Faraday's Law 423 FIGURE 9.2 A circuit showing emf-producing field Ef and electrostatic field Ee. B to oppose the flux producing it. This behavior is described as Lenz's law2. Lenz's law states the direction of current flow in the circuit is such that the induced magnetic field produced by the induced current opposes change in the original magnetic field. Recall that we described an electric field as one in which electric charges experience force. The electric fields considered so far are caused by electric charges; in such fields, the flux lines begin and end on the charges. However, electric fields of other kinds are not directly caused by electric charges.

These are emf-produced fields. Sources of emf include electric generators, batteries, thermocouples, fuel cells, and photovoltaic cells, which all convert nonelectrical energy into electrical energy. Consider the electric circuit of Figure 9.2, where the battery is a source of emf. The electrochemical action of the battery results in an emf-produced field Ef. Due to the accumulation of charge at the battery terminals, an electrostatic field Ee 1 5 2=V 2 also exists. The total electric field at any point is E 5 Ef 1 Ee(9.2) Note that Ef is zero outside the battery, Ef and Ee have opposite directions in the battery, and the direction of Ee inside the battery is opposite to that outside it. If we integrate eq. (9.2) over the closed circuit, we have # # # C E dl 5 C Ef dl 1 0 5 3 Ef dl P L L N 1 through battery 2 (9.3a) where A Ee # dl 5 0 because Ee is conservative. The emf of the battery is the line integral of the emf-produced field, that is, Vemf 5 3 Ef # dl 5 23 Ee # dl 5 IR(9.3b) P P N N since Ef and Ee are equal but opposite within the battery (see Figure 9.2).

It may also be regarded as the potential difference 1 VP 2 VN 2 between the battery's open-circuit terminals.

It is important to note the following facts. 1.

An electrostatic field Ee cannot maintain a steady current in a closed circuit, since # AL Ee dl 5 0 5 IR. 2. An emf-produced field Ef is nonconservative. 3.

Except in electrostatics, voltage and potential difference are usually not equivalent. 2 After Heinrich Friedrich Emil Lenz (1804–1865), a Russian professor of physics. 09_Sadiku_Ch09.indd 423 25/09/17 1:54 PM 424 CHAPTER 9 MAXWELL'S EQUATIONS 9.3 TRANSFORMER AND MOTIONAL ELECTROMOTIVE FORCES Having considered the connection between emf and electric field, we may examine how Faraday's law links electric and magnetic fields. For a circuit with a single turn 1 N 5 1 2 , eq. (9.1) becomes d Vemf 5 2 (9.4) dt In terms of E and B, eq. (9.4) can be written as d Vemf 5 C E # dl 5 2 3 B # dS(9.5) dt L S where has been replaced by e S B # dS and S is the surface area of

the circuit bounded by the closed path L. It is clear from eq. (9.5) that in a time-varying situation, both electric and magnetic fields are present and are interrelated. Note that d l and dS in eq. (9.5) are in accordance with the right-hand rule as well as Stokes's theorem. This should be observed in Figure 9.3. The variation of flux with time as in eq. (9.1) or eq.

(9.5) may be caused in three ways: 1. By having a stationary loop in a time-varying B field 2. By having a time-varying loop area in a static B field 3. By having a time-varying loop area in a time-varying B field Each of these will be considered separately. A. Stationary Loop in Time-Varying B Field (Transformer emf) In Figure 9.3 a stationary conducting loop is in a time-varying magnetic B field. Equation (9.5) becomes 'B # Vemf 5 C E # dl 5 23 dS(9.6) L S 't FIGURE 9.3 Induced emf due to a stationary loop in a timevarying B field. I 09_Sadiku_Ch09.indd 424 25/09/17 1:54 PM 9.3 Transformer and Motional Electromotive Forces 425 This emf induced by the time-varying current (producing the time-varying B field) in a stationary loop is often referred to as transformer emf in power analysis, since it is due to transformer action. By applying Stokes's theorem to the middle term in eq. (9.6), we obtain 'B # # dS(9.7) 3 1 = 3 E 2 dS 5 23 S S 't For the two integrals to be equal, their integrands must be equal; that is, =3E52 'B (9.8) 't This is one of the Maxwell's equations for time-varying fields. It shows that the time- varying E field is not conservative 1 = 3 E 2 0 2 . This does not imply that the principles of energy conservation are violated. The work done in taking a charge about a closed path in a time-varying electric field, for example, is due to the energy from the time-varying magnetic field. Observe that Figure 9.3 obeys Lenz's law: the induced current I flows such as to produce a magnetic field that opposes the change in B(t). B. Moving Loop in Static B Field (Motional emf) When a conducting loop is moving in a static B field, an emf is induced in the loop. We recall from eq. (8.2) that the force on a charge moving with uniform velocity u in a magnetic field B is Fm 5 Qu 3 B(8.2) We define the motional electric field Em as Em 5 Fm 5 u 3 B(9.9) Q If we consider a conducting loop, moving with uniform velocity u as consisting of a large number of free electrons, the emf induced in the loop is Vemf 5 C Em # dl 5 C 1 u 3 B 2 # dl(9.10) L L This type of emf is called motional emf or flux-cutting emf because it is due to motional action. It is the kind of emf found in electrical machines such as motors, generators, and alternators. Figure 9.4 illustrates a two-pole dc machine with one armature coil and a twobar commutator. Although the analysis of the dc machine is beyond the scope of this text, we can see that voltage is generated as the coil rotates within the magnetic field. Another 09_Sadiku_Ch09.indd 425 25/09/17 1:54 PM 426 CHAPTER 9 MAXWELL'S EQUATIONS FIGURE 9.4 A direct-current machine. N e xample of motional emf is illustrated in Figure 9.5, where a rod is moving between a pair of rails. In this example, B and u are perpendicular, so eq. (9.9) in conjunction with eq. (8.2) becomes Fm 5 I, , 3 B(9.11) or Fm 5 I,B(9.12) and eq. (9.10) becomes Vemf 5 uB,(9.13) By applying Stokes's theorem to eq. (9.10), we have # # 3 1 = 3 Em 2 dS 5 3 = 3 1 u 3 B 2 dS S or S = 3 Em 5 = 3 1 u 3 B 2 (9.14) Notice that unlike eq. (9.6), there is no need for a minus sign in eq. (9.10) because Lenz's law is already accounted for. FIGURE 9.5 Induced emf due to a moving loop in a static B field. 09_Sadiku_Ch09.indd 426 20/11/17 7:45 PM 9.3 Transformer and Motional Electromotive Forces 427 To apply eq. (9.10) is not always easy; some care must be exercised. The following points should be noted. 1. The integral in eq. (9.10) is zero along the portion of the loop where u 5 0. Thus dl is taken along the portion of the loop that is cutting the field (along the rod in Figure 9.5), where u has nonzero value. 2. The direction of the induced current is the same as that of Em or u 3 B. The limits of the integral in eq. (9.10) are selected in the direction opposite to the induced current, thereby satisfying Lenz's law. In Figure 9.5, for example, the integration over L is along 2ay, whereas induced current flows in the rod along ay. C. Moving Loop in Time-Varying Field In the general case, a moving conducting loop is in a time-varying magnetic field. Both transformer emf and motional emf are present. Combining eqs. (9.6) and (9.10) gives the total emf as 'B # Vemf 5 C E # dl 5 23 dS 1 C 1 u 3 B 2 # dl(9.15) L S 't L or from eqs. (9.8) and (9.14), =3E52 'B 1 = 3 1 u 3 B 2 (9.16) 't Note that eq. (9.15) is equivalent to eq.

(9.4), so Vemf can be found using either eq. (9.15) or (9.4). In fact, eq. (9.4) can always be applied in place of eqs. (9.6), (9.10), and (915). EXAMPLE 9.1 A conducting bar can slide freely over two conducting rails as shown in Figure 9.6. Calculate the induced voltage in the bar (a) If the bar is stationed at y 5 8 cm and B 5 4 cos 106taz mWb/m2 (b) If the bar slides at a velocity u 5 20ay m/s and B 5 4az mWb/m2 (c) If the bar slides at a velocity u 5 20ay m/s and B 5 4 cos 1 106t 2 y 2 az mWb/m2 FIGURE 9.6 For Example 9.1. 09_Sadiku_Ch09.indd 427 25/09/17 1:54 PM 428 CHAPTER 9 MAXWELL'S EQUATIONS Solution: (a) In this case, we have transformer emf given by Vemf 5 23 'B # dS 5 3 3 4 1 1023 2 1 106 2 sin 106t dx dy S 't y50 x50 0.08 0.06 5 4 1 103 2 1 0.08 2 1 0.06 2 sin 106t 5 19.2 sin 106t V The polarity of the induced voltage (according to Lenz's law) is such that point P on the bar is at lower potential than Q when B is increasing. (b) This is the case of motional emf: Vemf 5 3 1 u 3 B 2 # dl 5 3 L 0 x5, 1 uay 3 Baz 2 # dx ax 5 2uB, 5 220A4 3 1023 2 A0.06 2 5 24.8 mV (c) Both transformer emf and motional emf are present in this case.

This problem can be solved in two ways. Method 1: Using eq. (9.15), we write Vemf 5 23 53 'B # dS 1 3 1 u 3 B 2 # dl (9.1.1) S 't L 0.06 x50 13 23 6 6 3 4.10 1 10 2 sin 1 10 t 2 yr 2 dyr dx y 0 0 0.06 3 20ay 3 4.1023 cos 1 106t 2 y 2 az 4 # dx ax y 5 240 cos 1 106t 2 yr 2 ` 2 80 1 1023 2 1 0.06 2 cos 1 106t 2 y 2 0 5 240 cos 1 106t 2 y 2 2 240 cos 106t 2 4.8 1 1023 2 cos 1 106t 2 y 2 . 240 cos 1 106t 2 y 2 2 240 cos 106t (9.1.2) because the motional emf is negligible compared with the transformer emf. Using trigonometric identity, we write cos A 2 cos B 5 22 sin 09_Sadiku_Ch09.indd 428 A1B A2B sin 2 2 y y Vemf 5 2480 sina106t 2 b sin V 2 2 (9.1.3) 25/09/17 1:54 PM 9.3 Transformer and Motional Electromotive Forces 429 Method 2: Alternatively, we can apply eq.

(9.4), namely, Vemf 5 2 ' 't (9.1.4) where 5 3 B # dS y 53 0.06 3 4 cos 1 106t 2 y 2 dx dy y50 x50 5 24 1 0.06 2 sin 1 106t 2 y 2 ` y y50 5 20.24 sin 1 10 t 2 y 2 1 0.24 sin 106t mWb 6 But dy dt 5 u S y 5 ut 5 20t Hence, 5 20.24 sin 1 106t 2 20t 2 1 0.24 sin 106t mWb Vemf 5 2 ' 5 0.24 1 106 2 20 2 cos 1 106t 2 20t 2 2 0.24 1 106 2 cos 106t mV 't . 240 cos 1 106t 2 y 2 2 240 cos 106t V (9.1.5) which is the same result in (9.1.2). Notice that in eq. (9.1.1), the dependence of y on time is taken care of in e 1 u 3 B 2 # dl, and we should not be bothered by it in B/t. Why? Because in computing the transformer emf, the loop is assumed stationary. This is a subtle point one must keep in mind in applying eq. (9.1.1).

For the same reason, the second method is always easier. PRACTICE EXERCISE 9.1 Consider the loop of Figure 9.5. If B 5 0.5az Wb/m2, R 5 20 V, , 5 10 cm, and the rod is moving with a constant velocity of 8ax m/s, find (a) The induced emf in the rod (b) The current through the resistor (c) The motional force on the rod (d) The power dissipated by the resistor. Answer: (a) 0.4 V, (b) 20 mA, (c) 2ax mN, (d) 8 mW. 09_Sadiku_Ch09.indd 429 25/09/17 1:54 PM 430 CHAPTER 9 MAXWELL'S EQUATIONS EXAMPLE 9.2 The loop shown in Figure 9.7 is inside a uniform magnetic field B 5 50ax mWb/m2. If side DC of the loop cuts the flux lines at the frequency of 50 Hz and the loop lies in the yz-plane at time t 5 0, find (a) The induced emf at t 5 1 ms (b) The induced current at t 5 3 ms Solution: (a) Since the B field is time invariant, the induced emf is motional, that is, Vemf 5 3 1 u 3 B 2 # dl L where dl 5 dlDC 5 dz az, u5 r df dlr 5 a 5 rvaf dt dt f r 5 AD 5 4 cm, v 5 2pf 5 100p Because u and dl are in cylindrical coordinates, we transform B into cylindrical coordinates by using eq.

(2.9): where Bo 5 0.05. Hence, B 5 Boax 5 Bo 1 cos f ar 2 sin f af 2 ar u 3 B 5 †0 Bo cos f af rv 2Bo sin f az 0 † 5 2rvBo cos f az 0 FIGURE 9.7 For Example 9.2; polarity is for increasing emf. 09_Sadiku_Ch09.indd 430 25/09/17 1:54 PM 9.3 Transformer and Motional Electromotive Forces 431 and 1 u 3 B 2 # dl 5 2rvBo cos f dz 5 20.04 1 100p 2 1 0.05 2 cos f dz 5 20.2p cos f dz Vemf 5 3 0.03 z50 2 0.2p cos f dz 5 26p cos f mV To determine f, recall that v5 df S f 5 vt 1 Co dt where Co is an integration constant. At t 5 0, f 5 p/2 because the loop is in the yz-plane at that time, Co 5 p/2. Hence, f 5 vt 1 p 2 and Vemf 5 26p cosavt 1 p b 5 6p sin 1 100pt 2 mV 2 At t 5 1 ms, Vemf 5 6p sin 1 0.1p 2 5 5.825 mV (b) The current induced is i5 At t 5 3 ms, Vemf 5 60p sin 1 100pt 2 mA R i 5 60p sin 1 0.3p 2 mA 5 0.1525 A PRACTICE EXERCISE 9.2 Rework Example 9.2 with everything the same except that the B field is changed to: (a) B 5 50ay mWb/m2—that is, the magnetic field is oriented along the y-direction (b) B 5 0.02tax Wb/m2—that is, the magnetic field is time varying. Answer: (a) 217.93 mV, 20.1108 A, (b) 20.5 mV, 241.92 mA. EXAMPLE 9.3 09_Sadiku_Ch09.indd 431 The magnetic circuit of Figure 9.8 has a uniform cross section of 1023 m2. If the circuit is energized by a current i1 1 t 2 5 3 sin 100pt A in the coil of N1 5 200 turns, find the emf induced in the coil of N2 5 100 turns. Assume that m 5 500 mo. 25/09/17 1:54 PM 432 CHAPTER 9 MAXWELL'S EQUATIONS FIGURE 9.8 Magnetic circuit of Example 9.3. Solution: The flux in the circuit is 5 N1i1mS N1i1 5 5 ,/mS 2pro According to Faraday's law, the emf induced in the second coil is V2 5 2N2 52 N1N2mS di1 d 52 dt 2pro dt 100 # 1 200 2 # 1 500 2 # 1 4p 3 1027 2 # 1 1023 2 # 300p cos 100pt 2p 1 10 3 1022 2 5 26p cos 100pt V PRACTICE EXERCISE 9.3 A magnetic core of uniform cross section 4 cm2 is connected to a 120 V, 60 Hz generator as shown in Figure 9.9. Calculate the induced emf V2 in the secondary coil. Answer: 72 V. FIGURE 9.9 For Practice Exercise 9.3. 09_Sadiku_Ch09.indd 432 25/09/17 1:54 PM 9.4 Displacement Current 433 9.4 DISPLACEMENT CURRENT In Section 9.3 we have essentially reconsidered Maxwell's curl equation for electrostatic fields and modified it for time-varying situations to satisfy Faraday's law. We shall now reconsider Maxwell's curl equation for magnetic fields (Ampère's circuit law) for timevarying conditions. For static EM fields, we recall that = 3 H 5 J(9.17) But the divergence of the curl of any vector field is identically zero (see Example 3.10). Hence, = # 1 = 3 H 2 5 0 5 = # J(9.18) The continuity of current in eq. (5.43), however, requires that =#J52 'rv 2 0(9.19) 't Thus eqs. (9.18) and (9.19) are obviously incompatible for time-varying conditions. We must modify eq. (9.17) to agree with eq. (9.19). To do this, we add a term to eq. (9.17) so that it becomes = 3 H 5 J 1 Jd(9.20) where Jd is to be determined and defined. Again, the divergence of the curl of any vector is zero. Hence: = # 1 = 3 H 2 5 0 5 = # J 1 = # Jd(9.21) In order for eq.

(9.21) to agree with eq. (9.19), or 'rv ' 'D 1= # D2 5 = # (9.22a) 5 't 't 't Jd 5 = # Jd 5 2= # J 5 'D (9.22b) dt Substituting eq.

(9.22b) into eq.

(9.20) results in =3H5J1 'D (9.23) 't This is Maxwell's equation (based on Ampère's circuit law) for a time-varying field. The term Jd 5 'D/'t is known as displacement current density and J is the conduction current 09_Sadiku_Ch09.indd 433 25/09/17 1:54 PM 434 CHAPTER 9 MAXWELL'S EQUATIONS density 1 J 5 sE 2 .3 The insertion of Jd into eq. (9.17) was one of the major contributions of Maxwell. Without the term Jd, the propagation of electromagnetic waves (e.g., radio or TV waves) would be impossible. At low frequencies, Jd is usually neglected compared with J.

However, at radio frequencies, the two terms are comparable. At the time of Maxwell, high-frequency sources were not available and eq. (9.23) could not be verified experimentally. It was years later that Hertz succeeded in generating and detecting radio waves, thereby verifying eq. (9.23). This is one of the rare cases of a mathematical argument paving the way for experimental investigation. Based on the displacement current density, we define the displacement current as 'D # Id 5 3 Jd # dS 5 3 dS(9.24) S S 't We must bear in mind that displacement current is a result of time-varying electric field. A typical example of such current is the current through a capacitor when an alternating voltage source is applied to its plates. This example, shown in Figure 9.10, serves to illustrate the need for the displacement current. Applying an unmodified form of Ampère's circuit law to a closed path L shown in Figure 9.10(a) gives # # C H dl 5 3 J dS 5 Ienc 5 I (9.25) L S1 where I is the current through the conductor and S1 is the flat surface bounded by L. If we use the balloon-shaped surface S2 that passes between the capacitor plates, as in Figure 9.10(b), # # C H dl 5 3 J dS 5 Ienc 5 0(9.26) L S2 because no conduction current 1 J 5 0 2 flows through S2.

This is contradictory in view of the fact that the same closed path L is used. To resolve the conflict, we need to include FIGURE 9.10 Two surfaces of integration showing the need for Jd in Ampère's circuit law. 3 09_Sadiku_Ch09.indd 434 Recall that we also have J 5 rvu as the convection current density.

25/09/17 1:54 PM 9.4 Displacement Current 435 the displacement current in Ampère's circuit law. The total current density is J 1 Jd. In eq.

(9.25), Jd 5 0, so that the equation remains valid. In eq. (9.26), J 5 0, so that dQ d # # # C H dl 5 3 Jd dS 5 dt 3 D dS 5 dt 5 I (9.27) L S2 S2 So we obtain the same current for either surface, although it is conduction current in S1 and displacement current in S2. EXAMPLE 9.4 A parallel-plate capacitor with plate area of 5 cm2 and plate separation of 3 mm has a voltage 50 sin 103t V applied to its plates. Calculate the displacement current assuming e 5 2eo. Solution: D 5 eE 5 e Jd 5 V d 'D e dV 5 't d dt Hence, eS dV dV Id 5 Jd # S 5 5C d dt dt which is the same as the conduction current, given by drs dQ dD dE eS dV dV 5S 5S 5 eS 5 5C dt dt dt dt d dt dt 29 24 10 5 3 10 # # 103 3 50 cos 103t Id 5 2 # 36p 3 3 1023 5 147.4 cos 103t nA Ic 5 PRACTICE EXERCISE 9.4 In free space, E 5 20 cos 1 vt 2 50x 2 ay V/m. Calculate (a) Jd (b) H (c) v Answer: (a) 220veo sin 1 vt 2 50x 2 ay A/m2, (b) 0.4 veo cos 1 vt 2 50x 2 az A/m, (c) 1.5 3 1010 rad/s. 09_Sadiku_Ch09.indd 435 25/09/17 1:55 PM 436 CHAPTER 9 MAXWELL'S EQUATIONS 9.5 MAXWELL'S EQUATIONS IN FINAL FORMS The Scottish physicist James Clerk Maxwell (1831–1879) is regarded as the founder of electromagnetic theory in its present form. Maxwell's celebrated work led to the discovery of electromagnetic waves.4 Through his theoretical efforts when he was between 35 and 40 years old, Maxwell published the first unified theory of electricity and magnetism. The theory comprised all previously known results, both experimental and theoretical, on electricity and magnetism. It further introduced displacement current and predicted the existence of electromagnetic waves. Maxwell's equations were not fully accepted by many scientists until 1888, when they were confirmed by Heinrich Rudolf Hertz (1857–1894). The German physicist was successful in generating and detecting radio waves. The laws of electromagnetism that Maxwell put together in the form of four equations were presented in Table 7.2 in Section 7.6 for static conditions. The more generalized forms of these equations are those for time-varying conditions shown in Table 9.1. We notice from the table that the divergence equations remain the same, while the curl equations have been modified. The integral form of Maxwell's equations depicts the underlying physical laws, whereas the differential form is used more frequently in solving problems. For a field to "qualify" as an electromagnetic field, it must satisfy all four Maxwell's equations. The importance of Maxwell's equations cannot be overemphasized because they summarize all known laws of electromagnetism. We shall often refer to them in the remainder of this text.

Since this section is meant to be a compendium of our discussion in this text, it is worthwhile to mention other equations that go hand in hand with Maxwell's equations. The Lorentz force equation F 5 Q 1 E 1 u 3 B 2 (9.28) TABLE 9.1 Generalized Forms of Maxwell's Equations Differential Form Integral Form Remarks = # D 5 rv # C D dS 5 3 rv dv Gauss's law # C B dS 5 0 Nonexistence of isolated magnetic charge* S =#B50 v S =3E52 ' # # C E d l 5 2 't 3 B dS 'B 't =3H5J1 L 'D 't Faraday's law S 'D # # C H d l 5 3 aJ 1 't b dS L S Ampère's circuit law *This is also referred to as Gauss's law for magnetic fields. 4 09_Sadiku_Ch09.indd 436 Maxwell's work can be found in his two-volume Treatise on Electricity and Magnetism (New York: Dover, 1954). 25/09/17 1:55 PM 9.5 Maxwell's Equations in Final Forms 437 is associated with Maxwell's equations. Also the equation of continuity =#J52 'rv (9.29) 't is implicit in Maxwell's equations. The concepts of linearity, isotropy, and homogeneity of a material medium still apply for time-varying fields; in a linear, homogeneous, and isotropic medium characterized by s, «, and m, the constitutive relations D 5 eE 5 eoE 1 P B 5 mH 5 mo 1 H 1 M 2 (9.30b) J 5 sE 1 rvu (9.30a) (9.30c) hold for time-varying fields. Consequently, the boundary conditions remain valid for timevarying fields, where an is the unit normal vector to the boundary. 1 E1 2 E2 2 3 an 5 0(9.31a) E1t 2 E2t 5 0 H1t 2 H2t 5 K or 1 H1 2 H2 2 3 an 5 K (9.31b) or D1n 2 D2n 5 rs or 1 D1 2 D2 2 # an 5 rs (9.31c) B1n 2 B2n 5 0 or 1 B 2 2 B 1 2 # an 5 0 (9.31d) However, for a perfect conductor 1 s . ` 2 in a time-varying field, E 5 0, H 5 0, J 5 0 (9.32) and hence, Bn 5 0, Et 5 0 (9.33) For a perfect dielectric 1 s . 0 2 , eqs.

(9.31) hold except that K 5 0. Though eqs. (9.28) to (9.33) are not Maxwell's equations, they are associated with them.

To complete this summary section, we present a structure linking the various potentials and vector fields of the electric and magnetic fields in Figure 9.11. This electromagnetic flow diagram helps with the visualization of the basic relationships between field quantities. It also shows that it is usually possible to find alternative formulations, for a given problem, in a relatively simple manner. It should be noted that in Figure 9.11(b) and (c), we introduce rm as the free magnetic density (similar to rv), which is, of course, zero, Ae as the electric vector potential (analogous to A), and Jm as the magnetic current density (analogous to J). Using terms from stress analysis, the principal relationships are typified as follows: (a) compatibility equations 09_Sadiku_Ch09.indd 437 = # B 5 rm 5 0 (9.34) 25/09/17 1:55 PM 438 CHAPTER 9 MAXWELL'S EQUATIONS FIGURE 9.11 Electromagnetic flow diagrams showing the relationship between the potentials and vector fields: (a) electrostatic system, (b) magnetostatic system, (c) electromagnetic system. [Adapted with permission from the Publishing Department of the Institution of Electrical Engineers.] and =3E52 'B 5 Jm(9.35) 't (b) constitutive equations B 5 mH(9.36) and D 5 eE(9.37) (c) equilibrium equations = # D 5 rv(9.38) and 09_Sadiku_Ch09.indd 438 =3H5J1 'D (9.39) 't 25/09/17 1:55 PM 9.6 Time-Varying Potentials 439 † 9.6 TIME-VARYING POTENTIALS For static EM fields, we obtained the electric scalar potential as V53 rv dv (9.40) 4peR v and the magnetic vector potential as A53 mJ dv (9.41) v 4pR We would like to examine what happens to these potentials when the fields are time varying.

Recall that A was defined from the fact that = # B 5 0, which still holds for time- varying fields. Hence the relation B 5 = 3 A(9.42) holds for time-varying situations. Combining Faraday's law as expressed in eq. (9.8) with eq. (9.42) gives =3E52 or ' 1 = 3 A 2 (9.43a) 't = 3 aE 1 'A b 5 0(9.43b) 't Since the curl of the gradient of a scalar field is identically zero (see Practice Exercise 3.10), the solution to eq. (9.43b) is E1 'A 5 2=V (9.44) 't or E 5 2=V 2 'A (9.45) 't From eqs. (9.42) and (9.45), we can determine the vector fields B and E, provided the potentials A and V are known. However, we still need to find some expressions for A and V similar to those in eqs. (9.40) and (9.41) that are suitable for time-varying fields. From Table 9.1 or eq. (9.38) we know that = # D 5 rv is valid for time-varying conditions. By taking the divergence of eq. (9.45) and making use of eqs. (9.37) and (9.38), we obtain =#E5 09_Sadiku_Ch09.indd 439 rv ' 1= # A2 5 2=2V 2 e 't 25/09/17 1:55 PM 440 CHAPTER 9 MAXWELL'S EQUATIONS or =2V 1 r ' 1 = # A 2 5 2 v (9.46) e 't Taking the curl of eq. (9.42) and incorporating eqs.

(9.23) and (9.45) results in = 3 = 3 A 5 mJ 1 em ' 'A a2=V 2 b 't 't 5 mJ 2 me = a 'V '2A b 2 me 2 't 't (9.47) where D 5 eE and B 5 mH have been assumed. By applying the vector identity to eq. (9.47), = 3 = 3 A 5 = 1 = # A 2 2 =2A(9.48) =2A 2 = 1 = # A 2 5 2mJ 1 me = a 'V '2A b 1 me 2 (9.49) 't 't A vector field is uniquely defined when its curl and divergence are specified. The curl of A has been specified by eq. (9.42); for reasons that will be obvious shortly, we may choose the divergence of A as = # A 5 2me 'V (9.50) 't This choice relates A and V, and it is called the Lorenz condition for potentials.5 We had this in mind when we chose = # A 5 0 for magnetostatic fields in eq. (7.59). By imposing the Lorenz condition of eq. (9.50), eqs. (9.46) and (9.49), respectively, become =2V 2 me rv '2V 2 5 2 e (9.51) 't =2A 2 me '2 A 5 2mJ(9.52) 't2 and which are wave equations to be discussed in the next chapter. The reason for choosing the Lorenz condition becomes obvious as we examine eqs. (9.51) and (9.52). The Lorenz condition uncouples eqs.

(9.46) and (9.49) and also produces a symmetry between eqs. (9.51) and (9.52). It can be shown that the Lorenz condition can be obtained from the continuity equation; therefore, our choice of eq. (9.50) is not arbitrary. Notice that eqs. (6.4) and (7.60) are special static cases of eqs. (9.51) and (9.52), respectively. In other words, potentials V and A satisfy Poisson's equations for time-varying conditions. Just as 5 Not to be confused with Hendrick A. Lorentz, Ludvig V.

Lorenz (1829–1891) was a Danish m athematician and physicist. 09_Sadiku_Ch09.indd 440 25/09/17 1:55 PM 9.7 Time-Harmonic Fields 441 eqs. (9.40) and (9.41) are the solutions, or the integral forms of eqs. (6.4) and (7.60), it can be shown that the solutions6 to eqs. (9.51) and (9.52) are V53 v and 3 rv 4 dv 4peR (9.53) A53 m 3 J 4 dv v 4pR (9.54) The term [rv] (or [J]) means that the time t in rv 1 x, y, z, t 2 [or J 1 x, y, z, t 2] is replaced by the retarded time t given by R tr 5 t 2 (9.55) u where R 5 0 r 2 rr 0 is the distance between the source point r and the observation point r and u5 1 "me (9.56) is the velocity of wave propagation. In free space, u 5 c . 3 3 108 m/s is the speed of light in a vacuum. Potentials V and A in eqs. (9.53) and (9.54) are, respectively, called the retarded electric scalar potential and the retarded magnetic vector potential. Given rv and J, V and A can be determined by using eqs.

(9.53) and (9.54); from V and A, E and B can be determined by using eqs. (9.45) and (9.42), respectively. 9.7 TIME-HARMONIC FIELDS So far, our time dependence of EM fields has been arbitrary. To be specific, we shall assume that the fields are time harmonic. A time-harmonic field is one that varies periodically or sinusoidally with time. Not only is sinusoidal analysis of practical value, but also it can be extended to most waveforms by Fourier analysis. Sinusoids are easily expressed in phasors, which are more convenient to work with. Before applying phasors to EM fields, it is worthwhile to have a brief review of the concept of phasor. A phasor is a complex number that contains the amplitude and the phase of a sinusoidal oscillation. As a complex number, a phasor z can be represented as z 5 x 1 jy 5 r lf(9.57) 6 For example, see D. K.

Cheng, Fundamentals of Engineering Electromagnetics.

Reading, MA: A ddison-Wesley, 1993, pp. 253–254. 09_Sadiku_Ch09.indd 441 25/09/17 1:55 PM 442 CHAPTER 9 MAXWELL'S EQUATIONS or z 5 r e jf 5 r 1 cos f 1 j sin f 2 (9.58) where j 5 "21, x is the real part of z, y is the imaginary part of z, r is the magnitude of z, given by and f is the phase of z, given by r 5 0 z 0 5 "x2 1 y2(9.59) y f 5 tan21 (9.60) x Here x, y, z, r, and f should not be mistaken as the coordinate variables, although they look similar (different letters could have been used but it is hard to find better ones). The phasor z can be represented in rectangular form as z 5 x 1 jy or in polar form as z 5 r lf 5 r e jf. The two forms of representing z are related in eqs. (9.57) to (9.60) and illustrated in Figure 9.12. Addition and subtraction of phasors are better performed in rectangular form; multiplication and division are better done in polar form. Given complex numbers z 5 x 1 jy 5 r lf, z1 5 x1 1 jy1 5 r1 lf1, and z2 5 x2 1 jy2 5 r2 lf2 the following basic properties should be noted. addition: subtraction: multiplication: division: z1 1 z2 5 1 x1 1 x2 2 1 j 1 y1 1 y2 2 (9.61a) z1 2 z2 5 1 x1 2 x2 2 1 j 1 y1 2 y2 2 (9.61b) z1z2 5 r1r2 l f1 1 f2 iiii z1 r1 5 l f 1 2 f2 z2 r2 iiii (9.61c) (9.61d) FIGURE 9.12 Representation of a phasor z 5 x 1 jy 5 r lf. 09_Sadiku_Ch09.indd 442 25/09/17 1:55 PM 9.7 Time-Harmonic Fields 443 Square root: Complex conjugate: "z 5 "r l f/2 ii (9.61e) z* 5 x 2 jy 5 r l2f 5 re2jf(9.61f) ii Other properties of complex numbers can be found in Appendix A.2. To introduce the time element, we let 5 t 1 (9.62) where u may be a function of time or space coordinates or a constant. The real (Re) and imaginary (Im) parts of rej 5 re j(t1) 5 re j e jt (9.63) are respectively given by Re re jf 5 r cos(t 1) (9.64a) and Imre jf 5 r sin(t 1) (9.64b) Thus, a sinusoidal current I(t) 5 Io cos(t 1), for example, equals the real part of Ioe je jt. The current I(t) 5 Io sin(t 1), which is the imaginary part of Ioe je jt, can be represented as the real part of Ioe je jte 2j90 because sin 5 cos(2 90). However, in performing our mathematical operations, we must be consistent in our use of either the real part or the imaginary part of a quantity, but never both at the same time. The complex term Ioe j, which results from dropping the time factor e jt in I(t), is called the phasor current, denoted by Is; that is, Is 5 Ioe j 5 Io (9.65) where the subscript s denotes the phasor form of I(t). Thus I(t) 5 Io cos(t 1), the instantaneous form, can be expressed as I(t) 5 Re Ise jt (9.66) In general, a phasor is a complex quantity and could be a scalar or a vector. If a vector A(x, y, z, t) is a time-harmonic field, the phasor form of A is As(x, y, z); the two quantities are related as A(x, y, z, t) 5 ReAs (x, y, z)e jt (9.67) Note that the phasor is a function of position, not a function of time. For example, if A 5 Ao cos 1 vt 2 bx 2 ay, we can write A as 09_Sadiku_Ch09.indd 443 A 5 Re 1 Aoe2jbxaye jvt 2 (9.68) 25/09/17 1:55 PM 444 CHAPTER 9 MAXWELL'S EQUATIONS Comparing this with eq. (9.67) indicates that the phasor form of A is As 5 Aoe2jbxay(9.69) Notice from eq. (9.67) that ' 'A 5 Re 1 Ase jvt 2 't 't 5 Re 1 jvAse jvt 2 (9.70) showing that taking the time derivative of the instantaneous quantity is equivalent to multiplying its phasor form by jv. That is, 'A S jvAs(9.71) 't Similarly, 3 A 't S As (9.72) jv Note that the real part is chosen in eq. (9.67) as in circuit analysis; the imaginary part could equally have been chosen.

Also notice the basic difference between the instantaneous form A 1 x, y, z, t 2 and its phasor form As 1 x, y, z 2 : the former is time dependent and real, whereas the latter is time invariant and generally complex. It is easier to work with As and obtain A from As whenever necessary by using eq. (9.67). We shall now apply the phasor concept to time-varying EM fields. The field quantities E 1 x, y, z, t 2 , D 1 x, y, z, t 2 , H 1 x, y, z, t 2 , B 1 x, y, z, t 2 , J 1 x, y, z, t 2 , and rv 1 x, y, z, t 2 and their derivatives can be expressed in phasor form by using eqs. (9.67) and (9.71). Let us see how we can write Maxwell's equations in phasor form. For example, consider 3 E(x, y, z, t) 5 2 ' B(x, y, z, t)(9.73) 't We let E(x, y, z, t) 5 ReEs(x, y, z)e jt and B(x, y, z, t) 5 ReBs(x, y, z)e jt Substituting these in eq. (9.73) gives = 3 Re 3 Esejvt 4 f 5 2 ' e Re 3 Bsejvt 4 f 't (9.74) We consider the left-hand side of eq.

(9.74). The curl operation operates only on (x, y, z), 09_Sadiku_Ch09.indd 444 = 3 e Re 3 Esejvt 4 f 5 Re e 3 = 3 Es 4 ejvt f (9.75) 25/09/17 1:55 PM 9.7 Time-Harmonic Fields 445 We similarly consider the right-hand side of eq. (9.74), keeping in mind that Bs does not depend on time: 2 ' ' e Re 3 Bsejvt 4 f 5 2Re e Bs ejvt f 5 2Re 5 jvBsejvt 6 't 't (9.76) Comparing eqs. (9.75) and (9.76), we obtain = 3 Es 5 2jvBs(9.77) which is the phasor form of eq. (9.73). Other Maxwell's equations can be treated in a similar manner, and we obtain Table 9.2. From Table 9.2, note that the time factor e jvt disappears because it is associated with every term and therefore factors out, resulting in time-independent equations. Herein lies the justification for using phasors: the time factor can be suppressed in our analysis of time-harmonic fields and inserted when necessary. Also note that in Table 9.2, the time factor e jvt has been assumed. It is equally possible to have assumed the time factor e2jvt, in which case we would need to replace every j in Table 9.2 with 2j. TABLE 9.2 Time-Harmonic Maxwell's Equations Assuming Time Factor e jvt Point Form Integral Form = # Ds 5 rvs # C Ds dS 5 3 rvs dv = # Bs 5 0 # C Bs dS 5 0 = 3 Es 5 2jvBs # # C Es d l 5 2jv 3 Bs dS # # C Hs d l 5 3 1 Js 1 jvDs 2 dS = 3 Hs 5 Js 1 jvDs EXAMPLE 9.5 Evaluate the complex numbers (a) z1 5 j 1 3 2 j4 2 * 1 21 1 j6 2 1 2 1 j 2 2 (b) z2 5 c 11j 4 2 j8 d 1/2 Solution: (a) This can be solved in two ways: working with z in rectangular form or polar form. Method 1 (working in rectangular form): Let z1 5 09_Sadiku_Ch09.indd 445 z3z4 z5z6 25/09/17 1:55 PM 446 CHAPTER 9 MAXWELL'S EQUATIONS where z3 5 j z4 5 1 3 2 j4 2 * 5 the complex conjugate of 1 3 2 j4 2 5 3 1 j4 We note parenthetically that one can find the complex conjugate of a complex number simply by replacing every j with 2j: z5 5 21 1 j6 and Hence, z6 5 1 2 1 j 2 2 5 4 2 1 1 j4 5 3 1 j4 z3z4 5 j 3 1 j4 2 5 24 1 j3 z5z6 5 1 21 1 j6 2 1 3 1 j4 2 5 23 2 j4 1 j18 2 24 5 227 1 j14 Multiplying and dividing z1 by 227 2 j14 (rationalization), we have z1 5 1 24 1 j3 2 1 227 2 j14 2 150 2 j25 5 2 1 227 1 j14 2 1 227 2 j14 2 27 1 142 5 0.1622 2 j0.027 5 0.1644 l29.46° iii Method 2 (working in polar form): Hence, z3 5 j 5 1 l 90° ii z4 5 1 3 2 j4 2 * 5 1 5 l 253.132 * 5 5 l 53.13° iiii iii z5 5 1 21 1 j6 2 5 "37 l 99.46° iii z6 5 1 2 1 j 2 2 5 1 "5 l 26.56 2 2 5 5 l 53.13° iii iii 1 1 l 90° 2 1 5 l 53.13° 2 i i iii 1 "37 l 99.46° 2 1 5 l 53.13° 2 iii iii 1 5 l 90° 2 99.46° 5 0.1644 l29.46° iii i "37 iiiiii z1 5 5 0.1622 2 j0.027 as obtained before. 09_Sadiku_Ch09.indd 446 25/09/17 1:55 PM 9.7 Time-Harmonic Fields 447 (b) Let z2 5 c where z7 5 1 1 j 5 "2 l 45° ii and Hence z7 1/2 d z8 z8 5 4 2 j8 5 4"5 l 263.4° iii "2 l 45° z7 ii 5 "2 l45° 2 263.4° 5 z8 4"5 iiiii 4"5 l 263.4° iii 5 0.1581 l 108.4° iii and z2 5 "0.1581 l 108.4°/2 iiii 5 0.3976 l54.2° ii PRACTICE EXERCISE 9.5 Evaluate these complex numbers: 11j 2 (a) j3 c d 22j (b) 6 l30° 1 j5 2 3 1 e j45° Answer: (a) 0.24 1 j0.32, (b) 2.03 1 j8.707. EXAMPLE 9.6 Given that A 5 10 cos 1 108t 2 10x 1 60° 2 az and Bs 5 1 20/j 2 ax 1 10 e j2px/3 ay, express A in phasor form and Bs in instantaneous form. Solution: where v 5 108. Hence A 5 Re 3 10e j1vt210x160°2az 4 A 5 Re 3 10e j160º210x2 az e jvt 4 5 Re 1 Ase jvt 2 or As 5 10 e j160°210x2az 09_Sadiku_Ch09.indd 447 25/09/17 1:55 PM 448 CHAPTER 9 MAXWELL'S EQUATIONS If Bs 5 20 a 1 10e j2px/3ay 5 2j20ax 1 10e j2px/3ay j x 5 20e2jp/2ax 1 10e j2px/3ay B 5 Re 1 Bse jvt 2 5 Re c20e j1vt2p/22ax 1 10e j1vt12px/32ayd 5 20 cos 1 vt 2 p/2 2 ax 1 10 cosavt 1 5 20 sin vt ax 1 10 cosavt 1 2px bay 3 2px bay 3 PRACTICE EXERCISE 9.6 If P 5 2 sin 1 10t 1 x 2 p/4 2 ay and Qs 5 e jx 1 ax 2 az 2 sin py, determine the phasor form of P and the instantaneous form of Qs. 2e j1x23p/42ay, sin p y cos 1 vt 1 x 2 1 ax 2 az 2 . Answer: EXAMPLE 9.7 The electric field and the magnetic field in free space are given by E5 H5 50 cos 1 106t 1 bz 2 af V/m r Ho cos 1 106t 1 bz 2 ar A/m r Express these in phasor form and determine the constants Ho and b such that the fields satisfy Maxwell's equations. Solution: The instantaneous forms of E and H are written as E 5 Re 1 Ese jvt 2 , H 5 Re 1 Hse jvt 2 (9.7.1) where v 5 106 and phasors Es and Hs are given by 09_Sadiku_Ch09.indd 448 Es 5 50 jbz e af, r Hs 5 Ho jbz e ar(9.7.2) r 25/09/17 1:55 PM 9.7 Time-Harmonic Fields 449 For free space, rv 5 0, s 5 0, e 5 eo, and m 5 mo, so Maxwell's equations become = # D 5 eo= # E 5 0 S = # Es 5 0 (9.7.3) = # B 5 mo= # H 5 0 S = # Hs 5 0 (9.7.4) = 3 H 5 sE 1 eo 'E S = 3 Hs 5 jveoEs (9.7.5) 't = 3 E 5 2mo 'H 't S = 3 Es 5 2jvmoHs(9.7.6) Substituting eq (9.7.2) into eqs. (9.7.3) and (9.7.4), it is readily verified that two Maxwell's equations are satisfied; that is, = # Es 5 = # Hs 5 Now = 3 Hs 5 = 3 a 1 ' 1E 2 5 0 r 'f fs 1 ' 1 rHrs 2 5 0 r 'r jHob jbz Ho jbz e ar b 5 e af(9.7.7) r r Substituting eqs. (9.7.2) and (9.7.7) into eq. (9.7.5), we have jHob jbz 50 jbz e af 5 jveo e af r r or Hob 5 50 veo(9.7.8) Similarly, substituting eq. (9.7.2) into eq. (9.7.6) gives 2jb Ho jbz 50 jbz e ar 5 2jvmo e ar r r or Ho 50 (9.7.9) 5 vmo b Multiplying eq. (9.7.8) by eq. (9.7.9) yields H2o 5 1 50 2 2 09_Sadiku_Ch09.indd 449 eo mo 25/09/17 1:55 PM 450 CHAPTER 9 MAXWELL'S EQUATIONS or Ho 5 650"eo /mo 5 6 50 5 60.1326 120p Dividing eq. (9.7.8) by eq. (9.7.9), we get b2 5 v 2moeo or b 5 6v"moeo 5 6 v 106 56 c 3 3 108 5 63.33 3 1023 In view of eq. (9.7.8), Ho 5 0.1326, b 5 3.33 3 1023 or Ho 5 20.1326, b 523.33 3 1023; only these will satisfy Maxwell's four equations. PRACTICE EXERCISE 9.7 sin u cos 1 6 3 107t 2 br 2 af V/m. r Find b and H. In air, E 5 1 1 7 1 2 sin u 3 2 cos u sin 6 3 10 t 2 0.2r ar 2 120pr 12pr cos 1 6 3 107t 2 0.2r 2 au /m. Answer: 0.2 rad/m, 2 EXAMPLE 9.8 In a medium characterized by s 5 0, m 5 mo, e 5 4eo, and calculate b and H. E 5 20 sin 1 108t 2 bz 2 ay V/m Solution: This problem can be solved directly in time domain or by using phasors. As in Example 9.7, we find b and H by making E and H satisfy Maxwell's four equations. Method 1 (time domain): Let us solve this problem the harder way—in time domain. It is evident that Gauss's law for electric fields is satisfied; that is, =#E5 09_Sadiku_Ch09.indd 450 'Ey 'y 50 25/09/17 1:55 PM 9.7 Time-Harmonic Fields 451 From Faraday's law, = 3 E 5 2m 'H 't 1 H 5 2 3 1 = 3 E 2 dt m S But ay ' 'y Ey ax ' =3E5 ∞ 'x 0 az 'Ey 'Ey ' ∞ 5 2 ax 1 a 'z 'z 'x z 0 5 20b cos 1 108t 2 bz 2 ax 1 0 Hence, H52 52 20b cos 1 108t 2 bz 2 dt ax m 3 20b sin 1 108t 2 bz 2 ax m108 (9.8.1) It is readily verified that =#H5 'Hx 50 'x showing that Gauss's law for magnetic fields is satisfied. Lastly, from Ampère's law = 3 H 5 sE 1 e 'E 't S E5 1 1 = 3 H 2 dt(9.8.2) e3 because s 5 0. But ax ' =3H5 ∞ 'x Hx 5 09_Sadiku_Ch09.indd 451 ay ' 'y 0 az 'Hx 'Hx ' ∞ 5 a 2 a 'z 'z y 'y z 0 20b2 cos 1 108t 2 bz 2 ay 1 0 m108 25/09/17 1:55 PM 452 CHAPTER 9 MAXWELL'S EQUATIONS where H in eq. (9.8.1) has been substituted.

Thus eq. (9.8.2) becomes E5 5 20b2 8 3 cos 1 10 t 2 bz 2 dt ay me108 20b2 sin 1 108t 2 bz 2 ay me1016 Comparing this with the given E, we have 20b2 5 20 me1016 or 108 1 2 2 108 1 2 2 b 5 6108"me 5 6108"mo # 4eo 5 6 56 c 3 3 108 56 2 3 2 The b would be negative only in metamaterials, for an isotropic medium, b 5 . 3 From eq. (9.8.1), H51 or 20 1 2/3 2 2z 8 b ax 27 8 sin a10 t 2 # 3 4p 10 1 10 2 H51 Method 2 (using phasors): where v 5 108. 1 2z sin a108t 2 bax A/m 3p 3 E 5 Im 1 Ese jvt 2 S Es 5 20e2jbz ay(9.8.3) Again = # Es 5 'Eys 'y 50 = 3 Es 5 2jvmHs 09_Sadiku_Ch09.indd 452 S Hs 5 = 3 Es 2jvm 25/09/17 1:55 PM 9.7 Time-Harmonic Fields 453 or Hs 5 'Eys 20b 2jbz 1 a d 52 e ax (9.8.4) c2 vm 2jvm 'z x Notice that = # Hs 5 0 is satisfied. = 3 Hs 5 jveEs S Es 5 = 3 Hs (9.8.5) jve Substituting Hs in eq.

(9.8.4) into eq. (9.8.5) gives Es 5 20b2e2jbz 1 'Hxs ay 5 ay jve 'z v 2me Comparing this with the given Es in eq. (9.8.3), we have 20 5 20b2 v 2me or b 5 1v"me 5 1 2 3 as obtained before. From eq. (9.8.4), Hs 5 1 20 1 2/3 2 e 2 jbz 1 2 jbz ax 5 1 e ax 3p 108 1 4p 3 1027 2 H 5 Im 1 Hse jvt 2 5 ; 1 sin 1 108t 2 bz 2 ax A/m 3p as obtained before. It should be noticed that working with phasors is considerably simpler than working directly in time domain.

Also, notice that we have used A 5 Im 1 Ase jvt 2 because the given E is in sine form and not cosine. If we had used A 5 Re 1 Ase jvt 2 09_Sadiku_Ch09.indd 453 25/09/17 1:55 PM 454 CHAPTER 9 MAXWELL'S EQUATIONS sine would been expressed in terms of cosine, and eq. (9.8.3) would have been or E 5 20 cos 1 108t 2 bz 2 90º 2 ay 5 Re 1 Ese jvt 2 Es 5 20e2jbz2j90°ay 5 2j20e2jbzay and we follow the same procedure. PRACTICE EXERCISE 9.8 A medium is characterized by s 5 0, m 5 2mo and e 5 5eo.

If H 5 2 cos 1 vt 2 3y 2 az A/m, calculate and E. Answer: 2.846 3 108 rad/s, 2476.86 cos 1 2.846 3 108t 2 3y 2 ax V/m. † 9.8 APPLICATION NOTE—MEMRISTOR In 1971 Leon O. Chua of the University of California–Berkeley introduced the memristor (Figure 9.13) as one of the four basic circuit elements, coequal in importance with the other well-known circuit elements, namely, resistor (R), inductor (L), and capacitor (C). The new element had not been physically realized when Chua proposed it. However, he was the first to use this moniker.

Not until 2008 was a physical approximation of such an element fabricated, as a TiO2 nanodevice, by Stanley Williams's group at Hewlett-Packard (HP).

FIGURE 9.13 Schematic of a memristor.

Chua characterized the memristor in terms of the electric charge and the magnetic flux. He also linked this relationship with the quasi-static expansion of Maxwell's 09_Sadiku_Ch09.indd 454 25/09/17 1:55 PM 9.9 Application Note—Optical Nanocircuits 455 e quations. A charge-controlled memristor can be defined as a two-terminal element satisfying the constitutive relation 5 M(q), where and q are magnetic flux and electric charge, respectively, and M (qs) is a piecewise-differentiable function. Memristors have interesting circuit-theoretic properties different from those of the classical circuit elements R, L, and C. These properties, in turn, lead to remarkable applications not realizable with the earlier circuits.

A charge-controlled memristor behaves somewhat like a nonlinear resistor RM satisfying a q-dependent Ohm's law, v 5 RMi. The quantity RM is memresistance, measured in webers per coulomb, and for all passive memristors RM 5 0. Since the voltage v is related to dfM 1 q 2 dw , we can express the memresistance as RM 5 . When current through a memrisdt dq dq tor is turned off at t 5 t0, 5 0 implies q 5 q(t0). This allows us to view a memristor as dt a nonvolatile analog memory. In particular, it can be used as a nonvolatile binary memory, where two sufficiently different values of resistance are chosen to code binary states "\0" and "\1," respectively. The memristor reported by HP as well as many other nanodevices proposed recently can be scaled down to atomic dimensions. Thus the memristor offers immense potential for an ultra-low-power and ultradense nonvolatile memory technology that could replace flash memories and dynamic random-access memories (DRAMs). The most important common property of a memristor is the pinched hysteresis loop; that is, the loci of (v(t), i(t)) due to any bipolar periodic current source i(t) or periodic voltage source v(t) must always be pinched at the origin in the sense that (v(t), i(t)) 5 (0, 0) must always lie on the (v,i)-loci. The pinched hysteresis loop phenomenon of the memristor must hold for any bipolar periodic signal v(t), or i(t). Although memristors have become popular only recently, they are known to abound in many other forms. For example, the electric arc, dating back to 1801, has been identified as a memristor.

Also, a very interesting and scientifically significant example is the classic Hodgkin–Huxley axon circuit model of the squid giant axon. Chua showed that the Hodgkin–Huxley time-varying potassium conductance is in fact a first-order memristor, and the Hodgkin–Huxley time-varying sodium conductance is in fact a second-order memristor. Besides serving as nonvolatile memories, locally passive memristors have been used for switching electromagnetic devices, for field-programmable logic arrays, for synaptic memories, and for learning. In addition, locally passive memristors have been found to exhibit many exotic dynamical phenomena, such as oscillations, chaos, Hamiltonian vortices, and autowaves. † 9.9 APPLICATION NOTE—OPTICAL NANOCIRCUITS Circuit elements and electronic devices such as resistors, capacitors, inductors, switches, diodes, and transistors were developed at low frequencies; higher frequencies, even radio frequencies, were realized only later. With the development of metamaterials and nanotechnology, such elements have also been conceived at optical frequencies. Nader Engheta and his group at the University of Pennsylvania have recently proposed circuit elements at infrared and optical frequencies. The advantage of using lumped elements lies in their 09_Sadiku_Ch09.indd 455 25/09/17 1:55 PM 456 CHAPTER 9 MAXWELL'S EQUATIONS simplicity and modularity: when we want to use lumped elements by connecting them with one another, we need to know only their in-terminal behavior and the functionality of the overall circuit they comprise. Although it is difficult to pinpoint the frequency at which one might consider an element to be lumped, as long as it is smaller than the wavelength but larger than it is when the quantum effects begin to manifest, we can conveniently model an arbitrary particle as a lumped circuit element. FIGURE 9.14 A nanoparticle used as a circuit element. ⟨V⟩ufringe I uimp Optical lumped circuit elements with specific optical impedances have been realized by means of deep-subwavelength nanostructures. The use of gyroscopic nanospheres has permitted the identification of tunable circuit elements at infrared and optical frequencies. This new area is called optical metatronics—a portmanteau term derived from metamaterials, optical, and electronics.

The three fields of electronics, photonics, and magnetics can be brought together seamlessly under one umbrella. In this paradigm, for information processing and data storage at the nanoscale, the optical electric displacement currents or the optical magnetic displacement currents play a more important role than the conventional drift of charged particles. In such optical circuitry, nanostructures with specific values of permittivity (or permeability) may act as lumped circuit elements (e.g., nanocapacitors, nanoinductors, nanoresistors). By collecting properly arranging judiciously designed nanostructures, it is possible to achieve a new circuit platform in which optical signals can be tailored and manipulated, thus allowing optical information to be processed at the nanoscale. These nanostructures can be considered to be the modules and building blocks of metatronic circuits in which optical electrical fields and optical displacement currents are connected through the optical impedances of the nanoscale lumped elements. Such optical lumped circuit elements and metatronic circuitry afford the possibility of bringing many designs from RF electronics into the field of optics but with a much higher level of miniaturization and higher bandwidth. Just as electrons play the fundamental role in electronics, spins in spintronics, and photons in photonics, optical displacement current is of fundamental importance in the field of metatronics. If the real part of the permittivity of the material forming a given nanoparticle is positive, its optical impedance is capacitive. If the imaginary part of the permittivity is nonzero, then the particle impedance arises from a lumped conductance in parallel with 09_Sadiku_Ch09.indd 456 25/09/17 1:55 PM 9.10 Application Note—Wireless Power transfer and Qi Standard 457 the lumped capacitance. Similarly, a plasmonic particle with negative real part for its permittivity may possess an optical impedance that resembles the impedance of an inductor at that frequency. These are fixed elements. But if we wish to make them variable, since it is not possible to mechanically change their physical size, we can consider gyrotropic materials impressed with a dc magnetic field and still realize tunable circuit elements. These nanocircuit elements play a vital role in metatronics for the design of various nanodevices.

Actual fabrication of optical nanofilters, left-handed/right-handed nanotransmission lines, couplers, biosensors, information storage devices, and so on has become a reality, and more surprises are in the offing. † 9.10 APPLICATION NOTE—WIRELESS POWER TRANSFER AND QI STANDARD Rapid growth in the area of high-speed wireless data transfer has resulted in the proliferation of cell-phones and various mobile devices that include even biomedical implants. In turn, rapid charging of batteries and remote powering of electric circuits have become a high priority and a pressing need.

Especially the emergence of electric vehicles, aimed at reducing environmental pollution, became a greater impetus for more efficient ways of charging batteries. All along, recharging was done by connecting power cord battery, but to increase mobility and ease of handling, doing this task cordlessly or, if possible, dispensing of batteries altogether would be better. Toward this goal, operating mobile devices through wireless power transfer (WPT) became the preferred choice. The development in this area has been rather slows, although the idea dates as far back as a century to Tesla, who proposed that electric power can be transferred not only by means of radiation, but also by means of induction and resonant coupling. Induction machines, microwave heating, and similar power devices developed historically, are all based on WPT. Since the distance between the source and the receiver in these devices is usually small, the term wireless is not highlighted when we refer to them. We might achieve WPT in three broad ways: (i) near-field resonant reactive coupling, (ii) far-field directive power beaming, and (iii) far-field nondirective power transfer.

Nearfield or non-radiative WPT is based on the near-field magnetic coupling of conductive loops and can be either short range or mid-range in its applications. Far-field or radiative WPT takes place from a transmitting antenna and propagates through a medium such as air over distances that are several wavelengths long to a receiver where power is used to energize the mobile device. This method of transferring power can be highly directive if the locations of the receiver are predetermined or nondirective otherwise. In the latter case, the efficiency of transmission is very low. Wireless charging technology for portable electronic devices has escalated to the commercialization stage with the introduction of the Qi (pronounced "chee") Standard by the Wireless Power Consortium (WPC), now (at the time of this writing) growing with a membership of over 220 companies worldwide. The Qi system comprises a base station that appears as a charging pad, on which is placed a compatible device, which receives energy through resonant inductive coupling. The base station, connected to a power source, has 09_Sadiku_Ch09.indd 457 25/09/17 1:55 PM 458 CHAPTER 9 MAXWELL'S EQUATIONS planar coils that set up oscillating magnetic flux. Likewise, the mobile device has a receiver coil that harvests energy into a power receiver. Proper shielding of coils and selection of their parameters is done to ensure good inductive power transfer. To promote better coupling and higher power transfer, relative alignment of the device is made in a guided way by markings on the charging pad.

Also, free positioning is allowed by the careful design of coil geometry or using a technique that employs multiple cooperative flux generators. A typical WPT charging unit is depicted in Figure 9.15. It shows the base station with the charging pad on the top and a power transmitter section. It has a power conversion unit (PCU) and a communications and control unit (CCU). The transmitting coil of the PCU underneath the charging pad establishes the required oscillating magnetic flux. The Qi compatible mobile device is equipped with the power receiver section, which essentially has the power pickup unit (PPU) and a CCU similar to the one in the base station.

A receiver coil above the charging pad collects the energy induced and conveys to the PPU, which then drives the load. The CCUs are designed to regulate the transferred power to the required level at the highest possible transfer efficiency. Although Figure 9.15 does not show, in practice, the base station has an array of transmitting coils to facilitate charging of numerous mobile devices. The system unit in the base station contains additional user interfaces. Between the receiver and the transmitter, communication is established with the aid of backscatter modulation. In terms of the Qi standards, the low-power specification delivers up to 5 W, typically used to charge mobile devices, and the medium-power specification will deliver up to 120 W. Usually this is allocated for power displays and laptops. In 2015, WPC demonstrated a high-power specification that will deliver up to 1 kW, allowing the powering of kitchen utensils among other high-power utilities. The Qi logo is depicted in Figure 9.16. As the Qi standard gains popularity, it is forecast that Qi hotspots will begin to abound in all market places, coffee shops, airports, sports arenas, etc.

The technology of WPT developed for electric vehicles and medical implants and other consumer power devices has begun to explode and readers are encouraged to consult additional references. Sensing & Control Load Power Power Pick up Unit Communication & Control Unit Mobile Device Load FIGURE 9.15 A typical wireless power transfer system for charging a mobile device.

Power Receiver Charging Pad Power Conversion Unit Input Power System Unit 09_Sadiku_Ch09.indd 458 Communication & Control Unit Base Station Power Transmitter 25/09/17 1:55 PM 9.10 Application Note—Wireless Power transfer and Qi Standard 459 FIGURE 9.16 Qi logo MATLAB 9.1 % This script illustrates Matlab's complex arithmetic abilities % and assists the user to solve Practice Exercise 9.5 % clear % % % % z Matlab recognizes the input of complex numbers using i or j for example z = 7 - 6*j sets variable z to the complex value of 7 plus sqrt(-1) times 6, thus it is interactive with respect to entering and displaying complex values = input('Enter the complex number z in the format a+j*b... > '); disp(sprintf('The real part of z is %f', real(z))) % display the real part disp(sprintf('The imaginary part of z is %f', imag(z))) % display the imag part disp(sprintf('The magnitude of z is %f', abs(z))) % display the magnitude disp(sprintf('The phase of z is %f degrees', angle(z)*180/pi)) % display the phase (degrees) % Matlab also recognizes complex numbers in polar form % the exponential function accepts imaginary arguments, however it % interprets the value as being in radians, not degrees, so if % degrees are desired a conversion must be made disp('Enter the complex number z in the a*exp(j*b) where b is'); z = input(' in radians...

> '); disp(sprintf('The disp(sprintf('The disp(sprintf('The disp(sprintf('The % % % z real part of z is %f', real(z))) imaginary part of z is %f', imag(z))) magnitude of z is %f', abs(z))) phase of z is %f degrees', angle(z)*180/pi)) part a complex numbers may be handled with the same math operators as real numbers in matlab.... = j^3 * ((1+j)/(2-j))^2; disp(sprintf('Part (a)z = %0.2f ', real(z))) 09_Sadiku_Ch09.indd 459 25/09/17 2:44 PM 460 CHAPTER 9 MAXWELL'S EQUATIONS disp(sprintf(' + j%0.2f ', imag(z))) % % % z part b note the conversion from degrees to radians in the exponential = 6*exp(j*30*pi/180) + j*5 - 3 +exp(j*45*pi/180); disp(sprintf('Part (b)z = %0.3f ', real(z))) disp(sprintf(' + j%0.3f ', imag(z))) SUMMARY 1. In this chapter, we have introduced two fundamental concepts: electromotive force (emf), based on Faraday's experiments, and displacement current, which resulted from Maxwell's hypothesis. These concepts call for modifications in Maxwell's curl equations obtained for static EM fields to accommodate the time dependence of the fields. 2. Faraday's law states that the induced emf is given by 1 N 5 1 2 Vemf 5 2 For transformer emf, Vemf 5 23 ' 't 'B # dS 't and for motional emf, Vemf 5 3 1 u 3 B 2 # dl. 3. The displacement current Id 5 3 Jd # dS 'D (displacement current density) is a modification to Ampère's circuit 't law. This modification, attributed to Maxwell, predicted electromagnetic waves several years before the phenomenon was verified experimentally by Hertz. 4. In differential form, Maxwell's equations for dynamic fields are: where Jd 5 = # D 5 rv =#B50 'B 't 'D =3H5J1 't =3E52 Each differential equation has its integral counterpart (see Tables 9.1 and 9.2) that can be derived from the differential form by using Stokes's theorem or the divergence theorem. Any EM field must satisfy the four Maxwell's equations simultaneously.

09_Sadiku_Ch09.indd 460 25/09/17 1:55 PM Review Questions 461 5. Time-varying electric scalar potential V 1 x, y, z, t 2 and magnetic vector potential A 1 x, y, z, t 2 are shown to satisfy wave equations if Lorenz's condition is assumed. 6. Time-harmonic fields are those that vary sinusoidally with time. They are easily expressed in phasors, which are more convenient to work with. The cosine reference, can be used to show that the instantaneous vector quantity A 1 x, y, z, t 2 is related to its phasor form As 1 x, y, z 2 according to A 1 x, y, z, t 2 5 Re 3 As 1 x, y, z 2 e jvt 4 REVIEW QUESTIONS 9.1 The flux through each turn of a 100-turn coil is 1 t3 2 2t 2 mWb, where t is in seconds. The induced emf at t 5 2 s is (a) 1 V (d) 0.4 V (b) 21 V (c) 4 mV (e) 20.4 V 9.2 Assuming that each loop is stationary and the time-varying magnetic field B induces current I, which of the configurations in Figure 9.17 are incorrect? 9.3 Two conducting coils 1 and 2 (identical except that 2 is split) are placed in a uniform magnetic field that decreases at a constant rate as in Figure 9.18. If the plane of the coils is perpendicular to the field lines, which of the following statements is true? (a) An emf is induced in both coils. (b) An emf is induced in split coil 2. (c) Equal Joule heating occurs in both coils. (d) Joule heating does not occur in either coil. FIGURE 9.17 For Review Question 9.2. 09_Sadiku_Ch09.indd 461 FIGURE 9.18 For Review Question 9.3. 25/09/17 1:55 PM 462 CHAPTER 9 MAXWELL'S EQUATIONS FIGURE 9.19 For Review Question 9.5. 9.4 A loop is rotating about the y-axis in a magnetic field B 5 Bo sin vt ax Wb/m2. The voltage induced in the loop is due to (a) Motional emf (b) Transformer emf (c) A combination of motional and transformer emf (d) None of the above 9.5 A rectangular loop is placed in the time-varying magnetic field B 5 0.2 cos150ptaz Wb/m2 as shown in Figure 9.19. V1 is not equal to V2. (a) True (b) False 9.6 The concept of displacement current was a major contribution attributed to (a) Faraday (b) Lenz (c) Maxwell (d) Lorenz (e) Your professor 9.7 Identify which of the following expressions are not Maxwell's equations for time-varying fields: 'rv (a) = # J 1 50 't (b) = # D 5 rv 'B (c) = # E 5 2 't 'E # (d) C H # d l 5 3 asE 1 e b dS 't L S (e) C B # dS 5 0 S 09_Sadiku_Ch09.indd 462 15/11/17 4:16 PM Problems 463 9.8 An EM field is said to be nonexistent or not Maxwellian if it fails to satisfy Maxwell's equations and the wave equations derived from them. Which of the following fields in free space are not Maxwellian? (a) H 5 cos x cos 106t ay (b) E 5 100 cos vt ax (c) D 5 e210y sin 1 105t 2 10y 2 az (d) B 5 0.4 sin 104t az (e) H 5 10 cos a105t 2 (f) E 5 z ba 10 x sin u cos 1 vt 2 rv"moeo 2 au r (g) B 5 1 1 2 r2 2 sin vt az 9.9 Which of the following statements is not true of a phasor? (a) It may be a scalar or a vector. (b) It is a time-dependent quantity. (c) A phasor Vs may be represented as Vo lu or Voe jv where Vo 5 0 Vs 0 . (d) It is a complex quantity. 9.10 If Es 5 10 e j4x ay, which of these is not a correct representation of E? (a) Re 1 Ese jvt 2 (b) Re 1 Ese2jvt 2 (c) Im 1 Ese jvt 2 (d) 10 cos 1 vt 1 j4x 2 ay (e) 10 sin 1 vt 1 4x 2 ay Answers: 9.1b, 9.2b, d, 9.3a, 9.4c, 9.5a, 9.6c, 9.7a,c, 9.8b, d, 9.9b, 9.10d. PROBLEMS Sections 9.2 and 9.3—Faraday's Law and Electromotive Forces 9.1 A conducting circular loop of radius 20 cm lies in the z 5 0 plane in a magnetic field B 5 10 cos 377t az mWb/m2. Calculate the induced voltage in the loop.

9.2 The loop in Figure 9.20 exists in a magnetic field B 4cos(20t)az Wb/m2, where az is directed out of the page. If the area enclosed by the circuit is 2 cm2, find the current i(t). 9.3 A circuit conducting loop lies in the xy-plane as shown in Figure 9.21. The loop has a radius of 0.2 m and resistance R 4 . If B 40 sin 104 taz mWb/m2, find the currrent. 09_Sadiku_Ch09.indd 463 15/11/17 4:16 PM 464 CHAPTER 9 MAXWELL'S EQUATIONS FIGURE 9.20 For Problem 9.2. i(t) 30 Ω B 20 Ω FIGURE 9.21 For Problem 9.3. z y x R 9.4 Two conducting bars slide over two stationary rails, as illustrated in Figure 9.22. If B 0.2az Wb/m2, determine the induced emf in the loop thus formed. 9.5 A conductor located at 0 , y , 1.6 m moves with velocity 2ax m/s in a magnetic field, B 5 10 cos byaz Wb/m2 where b is a constant. Determine the induced voltage. 9.6 A square loop of side a recedes with a uniform velocity uoay from an infinitely long filament carrying current I along az as shown in Figure 9.23. Assuming that r 5 ro at time t 5 0, show that the emf induced in the loop at t . 0 is Vemf 5 uoa2moI 2pr 1 r 1 a 2 9.7 A conducting rod moves with a constant velocity of 3 az m/s parallel to a long straight wire carrying a current of 15 A as in Figure 9.24. Calculate the emf induced in the rod and state which end is at the higher potential. 9.8 A conducting rod has one end grounded at the origin, while the other end is free to move in the z 5 0 plane. The rod rotates at 30 rad/s in a static magnetic field B 5 60az mWb/m2. If the rod is 8 cm long, find the voltage induced in the rod. 09_Sadiku_Ch09.indd 464 15/11/17 7:19 PM Problems 465 FIGURE 9.22 For Problem 9.4. y B 1.2 m 5 m/s x 15 m/s FIGURE 9.23 For Problem 9.6. y 30° FIGURE 9.24 For Problem 9.7. u x FIGURE 9.25 For Problem 9.10. 9.9 A rectangular coil has a cross-sectional area of 30 cm2 and 50 turns. If the coil rotates at 60 rad/s in a magnetic field of 0.2 Wb/m2 such that its axis of rotation is perpendicular to the direction of the field, determine the induced emf in the coil. 9.10 Determine the induced emf in the V-shaped loop of Figure 9.25. Take B 5 0.6xaz Wb/m2 and u 5 5ax m/s. Assume that the sliding rod starts at the origin when t = 0. 9.11 A car travels at 120 km/hr. If the earth's magnetic field is 4.3 3 1025 Wb/m2, find the induced voltage in the car bumper of length 1.6 m. Assume that the angle between the earth's magnetic field and the normal to the car is 65°.

09_Sadiku_Ch09.indd 465 15/11/17 4:16 PM 466 CHAPTER 9 MAXWELL'S EQUATIONS 9.12 An airplane with a metallic wing of span 36 m flies at 410 m/s in a region where the vertical component of the earth's magnetic field is 0.4 mWb/m2. Find the emf induced on the airplane wing. 9.13 As portrayed in Figure 9.26, a bar magnet is thrust toward the center of a coil of 10 turns and resistance 15 . If the magnetic flux through the coil changes from 0.45 Wb to 0.64 Wb in 0.02 s, find the magnitude and direction (as viewed from the side near the magnet) of the induced current. 9.14 The cross section of a homopolar generator disk is shown in Figure 9.27. The disk has inner radius r1 5 2 cm and outer radius r2 5 10 cm and rotates in a uniform magnetic field 15 mWb/m2 at a speed of 60 rad/s. Calculate the induced voltage. Section 9.4—Displacement Current 9.15 A 50 V voltage generator at 20 MHz is connected to the plates of an air dielectric parallelplate capacitor with a plate area of 2.8 cm2 and a separation distance of 0.2 mm. Find the maximum value of displacement current density and displacement current. 9.16 A dielectric material with 5 o, « 5 9«o 5 4 S/m is placed between the plates of a parallel-plate capacitor. Calculate the frequency at which the conduction and displacement currents are equal. FIGURE 9.26 For Problem 9.13. FIGURE 9.27 For Problem 9.14. B S 09_Sadiku_Ch09.indd 466 C 15/11/17 4:16 PM Problems 467 9.17 The ratio J/Jd (conduction current density to displacement current density) is very important at high frequencies. Calculate the ratio at 1 GHz for: (a) distilled water 1 m 5 mo, e 5 81eo, s 5 2 3 1023 S/m 2 (b) seawater 1 m 5 mo, e 5 81eo, s 5 25 S/m 2 (c) limestone 1 m 5 mo, e 5 5eo, s 5 2 3 1024 S/m 2 9.18 In seawater (5 4 S/m, « 5 81«o, o), find the ratio of the conduction to the displacement currents at 10 MHz. 9.19 Assume that dry soil has 5 104 S/m, « 3«o, and o. Determine the frequency at which the ratio of the magnitudes of the conduction current density and the displacement current density is unity. 9.20 In a dielectric (5 1024 S/m, mr 5 1, «r 5 4.5), the conduction current density is given as Jc 5 0.4 cos 1 2p 3 108 t 2 A/m2. Determine the displacement current density. 9.21 In a source-free region, H 5 Ho cos(wt 2 bz)ax A/m. Find the displacement current density. 9.22 An ac voltage source is connected across the plates of a parallel-plate capacitor so that E 5 25sin(103t)az V/m. Calculate the total current crossing a 2 3 5 m area placed perpendicular to the electric field. Assume that the capacitor is air filled. Section 9.5—Maxwell's Equations 9.23 (a) Write Maxwell's equations for a linear, homogeneous medium in terms of Es and Hs, assuming only the time factor e2jvt. (b) In Cartesian coordinates, write the point form of Maxwell's equations in Table 9.2 as eight scalar equations. 9.24 Show that in a source-free region 1 J 5 0, rv 5 0 2 , Maxwell's equations can be reduced to two. Identify the two all-embracing equations.

9.25 Show that fields E Eo cos x cos tay and H 5 Eo sin x sin taz mo do not satisfy all of Maxwell's equations. 9.26 Assuming a source-free region, derive the diffusion equation =2E 5 ms 'E 't 9.27 In a certain region, J 5 1 2yax 1 xz ay 1 z3az 2 sin 104t A/m find rv if rv 1 x, y, 0, t 2 5 0.

09_Sadiku_Ch09.indd 467 15/11/17 4:16 PM 468 CHAPTER 9 MAXWELL'S EQUATIONS 9.28 In free space, the electric field is given by E 5 Eo cosz costaz Find the charge density ρv that will produce this field. 9.29 In free space, H 5 10 sin(108 t 1 x)ay A/m. Find E and . 9.30 In free space, E5 50 cos 1 108 t 2 kz 2 arV/m r Find k, Jd, and H. 9.31 The electric field intensity of a spherical wave in free space is given by E5 10 sin u cos 1 vt 2 br 2 au V/m r Find the corresponding magnetic field intensity H. 9.32 In a certain region for which 0, 2o, and « 10«o J 60 sin(109t 2 z)ax mA/m2 (a) Find D and H. (b) Determine . 9.33 Use Maxwell's equations to derive the continuity equation.

9.34 In a source-free region, show that ,2E 2 ms 'E '2E 2 me 2 5 0 't 't 9.35 Check whether the following fields are genuine EM fields (i.e., they satisfy Maxwell's equations). Assume that the fields exist in charge-free regions. (a)   A 5 40 sin 1 vt 1 10x 2 az (b)   B 5 10 cos 1 vt 2 2r 2 af r (c)   C 5 a3r2 cot f ar 1 (d)   D 5 09_Sadiku_Ch09.indd 468 cos f af b sin vt r 1 sin u sin 1 vt 2 5r 2 au r 15/11/17 4:16 PM Problems   469 9.36 Given the total electromagnetic energy W5 1 1 E # D 1 H # B 2 dv 23 show from Maxwell's equations that 'W 5 2 C 1 E 3 H 2 # dS 2 3 E # J dv 't S v 9.37 In air, E 5 cos(12px)sin(1011t 2 ay)az V/m. Find H and a. 9.38 An AM radio signal propagating in free space has E 5 Eo sin(1200pt 2 bz)ax H5 Eo h sin(1200pt 2 bz)ay Determine b and h. 9.39 An antenna radiates in free space and 12 sin u cos 1 2p 3 108t 2 br 2 au mA/m r Find the corresponding E in terms of b. H5 Section 9.6—Time-Varying Potentials 9.40 In free space Arv 5 0, J 5 0 2 , show that m0 1 cos u ar 2 sin u au 2 e jv1t2r/c2 4pr satisfies the wave equation in eq. (9.52). Find the corresponding V.

Take c as the speed of light in free space.

A5 9.41 Retrieve Faraday's law in differential form from E 5 2=V 2 'A 't 9.42 In free space, the retarded potentials are given by V x(z 2 ct)V, where c A x(z/c 2 t)az Wb/m 1 "moeo (a)   Prove that = # A 5 moeo 'V . 't (b) Determine E. 9.43 Let A 5 Ao sin(t 2 z)ax Wb/m in free space. (a) Find V and E. (b) Express b in terms of , «o, and o. 09_Sadiku_Ch09.indd 469 15/11/17 4:16 PM 470   CHAPTER 9   MAXWELL'S EQUATIONS Section 9.7—Time-Harmonic Fields 9.44 Evaluate the following complex numbers and express your answers in polar form: (a)   1 4 l 30° 2 10 l 50° 2 1/2 i i 1 1 j2 (b)   6 1 j8 2 7 l 15° i i (c)   1 3 1 j4 2 2 12 2 j7 1 1 26 1 j10 2 * 1 3.6 l2200° 2 1/2 ii 1 2.4 l 45° 2 2 1 25 1 j8 2 * ii 9.45 Determine the phasor forms of the following instantaneous vector fields: (d)   (a)   H 5 210cos(106t 1 p/3)ax (b)   E 5 4cos(4y)cos(104t 2 2x)az (c)   D 5 5sin(104t 1 p/3)ax 2 8cos(104t 2 p/4)ay 9.46 Find the instantaneous form for each of the following phasors: (a)   As 5 j10ax 1 20 a j y (b)   Bs 5 j4e 2j2xax 1 6e 1j2xaz (c)   Cs 5 j2e 220ze 2jp/4az 9.47 In a source-free vacuum region, H5 (a) Express H in phasor form. 1 cos 1 vt 2 3z 2 af A/m r (b)   Find the associated E field. (c) Determine . 9.48 In a certain homogeneous medium, « 5 81«o, and 5 o, Es 5 10e j(t 1 z)ay V/m Hs 5 Hoe j(t 1 z)ax A/m If 5 2 3 109 rad/m, find b and Ho. 09_Sadiku_Ch09.indd 470 15/11/17 4:16 PM Problems 471 9.49 The magnetic phasor of a plane wave propagating in air is Hs(x) 5 12ejaxaz Determine a and Es(x).

Assume v = 109 rad/s. 9.50 Given that d2y dt2 14 dy dt 1 y 5 2 cos 3t Solve for y by using phasors. 9.51 Show that in a linear homogeneous, isotropic source-free region, both Es and Hs must satisfy the wave equation =2As 1 g2As 5 0 where g2 5 v 2me 2 jvms and As 5 Es or Hs. 09_Sadiku_Ch09.indd 471 15/11/17 7:19 PM Hermann von Helmholtz (1821–1894), a German physicist, extended Joule's results to a general principle and derived the wave equation (to be discussed in this chapter). Helmholtz was born in Potsdam, and his youth was marred by illness. He graduated from the Medical Institute in Berlin in 1843 and was assigned to a military regiment at Potsdam, but spent all his spare time doing research. In 1858 he became professor of anatomy and physiology at Bonn. In 1871 he became professor of physics at Berlin. Helmholtz made important con tributions in all major fields of science, not only unifying the diverse fields of medicine, physiology, anatomy, and physics, but also relating this universal view to the fine arts. Helmholtz expressed the relationship between mechanics, heat, light, electricity, and magnetism by treating them all as manifestations of a single force. He sought to synthesize Maxwell's electromagnetic theory of light with the central force theorem. Heinrich Rudolf Hertz   (1857–1894), a German experimental physicist, demonstrated that electromagnetic waves obey the same fundamental laws that g overn light. His work confirmed James Clerk Maxwell's celebrated theory and prediction that such waves existed. Hertz was born into a prosperous family in Hamburg, Germany. He attended the University of Berlin and did his doctorate under Hermann von Helmholtz. He became a professor at Karlsruhe, where he began his quest for electromagnetic waves. Hertz successively generated and detected elec tromagnetic waves; he was first to show that light is electromagnetic energy. In 1887 Hertz noted for the first time the photoelectric effect of electrons in a molecular structure. Although Hertz died at the age of 37, his discovery of electromagnetic waves paved the way for the practical use of such waves in radio, television, and other communication systems.

The unit of fre quency, the hertz (Hz), bears his name. 10_Sadiku_Ch10.indd 472 25/09/17 3:16 PM CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION Young people tell what they are doing, old people what they have done, and fools what they wish to do. —FRENCH PROVERB 10.1 INTRODUCTION Our first application of Maxwell's equations will be in relation to electromagnetic wave propagation. The existence of EM waves, predicted by Maxwell's equations, was first inves tigated by Heinrich Hertz. After several calculations and experiments, Hertz succeeded in generating and detecting radio waves, which are sometimes called Hertzian waves in his honor. In general, waves are means of transporting energy or information. Typical examples of EM waves include radio waves, TV signals, radar beams, and light rays. All forms of EM energy share three fundamental characteristics: they all travel at high velocity; in traveling, they assume the properties of waves; and they radiate outward from a source, without benefit of any discernible physical vehicles. The problem of radiation will be addressed in Chapter 13. In this chapter, our major goal is to solve Maxwell's equations and describe EM wave motion in the following media: 1. 2. 3. 4. Free space 1 s 5 0, e 5 eo, m 5 mo 2 Lossless dielectrics 1 s . 0, e 5 ereo, m 5 mrmo, or s V ve 2 Lossy dielectrics 1 s 2 0, e 5 ereo, m 5 mrmo 2 Good conductors 1 s . `, e 5 eo, m 5 mrmo, or s W ve 2 where v is the angular frequency of the wave. Case 3, for lossy dielectrics, is the most general case and will be considered first. Once this general case has been solved, we simply derive the other cases (1, 2, and 4) from it as special cases by changing the values of s, , and «. However, before we consider wave motion in those different media, it is appro priate that we study the characteristics of waves in general. This is important for proper understanding of EM waves. The reader who is conversant with the concept of waves may skip Section 10.2. Power considerations, reflection, and transmission between two different media will be discussed later in the chapter. 473 10_Sadiku_Ch10.indd 473 25/09/17 3:16 PM 474   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION † 10.2 WAVES IN GENERAL A clear understanding of EM wave propagation depends on a grasp of what waves are in general. A wave is a function of both space and time.

Wave motion occurs when a disturbance at point A, at time to, is related to what happens at point B, at time t . to. A wave equation, as exemplified by eqs. (9.51) and (9.52), is a partial differential equation of the second order. In one dimension, a scalar wave equation takes the form of 2 '2E 2 ' E 2 u 5 0 (10.1) 't2 'z2 where u is the wave velocity. Equation (10.1) is a special case of eq. (9.51) in which the medium is source free 1 rv 5 0, J 5 0 2 . It can be solved by following a procedure similar to that in Example 6.5. Its solutions are of the form E1 5 f (z 2 ut)(10.2a) or E 5 g (z 1 ut)(10.2b)   E 5 f (z 2 ut) 1 g (z 1 ut)(10.2c) where f and g denote any function of z 2 ut and z 1 ut , respectively. Examples of such functions include z 6 ut , sin k 1 z 6 ut 2 , cos k 1 z 6 ut 2 , and e jk1z6ut2, where k is a constant. It can easily be shown that these functions all satisfy eq. (10.1). If we particularly assume harmonic (or sinusoidal) time dependence e jvt, eq. (10.1) becomes d2Es 1 b2Es 5 0 (10.3) dz2 where b 5 v/u and Es is the phasor form of E. The solution to eq. (10.3) is similar to Case 3 of Example 6.5 [see eq.

(6.5.12)]. With the time factor inserted, the possible solu tions to eq. (10.3) are E1 5 Ae j1vt2bz2 (10.4a) E2 5 Be j1vt1bz2 (10.4b) where E means positive z-travel and E means negative travel. Combining E and E leads to E 5 Ae j 1vt2bz2 1 Be j 1vt1bz2 (10.4c) where A and B are real constants. 10_Sadiku_Ch10.indd 474 25/09/17 3:16 PM 10.2 Waves in General   475 For the moment, let us consider the solution in eq. (10.4a). Taking the imaginary part of this equation, we have E 5 A sin 1 vt 2 bz 2 (10.5) This is a sine wave chosen for simplicity; a cosine wave would have resulted had we taken the real part of eq. (10.4a). Note the following characteristics of the wave in eq. (10.5): 1. It is time harmonic because we assumed time dependence of the form e jvt to arrive at eq. (10.5). 2. The amplitude of the wave A has the same units as E. 3. The phase (in radians) of the wave depends on time t and space variable z, it is the term 1 vt 2 bz 2 . 4. The angular frequency v is given in radians per second; b, the phase constant or wave number, is given in radians per meter. Because E varies with both time t and the space variable z, we may plot E as a func tion of t by keeping z constant and vice versa. The plots of E 1 z, t 5 constant 2 and E 1 t, z 5 constant 2 are shown in Figure 10.1(a) and (b), respectively. From Figure 10.1(a), we observe that the wave takes distance l to repeat itself and hence l is called the FIGURE 10.1   Plot of E 1 z, t 2 5 A sin 1 vt 2 bz 2 : (a) with constant t, (b) with constant z. 10_Sadiku_Ch10.indd 475 25/09/17 3:16 PM 476   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION wavelength (in meters). From Figure 10.1(b), the wave takes time T to repeat itself; consequently T is known as the period, in seconds. Since it takes time T for the wave to travel distance l at the speed u, we expect l 5 uT (10.6a) But T 5 1/f , where f is the frequency (the number of cycles per second) of the wave in hertz (Hz). Hence, u 5 f l (10.6b) Because of this fixed relationship between wavelength and frequency, one can identify the position of a radio station within its band by either the frequency or the wavelength. Usually the frequency is preferred. Also, because v 5 2pf (10.7a) b5 and T5 v (10.7b) u 1 2p (10.7c) 5 v f we expect from eqs. (10.6) and (10.7) that b5 2p v 5 (10.8) u l Equation (10.8) shows that for every wavelength of distance traveled, a wave undergoes a phase change of 2p radians. We will now show that the wave represented by eq. (10.5) is traveling with a veloc ity u in the 1z-direction. To do this, we consider a fixed point P on the wave. We sketch eq. (10.5) at times t 5 0, T/4, and T/2 as in Figure 10.2. From the figure, it is evident that as the wave advances with time, point P moves along the 1zdirection. Point P is a point of constant phase, therefore vt 2 bz 5 constant or dz v 5 u (10.9) dt b which is the same as eq.

(10.7b). Equation (10.9) shows that the wave travels with veloc ity u in the 1z-direction. Similarly, it can be shown that the wave B sin 1 vt 1 bz 2 in eq. (10.4b) is traveling with velocity u in the 2z-direction.

In summary, we note the following: 1. A wave is a function of both time and space. 2. Though time t 5 0 is arbitrarily selected as a reference for the wave, a wave is without beginning or end. 10_Sadiku_Ch10.indd 476 25/09/17 3:16 PM 10.2 Waves in General   477 FIGURE 10.2   Plot of E 1 z, t 2 5 A sin 1 vt 2 bz 2 at time (a) t 5 0, (b) t 5 T/4, (c) t 5 T/2; P moves in the 1z-direction with velocity u. 3. A negative sign in 1 vt 6 bz 2 is associated with a wave propagating in the 1z-direction (forward-traveling or positive-going wave), whereas a positive sign indicates that a wave is traveling in the 2z-direction (backward-traveling or negative-going wave). 4. Since sin 1 2c 2 5 2sin c 5 sin 1 c 6 p 2 , whereas cos 1 2c 2 5 cos c, sin 1 c 6 p/2 2 5 6cos c (10.10a) sin 1 c 6 p 2 5 2sin c (10.10b) cos 1 c 6 p/2 2 5 7sin c (10.10c) cos 1 c 6 p 2 5 2cos c (10.10d) where c 5 vt 6 bz. One of the relations in eqs. (10.10) can be used to represent any time-harmonic wave in the form of sine or cosine. 5. E and H are called uniform waves if they lie in a plane and are constant over such planes. A large number of frequencies visualized in numerical order constitute a spectrum. Table 10.1 shows the frequencies at which various types of energy in the EM spectrum occur. Frequencies usable for radio communication occur near the lower end of the EM spectrum. As frequency increases, the manifestation of EM energy becomes dangerous to human beings. Microwave ovens, for example, can pose a hazard if not properly shielded. The practical difficulties of using EM energy for communication purposes also increase as frequency increases, until finally it can no longer be used. As communication methods improve, the limit to usable frequency has been pushed higher. Today communication satellites use frequencies near 14 GHz. This is still far below light frequencies, but in the enclosed environment of fiber optics, light itself can be used for radio communication. 10_Sadiku_Ch10.indd 477 25/09/17 3:16 PM 478   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION TABLE 10.1 Electromagnetic Spectrum EM Phenomena Examples of Uses Approximate Frequency Range Cosmic rays Gamma rays X-rays Ultraviolet radiation Visible light Infrared radiation Microwave waves Physics, astronomy Cancer therapy X-ray examination Sterilization Human vision Photography Radar, microwave relays,   satellite communication UHF television VHF television, FM radio Short-wave radio AM radio 1014 GHz and above 1010–1013 GHz 108–109 GHz 106–108 GHz 105–106 GHz 103–104 GHz 3–300 GHz Radio waves EXAMPLE 10.1 470–806 MHz 54–216 MHz 3–26 MHz 535–1605 kHz An electric field in free space is given by E 5 50 cos 1 108t 1 bx 2 ay V/m (a) Find the direction of wave propagation. (b) Calculate b and the time it takes to travel a distance of l/2. (c) Sketch the wave at t 5 0, T/4, and T/2. Solution: (a)   From the positive sign in 1 vt 1 bx 2 , we infer that the wave is propagating along 2ax. This will be confirmed in part (c) of this example. (b) In free space, u 5 c: b5 v 108 1 5 8 5 c 3 3 3 10 or b 5 0.3333 rad/m If T is the period of the wave, it takes T seconds to travel a distance l at speed c. Hence to travel a distance of l/2 will take t1 5 T 1 2p p 5 5 8 5 31.42 ns 2 2 v 10 Alternatively, because the wave is traveling at the speed of light c, l l 5 ct1 or t1 5 2 2c But l5 10_Sadiku_Ch10.indd 478 2p 5 6p b 25/09/17 3:16 PM 10.2 Waves in General   479 Hence, t1 5 as obtained before. (c) At At At t 5 0, 6p 5 31.42 ns 2 1 3 3 108 2 Ey 5 50 cos bx 2p t 5 T/4, Ey 5 50 cosav # 1 bxb 5 50 cos 1 bx 1 p/2 2 4v 5 250 sin bx 2p t 5 T/2, Ey 5 50 cosav # 1 bxb 5 50 cos 1 bx 1 p 2 2v 5 250 cos bx Ey at t 5 0, T/4, T/2 is plotted against x as shown in Figure 10.3. Notice that a point P (arbitrarily selected) on the wave moves along 2ax as t increases with time. This shows that the wave travels along 2ax.   FIGURE 10.3   For Example 10.1; wave travels along ax. 10_Sadiku_Ch10.indd 479 25/09/17 3:16 PM 480   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION FIGURE 10.4   For Practice Exercise 10.1(c). PRACTICE EXERCISE   10.1 In free space, H 5 0.1 cos 1 2 3 108t 2 kx 2 ay A/m.

(a) Calculate k, l, and T. (b)   Calculate the time t1 it takes the wave to travel l/8.

(c)   Sketch the wave at time t1. Answer: (a) 0.667 rad/m, 9.425 m, 31.42 ns, (b) 3.927 ns, (c) see Figure 10.4. 10.3   WAVE PROPAGATION IN LOSSY DIELECTRICS As mentioned in Section 10.1, wave propagation in lossy dielectrics is a general case from which wave propagation in media of other types can be derived as special cases. Therefore, this section is foundational to the next three setions. A lossy dielectric is a medium in which an EM wave, as it propagates, loses power owing to imperfect dielectric. In other words, a lossy dielectric is a partially conducting medium (imperfect dielectric or imperfect conductor) with s 2 0, as distinct from a lossless dielectric (perfect or good dielectric) in which s 5 0.

Consider a linear, isotropic, homogeneous, lossy dielectric medium that is charge free 1 macroscopic rv 5 0 2 . Assuming and suppressing the time factor e jvt, Maxwell's equations (see Table 9.2) become 10_Sadiku_Ch10.indd 480 = # Es 5 0 (10.11) = # Hs 5 0 (10.12) = 3 Es 5 2jvmHs = 3 Hs 5 1 s 1 jve 2 Es(10.14) (10.13) 25/09/17 3:16 PM 10.3 Wave Propagation in Lossy Dielectrics   481 Taking the curl of both sides of eq. (10.13) gives Applying the vector identity = 3 = 3 Es 5 2jvm 1 = 3 Hs 2 (10.15) = 3 1 = 3 A 2 5 = 1 = # A 2 2 =2A(10.16) to the left-hand side of eq. (10.15) and invoking eqs.

(10.11) and (10.14), we obtain = 1 = # Es 2 2 =2Es 5 2jvm 1 s 1 jve 2 Es 0 or → =2Es 2 g2Es 5 0(10.17) where g2 5 jvm 1 s 1 jve 2 (10.18) and g, in reciprocal meters, is called the propagation constant of the medium. By a similar procedure, it can be shown that for the H field, =2Hs 2 g2Hs 5 0(10.19) Equations (10.17) and (10.19) are known as homogeneous vector Helmholtz's equations or simply vector wave equations. In Cartesian coordinates, eq. (10.17), for example, is equiva lent to three scalar wave equations, one for each component of E along ax, ay, and az. Since g in eqs.

(10.17) to (10.19) is a complex quantity, we may let g 5 a 1 jb(10.20) We obtain a and b from eqs. (10.18) and (10.20) by noting that 2Re g2 5 b2 2 a2 5 v 2me(10.21) and 0 g2 0 5 b2 1 a2 5 vm "s2 1 v 2e2(10.22) From eqs. (10.21) and (10.22), we obtain me s 2 a 5 v Ç 2 c Å 1 1 c ve d 2 1d b5v me s 2 c 1 1 c d 1 1d Ç 2 Å ve (10.23) (10.24)
10_Sadiku_Ch10.indd 481 25/09/17 3:16 PM 482   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION Without loss of generality, if we assume that a wave propagates along 1az and that Es has only an x-component, then Es 5 Exs 1 z 2 ax(10.25) We then substitute into eq. (10.17), which yields 1 =2 2 g2 2 Exs 1 z 2 5 0(10.26) Without loss of generality, if we assume that a wave propagates in an unbounded medium along az and that E has only an x-component that does not vary with x and y, then → → or   '2Exs 1 z 2 '2Exs 1 z 2 '2Exs 1 z 2 1 1 2 g2Exs 1 z 2 5 0 2 2 'x 'y 'z2 0 0 c d2 2 g2 d Exs 1 z 2 5 0(10.27) dz2 This is a scalar wave equation, a linear homogeneous differential equation, with solution (see eq. 6.5.13a in Case 3 of Example 6.5), Exs 1 z 2 5 Eoe2gz 1 Eroegz(10.28) where Eo and Ero are constants. The fact that the field must be finite at infinity requires that Ero 5 0. Alternatively, because egz denotes a wave traveling along 2az, whereas we assume wave propagation along az, Ero 5 0.

Whichever way we look at it, Ero 5 0. Inserting the time factor e jvt into eq. (10.28) and using eq. (10.20), we obtain or E z, t 2 5 Re 3 Exs z 2 e jvtax 4 5 Re Eoe2aze jvt2bz2ax 2 E 1 z, t 2 5 Eoe2azcos 1 vt 2 bz 2 ax(10.29) A sketch of |E 0 at times t 5 0 and t 5 Dt is portrayed in Figure 10.5, where it is evident that E has only an x-component and it is traveling in the 1z-direction. Having obtained E 1 z, t 2 , we obtain H 1 z, t 2 either by taking similar steps to solve eq. (10.19) or by using eq. (10.29) in conjunction with Maxwell's equations, as we did in Example 9.8. We will eventually arrive at where H 1 z, t 2 5 Re 1 Hoe2aze j1vt2bz2 ay 2 (10.30) Ho 5 Eo (10.31) h and h is a complex quantity known as the intrinsic impedance, in ohms, of the medium. It can be shown by following the steps taken in Example 9.8 that 10_Sadiku_Ch10.indd 482 11/11/17 2:47 PM 10.3 Wave Propagation in Lossy Dielectrics   483 FIGURE 10.5   An E-field with an x-component traveling in the 1z-direction at times t 5 0 and t 5 Dt ; arrows indicate instantaneous values of E. A A h5 jvm Å s 1 jve 5 0 h 0 lu h 5 0 h 0 e juh (10.32) with 0h0 5 "m/e , s 2 1/4 c1 1 a b d ve tan 2u h 5 s (10.33) ve where 0 # u h # 45º. Substituting eqs. (10.31) and (10.32) into eq.

(10.30) gives or H 5 Re c H5 Eo e2aze j 1vt2bz2 ay d 0 h 0 e juh Eo 2az e cos 1 vt 2 bz 2 u h 2 ay(10.34) 0h0 Notice from eqs. (10.29) and (10.34) that as the wave propagates along az, it decreases or attenuates in amplitude by a factor e2az, and hence a is known as the attenuation constant, or attenuation coefficient, of the medium. It is a measure of the spatial rate of decay of the wave in the medium, measured in nepers per meter (Np/m), and can be expressed in decibels per meter (dB/m). An attenuation of 1 neper denotes a reduction to e21 of the original value, whereas an increase of 1 neper indicates an increase by a factor of e. Hence, for voltages 1 Np 5 20 log10 e 5 8.686 dB(10.35) From eq. (10.23), we notice that if s 5 0, as is the case for a lossless medium and free space, a 5 0 and the wave is not attenuated as it propagates. The quantity b is a measure of the phase shift per unit length in radians per meter and is called the phase constant or 10_Sadiku_Ch10.indd 483 25/09/17 3:16 PM 484   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION wave number. In terms of b, the wave velocity u and wavelength l are, respectively, given by [see eqs. (10.7b) and (10.8)] u5 v 2p (10.36) , l 5 b b We also notice from eqs. (10.29) and (10.34) that E and H are out of phase by uh at any instant of time due to the complex intrinsic impedance of the medium. Thus at any time, E leads H(or H lags E) by uh. Finally, we notice that the ratio of the magnitude of the con duction current density Jc to that of the displacement current density Jd in a lossy medium is 0 Jcs 0 0 sEs 0 s 5 5 5 tan u ve 0 Jds 0 0 jveEs 0 or tan u 5 s ve (10.37) where tan u is known as the loss tangent and u is the loss angle of the medium as illustrated in Figure 10.6. Although a line of demarcation between good conductors and lossy dielec trics is not easy to make, tan u or u may be used to determine how lossy a medium is.

A medium is said to be a good (lossless or perfect) dielectric if tan u is very small 1 s V ve 2 or a good conductor if tan u is very large 1 s W ve 2 . From the viewpoint of wave propaga tion, the characteristic behavior of a medium depends not only on its constitutive param eters s, «, and m but also on the frequency of operation. A medium that is -regarded as a good conductor at low frequencies may be a good dielectric at high frequencies. Note from eqs. (10.33) and (10.37) that u 5 2u h(10.38) From eq. (10.14) = 3 Hs 5 1 s 1 jve 2 Es 5 jve c1 2   5 jvecEs js dE ve s (10.39) FIGURE 10.6   Loss angle of a lossy medium. 10_Sadiku_Ch10.indd 484 25/09/17 3:16 PM 10.3 Wave Propagation in Lossy Dielectrics   485 where or ec 5 e c1 2 j s d 5 e 3 12j tan u 4 (10.40a) ve ec 5 er 2 jes (10.40b) with er 5 e, es 5 s/v , e 5 eoer; «c is called the complex permittivity of the medium. We observe that the ratio of « to « is the loss tangent of the medium; that is, tan u 5 es s 5 (10.41) ve er In subsequent sections, we will consider wave propagation in media of other types that may be regarded as special cases of what we have considered here. Thus we will simply deduce the governing formulas from those obtained for the general case treated in this section. The student is advised not just to memorize the formulas but to observe how they are easily obtained from the formulas for the general case. EXAMPLE 10.2 A lossy dielectric has an intrinsic impedance of 200 l 30° V at a particular radian frequency i v. If, at that frequency, the plane wave propagating through the dielectric has the magnetic field component H 5 10 e2ax cosavt 2 1 xbay A/m 2 find E and a. Solution: The given wave travels along ax so that ak 5 ax; aH 5 ay, so 2aE 5 ak 3 aH 5 ax 3 ay 5 az or aE 5 2az Also Ho 5 10, so Eo 5 h 5 200 l30° 5 200 e jp/6 S Eo 5 2000e jp/6 Ho i Except for the amplitude and phase difference, E and H always have the same form. Hence E 5 Re 1 2000e jp/6e2gxe jvtaE 2 10_Sadiku_Ch10.indd 485 25/09/17 3:16 PM 486   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION or E 5 22e2ax cosavt 2 x p 1 baz kV/m 2 6 Knowing that b 5 1/2, we need to determine .

Since a5v me s 2 c 1 1 c d 2 1d ve Ç 2 Å and b5v me s 2 c 1 1 c d 1 1d ve Ç 2 Å a Å 5 ≥ b 11 c s 2 d 21 ve s 2 11 c d 11 ve Å But 1/2 ¥ s 5 tan 2u h 5 tan 60° 5 "3. Hence, ve a 2 2 1 1/2 1 5 c d 5 b 211 "3 or a5 b "3 5 1 2"3 5 0.2887 Np/m PRACTICE EXERCISE   10.2 A plane wave propagating through a medium with er 5 8, mr 5 2 has E 5 0.5 e2z/3 sin 1 108t 2 bz 2 ax V/m. Determine (a) b (d)   Wave velocity (b)   The loss tangent (e)   H field (c)   Intrinsic impedance Answer: (a) 1.374 rad/m, (b) 0.5154, (c) 177.72 l 13.63° V, (d) 7.278 3 107 m/s, iii (e) 2.817e 2z/3 sin(108 t 2 bz 2 13.63° 2 ay mA/m. 10_Sadiku_Ch10.indd 486 25/09/17 3:16 PM 10.5 Plane Waves in Free Space   487 10.4   PLANE WAVES IN LOSSLESS DIELECTRICS In a lossless dielectric, s V ve. It is a special case of that in Section 10.3 except that s . 0, e 5 eoer, m 5 momr(10.42) Substituting these into eqs. (10.23) and (10.24) gives a 5 0, u5 Also b 5 v"me(10.43a) v 1 2p 5 , l 5 (10.43b) b b "me h5 m l0°(10.44) Åe and thus E and H are in time phase with each other. 10.5 PLANE WAVES IN FREE SPACE Plane waves in free space comprise a special case of what we considered in Section 10.3. In this case, s 5 0, e 5 eo, m 5 mo(10.45) This may also be regarded as a special case of Section 10.4. Thus we simply replace « by «o and by o in eq. (10.43), or we substitute eq. (10.45) directly into eqs.

(10.23) and (10.24). Either way, we obtain a 5 0, b 5 v"moeo 5 u5 1 "moeo 5 c, l 5 v (10.46a) c 2p (10.46b) b where c . 3 3 108 m/s, the speed of light in a vacuum. The fact that EM waves travel in free space at the speed of light is significant. It provides some evidence that light is the manifestation of an EM wave. In other words, light is characteristically electromagnetic. 10_Sadiku_Ch10.indd 487 25/09/17 3:16 PM 488   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION By substituting the constitutive parameters in eq. (10.45) into eq. (10.33), u h 5 0 and h 5 ho, where ho is called the intrinsic impedance of free space and is given by ho 5 mo 5 120p . 377 V Å eo (10.47) E 5 Eo cos 1 vt 2 bz 2 ax(10.48a) then H 5 Ho cos 1 vt 2 bz 2 ay 5 Eo cos 1 vt 2 bz 2 ay(10.48b) ho The plots of E and H are shown in Figure 10.7(a). In general, if aE, aH, and ak are unit vectors along the E field, the H field, and the direction of wave propagation; it can be shown that (see Problem 10.69). ak 3 aE 5 aH or ak 3 aH 5 2aE or aE 3 aH 5 ak(10.49) Both E and H fields (or EM waves) are everywhere normal to the direction of wave propa gation, ak. That means that the fields lie in a plane that is transverse or orthogonal to the FIGURE 10.7   Plots of E and H (a) as functions of z at t 5 0; and (b) at z 5 0. The arrows indicate instantaneous values. 10_Sadiku_Ch10.indd 488 25/09/17 3:16 PM 10.6 Plane Waves in Good Conductors   489 direction of wave propagation. They form an EM wave that has no electric or magnetic field components along the direction of propagation; such a wave is called a transverse electromagnetic (TEM) wave. A combination of E and H is called a uniform plane wave because E (or H) has the same magnitude throughout any transverse plane, defined by z 5 constant. The direction in which the electric field points is the polarization of a TEM wave.1 The wave in eq. (10.29), for example, is polarized in the x-direction. This should be observed in Figure 10.7(b), which illustrates uniform plane waves.

A uniform plane wave cannot exist physically because it stretches to infinity and would represent an infinite energy. Such waves are characteristically simple and fundamentally important. They serve as approximations to practical waves such as those from a radio antenna at distances suf ficiently far from radiating sources. Although our discussion after eq. (10.48) deals with free space, it also applies for any other isotropic medium. 10.6   PLANE WAVES IN GOOD CONDUCTORS Plane waves in good conductors comprise another special case of that considered in s W 1; that is, Section 10.3. A perfect, or good conductor, is one in which s W ve, so that ve, s . `, e 5 eo, m 5 momr(10.50) Hence, eqs. (10.23) and (10.24) become a5b5 u5 vms 5 "pfms Å 2 2v v 5 , b Å ms l5 2p b (10.51a) (10.51b) Also, from eq. (10.32), h5 and thus E leads H by 45°. If E 5 Eoe2azcos 1 vt 2 bz 2 ax(10.53a) then 1 10_Sadiku_Ch10.indd 489 jvm vm 5 l45°(10.52) Å s Å s H5 Eo vm Å s e2az cos 1 vt 2 bz 2 45° 2 ay(10.53b) Polarization will be covered in Section 10.7. 25/09/17 3:16 PM 490   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION Therefore, as the E (or H) wave travels in a conducting medium, its amplitude is attenuated by the factor e2az. The distance d, shown in Figure 10.8, through which the wave amplitude decreases to a factor e21 (about 37% of the original value) is called skin depth or penetration depth of the medium; that is, Eoe2ad 5 Eoe21 or 1 d 5 (10.54a) a The skin depth is a measure of the depth to which an EM wave can penetrate the medium. Equation (10.54a) is generally valid for any material medium. For good conductors, eqs. (10.51a) and (10.54a) give d5 1 1 5 a !pfms (10.54b) The illustration in Figure 10.8 for a good conductor is exaggerated. However, for a partially conducting medium, the skin depth can be quite large. Note from eqs. (10.51a), (10.52), and (10.54b) that for a good conductor, h5 11j 1 "2 e jp/4 5 sd sd (10.55) 1 Noting that for good conductors we have a 5 b 5 , eq. (10.53a) can be written as d z E 5 Eoe2z/d cosavt 2 bax d showing that d measures the exponential damping of the wave as it travels through the conductor. The skin depth in copper at various frequencies is shown in Table 10.2. From Table 10.2, we notice that the skin depth decreases with increasing frequency. Thus, E and H can hardly propagate through good conductors. The phenomenon whereby field intensity in a conductor rapidly decreases is known as the skin effect. It is a tendency of charges to migrate from the bulk of the conducting material to the surface, resulting in higher resistance. The fields and associated currents are confined to a very thin layer (the skin) of the conductor surface. For a wire of radius a, for example, it is a good approximation at high frequencies to assume that all of the current 10_Sadiku_Ch10.indd 490 25/09/17 3:16 PM 10.6 Plane Waves in Good Conductors   491 FIGURE 10.8   Illustration of skin depth. flows in the circular ring of thickness d as shown in Figure 10.9. The skin effect appears in different guises in such problems as attenuation in waveguides, effective or ac resistance of transmission lines, and electromagnetic shielding. It is used to advantage in many applica tions. For example, because the skin depth in silver is very small, the difference in perfor mance between a pure silver component and a silver-plated brass component is negligible, so silver plating is often used to reduce the material cost of waveguide components. For the same reason, hollow tubular conductors are used instead of solid conductors in out door television antennas. Effective electromagnetic shielding of electrical devices can be provided by conductive enclosures a few skin depths in thickness. The skin depth is useful in calculating the ac resistance due to skin effect. The resis tance in eq. (5.16) is called the dc resistance, that is, Rdc 5 , sS We define the surface or skin resistance Rs (in V) as the real part of h for a good conductor. Thus from eq. (10.55) Rs 5 pfm 1 5 sd Å s (10.56) This is the resistance of a unit width and unit length of the conductor. It is equivalent to the dc resistance for a unit length of the conductor having cross-sectional area 1 3 d. Thus TABLE 10.2   Skin Depth in Copper* Frequency (Hz) 10 60 100 500 104 108 1010 Skin depth (mm) 20.8 8.6 6.6 2.99 0.66 6.6 3 1023 6.6 3 1024 For copper, s 5 5.8 3 107 S/m, m 5 mo, d 5 66.1/ !f (in mm). * 10_Sadiku_Ch10.indd 491 25/09/17 3:16 PM 492   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION FIGURE 10.9   Skin depth at high frequencies, d V a. for a given width w and length , the ac resistance is calculated by using the familiar dc resistance relation of eq. (5.16) and assuming a uniform current flow in the conductor of thickness d; that is, Rac 5 Rs, , 5 (10.57) w sdw where S . dw. For a conductor wire of radius a (see Figure 10.9), w 5 2pa, so , Rac s2pad a a 5 5 5 !pfms Rdc , 2d 2 2 spa Since d V a at high frequencies, this shows that Rac is far greater than Rdc.

In general, the ratio of the ac to the dc resistance starts at 1.0 for dc and very low frequencies and increases as the frequency increases. Also, although the bulk of the current is nonuniformly distrib uted over a thickness of 5d of the conductor, the power loss is the same as though it were uniformly distributed over a thickness of d and zero elsewhere. This is one more reason that d is referred to as the skin depth. For easy reference, the formulas for propagation constants for different media are summarized in Table 10.3. TABLE 10.3   Formulas, for a, b, h, n, and l Lossy Medium Lossless Medium 1/2 a5 0 Free Space 0 "pfms s 2 v D D 1 1 a b 2 1T T 2 Ñ ve me 1/2 b5 me v"me 2 v"moeo s v D D 1 1 a b 1 1T T 2 Ñ ve jvm h5 n5 10_Sadiku_Ch10.indd 492 v , b Å s 1 jve l5 2p b m mo Åe Å eo Conductor . 377 "pfms (1 1 j) a s 11/11/17 2:47 PM 10.6 Plane Waves in Good Conductors   493 EXAMPLE 10.3 In a lossless dielectric for which h 5 60p, mr 5 1, and H 5 20.1 cos 1 vt 2 z 2 ax 1 0.5 sin 1 vt 2 z 2 ay A/m, calculate «r, v, and E. Solution: In this case, s 5 0, a 5 0, and b 5 1, so h 5 "m/e 5 mo mr 120p 5 Å eo Å er "er or "er 5 120p 120p 52 5 h 60p S b 5 v"me 5 v"moeo "mrer 5 er 5 4 v"er 4 "4 5 c c or v5 bc 1 1 3 3 108 2 5 5 1.5 3 108 rad/s 2 2 From the given H field, E can be calculated in two ways: by using the techniques (based on Maxwell's equations) developed in this chapter or directly, by using Maxwell's equations as in Chapter 9. Method 1:   To use the techniques developed in the present chapter, we let H 5 H1 1 H2 where H1 5 20.1 cos 1 vt 2 z 2 ax and H2 5 0.5 sin 1 vt 2 z 2 ay and the corresponding electric field E 5 E1 1 E2 where E1 5 E1o cos 1 vt 2 z 2 aE1 and E2 5 E2o sin 1 vt 2 z 2 aE2. Notice that although H has components along ax and ay, it has no component along the direction of propagation; it is therefore a TEM wave. For E1, aE1 5 2 1 ak 3 aH1 2 5 2 1 az 3 2ax 2 5 ay Hence E1o 5 hH1o 5 60p 1 0.1 2 5 6p E1 5 6p cos 1 vt 2 z 2 ay 10_Sadiku_Ch10.indd 493 25/09/17 3:16 PM 494   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION For E2, aE2 5 2 1 ak 3 aH 2 2 5 2 1 a z 3 ay 2 5 ax E2o 5 hH2o 5 60p 1 0.5 2 5 30p Hence E2 5 30p sin 1 vt 2 z 2 ax Adding E1 and E2 gives E; that is, E 5 94.25 sin 1 1.5 3 108t 2 z 2 ax 1 18.85 cos 1 1.5 3 108t 2 z 2 ay V/m Method 2:   We may apply Maxwell's equations directly = 3 H 5 sE 1 e → 'E 't S E5 1 = 3 H dt e3 0 because s 5 0. But ax ' =3H5 ∞ 'x Hx 1 z 2 ay ' 'y Hy 1 z 2 az 'Hy 'Hx ' ∞ 52 ax 1 a 'z 'z 'z y 0 5 H2o cos 1 vt 2 z 2 ax 1 H1o sin 1 vt 2 z 2 ay where H1o 5 20.1 and H2o 5 0.5. Hence H2o H1o 1 = 3 H dt 5 sin 1 vt 2 z 2 ax 2 cos 1 vt 2 z 2 ay e3 ev ev 5 94.25 sin 1 vt 2 z 2 ax 1 18.85 cos 1 vt 2 z 2 ay V/m E5 as expected. PRACTICE EXERCISE   10.3 A plane wave in a nonmagnetic medium has E 5 50 sin 1 108t 1 2z 2 ay V/m. Find (a)   The direction of wave propagation (b)   l, f, and er (c)   H Answer: (a) in the 2z-direction, (b) 3.142 m, 15.92 MHz, 36, (c) 0.7958 sin 1 108t 1 2z 2 ax A/m. 10_Sadiku_Ch10.indd 494 25/09/17 3:16 PM 10.6 Plane Waves in Good Conductors   495 EXAMPLE 10.4 A uniform plane wave propagating in a medium has E 5 2e2az sin 1 108t 2 bz 2 ay V/m If the medium is characterized by er 5 1, mr 5 20, and s 5 3 S/m, find a, b, and H. Solution: We need to determine the loss tangent to be able to tell whether the medium is a lossy dielectric or a good conductor.

s 5 ve 3 1029 10 3 1 3 36p 8 5 3393 W 1 showing that the medium may be regarded as a good conductor at the frequency of operation. Hence, a5b5 mvs 4p 3 1027 3 20 1 108 2 1 3 2 1/2 5 c d Å 2 2 5 61.4 a 5 61.4 Np/m, b 5 61.4 rad/m Also 0h0 5 mv 4p 3 1027 3 20 1 108 2 1/2 5 c d Å s 3 800p Å 3 s tan 2u h 5 5 3393 ve 5 u h 5 45º 5 S p 4 Hence H 5 Hoe2az sinavt 2 bz 2 p ba 4 H where aH 5 ak 3 aE 5 az 3 ay 5 2ax and Ho 5 Eo 3 52 5 69.1 3 1023 0h0 Å 800p Thus H 5 269.1 e261.4z sina108t 2 61.42z 2 10_Sadiku_Ch10.indd 495 p ba mA/m 4 x 25/09/17 3:16 PM 496   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION PRACTICE EXERCISE   10.4 A plane wave traveling in the 1y-direction in a lossy medium 1 er 5 4, mr 5 1, s 5 1022 S/m 2 has E 5 30 cos 1 109p t 1 p/4 2 az V/m at y 5 0.

Find (a)   E at y 5 1 m, t 5 2 ns (b)   The distance traveled by the wave to have a phase shift of 10 (c)   The distance traveled by the wave to have its amplitude reduced by 40% (d)   H at y 5 2 m, t 5 2 ns Answer: (a) 2.844az V/m,   (b) 8.349 mm,   (c) 542 mm,   (d) 222.6ax mA/m.

EXAMPLE 10.5 A plane wave E 5 Eo cos 1 vt 2 bz 2 ax is incident on a good conductor at z $ 0. Find the current density in the conductor. Solution: Since the current density J 5 sE, we expect J to satisfy the wave equation in eq. (10.17); that is, we expect to find =2Js 2 g2Js 5 0 Also the incident E has only an x-component and varies with z. Hence J 5 Jx 1 z, t 2 ax and d2 Jsx 2 g2Jsx 5 0 dz2 which is an ordinary differential equation with solution (see Case 2 of Example 6.5) Jsx 5 Ae2gz 1 Be1gz The constant B must be zero to have Jsx is finite as z S ` . But in a good conductor, s W ve, so that a 5 b 5 1/d. Hence and g 5 a 1 jb 5 a 1 1 1 j 2 5 11 1 j2 d Jsx 5 Ae2z111j2/d or Jsx 5 Jsx 1 0 2 e2z111j2/d where Jsx (0) is the current density on the conductor surface.   10_Sadiku_Ch10.indd 496 25/09/17 3:16 PM 10.6 Plane Waves in Good Conductors   497 PRACTICE EXERCISE   10.5 Given the current density of Example 10.5, find the magnitude of the total current through a strip of the conductor of infinite depth along z and width w along y. Jsx 1 0 2 wd Answer: .

"2 EXAMPLE 10.6 For the copper coaxial cable of Figure 7.12, let a 5 2 mm, b 5 6 mm, and t 5 1 mm. Calculate the resistance of a 2 m length of the cable at dc and at 100 MHz. Solution: Let R 5 Ro 1 Ri where Ro and Ri are the resistances of the inner and outer conductors. At dc, , , 2 5 5 2.744 mV 2 5 7 sS spa 5.8 3 10 p 3 2 3 1023 4 2 , , , 5 Ro 5 5 sS sp 3 3 b 1 t 4 2 2 b2 4 sp 3 t2 1 2bt 4 2 5 7 5.8 3 10 p 3 1 1 12 4 3 1026 5 0.8429 mV Ri 5 Hence Rdc 5 2.744 1 0.8429 5 3.587 mV. At f 5 100 MHz, Ri 5 pfm Rs, , , 5 5 w sd2pa 2pa Å s 2 p 3 108 3 4p 3 1027 23 5.8 3 107 2p 3 2 3 10 Å 5 0.41 V 5 Since d 5 6.6 mm V t 5 1 mm, w 5 2pb for the outer conductor. Hence, pfm Rs, , 5 w 2pb Å s 2 p 3 108 3 4p 3 1027 5 5.8 3 107 2p 3 6 3 1023 Å 5 0.1384 V Ro 5 10_Sadiku_Ch10.indd 497 25/09/17 3:16 PM 498   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION Hence, Rac 5 0.41 1 0.1384 5 0.5484 V which is about 150 times greater than Rdc.

Thus, for the same effective current i, the ohmic loss (i2R) of the cable at 100 MHz is greater than the dc power loss by a factor of 150.   PRACTICE EXERCISE   10.6 For an aluminum wire having a diameter 2.6 mm, calculate the ratio of ac to dc resistance at (a)   10 MHz (b)   2 GHz Answer: (a) 24.16,   (b) 341.7. 10.7   WAVE POLARIZATION It is a common practice to describe an EM wave by its polarization.

Polarization is an important property of an EM wave, and the concept has been developed to describe the various types of electric field variation and orientation. The polarization of an EM wave depends on the transmitting antenna or source. It is determined by the direction of the electric field for fields having more than one component. Polarization may be regarded as the locus of the tip of the electric field (in a plane perpendicular to the direction of propagation) at a given point as a function of time. There are three types of polarization: linear or plane, circular, and elliptical. That means that the tip of the electric field can describe a straight line, a circle, or an ellipse with time, as shown in Figure 10.10. Wave polarization is important for radio and TV broad casting. Amplitude modulation (AM) radio broadcasting is with polarization vertical to the earth's surface, while frequency modulation (FM) broadcasting is generally circularly polarized. A uniform plane wave is linearly polarized if it has only one component or when its transverse components are in phase. For a wave traveling in the +z-direction, we may have Ex 5 Eox cos(t 2 bz 1 x)(10.58) Ey 5 Eoy cos(t 2 bz 1 y) where Eox and Eoy are real. The composite wave 10_Sadiku_Ch10.indd 498 25/09/17 3:16 PM (a) (a) 10.7 Wave Polarization   499 y y y E E E x x (b) (a) x (b) y y y E E E x x x (c) (c) FIGURE 10.10 (b)Wave polarizations: (a) linear, (b) circular, (c) elliptical. y E 5 Eox cos(t 2 bz 1 x)ax 1 Eoy cos(t 2 bz 1 y)ay(10.59) is linearly polarized when the phase difference Df is Df 5 fy 2 fx 5 np,   n 5 0, 1, 2, .

. . (10.60) This allows the two components to maintain the same ratio at all times. If we observe the wave in the direction of propagation (z in this case), we will notice that the tip of the elec tric field follows a line—hence, the term linear polarization. Linearly polarized plane waves can be generated by simple antennas (such as dipole antennas) or lasers. Circular polarization takes place when the x and ycomponents are the same in magnitude (Eox 5 Eoy 5 Eo) and the phase difference between them is an odd multiple of p/2; that is, 10_Sadiku_Ch10.indd 499 Df 5 fy 2 fx 5 (2n 1 1)p/2 ,   n 5 0, 1, 2, . . . (10.61) 25/09/17 3:16 PM 500   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION For example, the x- and y-components may be of the form Ex 5 Eo cos(t 2 bz)(10.62) Ey 5 Eo cos(t 2 bz 1 /2) The tip of the composite electric field as observed as a fixed point in the xy-plane moves along a circle as time progresses. Circularly polarized waves can be generated by a helically wound wire antenna or by two linear sources that are oriented perpendicular to each other and fed with currents that are out of phase by 90. That the locus of total field traces a circle can be seen if we examine the components at a point, say z 5 0, Ex 5 Eo cos(t),   Ey 5 Eo cos(t 1 /2) 5 2Eo sin(t)(10.63) E2 5 E2x 1 E2y 5 E2o (10.64) which is the equation of a circle. Linear and circular polarizations are special cases of the more general case of the ellip tical polarization. An elliptically polarized wave is one in which the tip of the field traces an elliptic locus in a fixed transverse plane as the field changes with time. Elliptical polariza tion is achieved when the x- and y-components are not equal in magnitude Eox Eoy and the phase difference between them is an odd multiple of p/2; that is, Df 5 fy 2 fx 5 (2n 1 1)p/2,   n 5 0, 1, 2, . .

. (10.65) This allows the tip of the electric field to trace an ellipse in the xy-plane. To show that this is the case, consider eq. (10.58) when z 5 0 and Df 5 fy 2 fx 5 p/2, Ex 5 Eoz cos(t)   →   cos(t) 5 Ex Eox Ey 5 Eoy cos(t 1 /2) 5 2Eo sin(t)   →   2sin(t) 5 Ey (10.66) Eoy Squaring and adding these equations yields cos2 (t) 1 sin2 (t) 5 1   →   E2y E2x 1 5 1 E2ox E2oy (10.67) which is the equation of an ellipse, as shown in Figure 10.10. Notice that if Eox 5 Eoy , we have circular polarization. Thus, circular polarization is a special case of elliptical polariza tion. In fact, we can show that linear polarization is also a special case of elliptical polariza tion. Thus, the most general case is elliptical polarization. EXAMPLE 10.7 Determine the polarization of a plane wave with: (a)   E(z, t) 5 4e20.25z cos(t 2 0.8z)ax 1 3e20.25z sin(t 2 0.8z)ay V/m (b)   Hs(z) 5 Ho e2jbz ax 2 2Ho e2jbz ay   Solution: (a)   From the given E, Ex5 4e20.25z cos(t 2 0.8z) 10_Sadiku_Ch10.indd 500 25/09/17 3:16 PM 10.7 Wave Polarization   501 Ey5 3e20.25z sin(t 2 0.8z) In the z 5 0 plane, we have 1 E (0, t) 5 cos(t) 4 x 1 Ex(0, t) 5 sin(t) 3 Squaring and adding gives 1 2 1 Ex 1 0, t 2 1 E2y 1 0, t 2 5 1 16 9 which describes an ellipse. Hence, the wave is elliptically polarized. (b) The two components of H are in phase; hence, the polarization is linear. For proper characterization, it is expedient to find the electric field component. This can be done in many ways. Using Maxwell's equation,   Hs jEs → Es 5 ax ' 3 Hs 5 ∞ 'x Hoe2jbz ay ' 'y 22Hoe2jbz 1   Hs jve az ' ∞ 5 j2bHoe2jbz ax 2 jbHoe2jbz ay 'z 0 Dividing both sides by j« and setting h = b/« yields Es 5 2Hoe2jbz ax 2 Hoe2jbz ay In the time domain, E(z, t)5 Re[Ese2jt] 5 2Hocos(t 2 bz)ax 2 Hocos(t 2 bz)ay If we set z 5 0, E(0, t) 5 (2Hoax 2 Hoay) cos(t) At t 5 0, E has components 2hHo in the x-direction and hHo in the y-direction.

The ratio Ey/Ex remains the same as t changes. Hence, E is linearly polarized. PRACTICE EXERCISE   10.7 Given that Es5 Eo (ay 2 jay)e2jbz, determine the polarization. Answer: Circular polarization. 10_Sadiku_Ch10.indd 501 25/09/17 3:16 PM 502   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION 10.8   POWER AND THE POYNTING VECTOR As mentioned before, energy can be transported from one point (where a transmitter is located) to another point (with a receiver) by means of EM waves. The rate of such energy transportation can be obtained from Maxwell's equations: 'H (10.68a) 't 'E = 3 H 5 sE 1 e (10.68b) 't = 3 E 5 2m Dotting both sides of eq. (10.68b) with E gives 'E E # 1 = 3 H 2 5 sE2 1 E # e (10.69) et But for any vector fields A and B (see Appendix A.10) = # 1A 3 B2 5 B # 1= 3 A2 2 A # 1= 3 B2 Applying this vector identity to eq. (10.69) 1 letting A 5 H and B 5 E 2 gives 'E H # 1 = 3 E 2 1 = # 1 H 3 E 2 5 sE2 1 E # e (10.70) 't 1 ' 5 sE2 1 e E2 2 't Dotting both sides of eq. (10.68a) with H, we write H # 1 = 3 E 2 5 H # a2m and thus eq. (10.70) becomes 2 m ' 'H 1 H # H 2 (10.71) b52 't 2 't m 'H2 1 'E2 2 = # 1 E 3 H 2 5 sE2 1 e 2 't 2 't Rearranging terms and taking the volume integral of both sides, ' 1 2 1 2 2 # 3 = 1 E 3 H 2 dv 5 2 3 c eE 1 mH d dv 2 3 sE dv(10.72) v 't v 2 2 v Applying the divergence theorem to the left-hand side gives 10_Sadiku_Ch10.indd 502 ' 1 2 1 2 2 # C 1 E 3 H 2 dS 5 2 't 3 c 2 eE 1 2 mH d dv 2 3 sE dv(10.73) S v v T T T total power rate of decrease in ohmic power (10.74) leaving the volume 5 energy stored in electric 2 dissipated and magnetic fields 25/09/17 3:16 PM 10.8 Power and the Poynting Vector   503 Equation (10.73) is referred to as Poynting's theorem.2 The various terms in the equation are identified using energy-conservation arguments for EM fields. The first term on the right-hand side of eq. (10.73) is interpreted as the rate of decrease in energy stored in the electric and magnetic fields. The second term is the power dissipated because the medium is conducting 1 s 2 0 2 .

The quantity E 3 H on the left-hand side of eq. (10.73) is known as the Poynting vector , measured in watts per square meter (W/m2); that is, 5 E 3 H(10.75) It represents the instantaneous power density vector associated with the EM field at a given point. The integration of the Poynting vector over any closed surface gives the net power flowing out of that surface. Poynting's theorem states that the net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within v minus the ohmic losses.

The theorem is illustrated in Figure 10.11.

It should be noted that is normal to both E and H and is therefore along the direction of wave propagation ak for uniform plane waves. Thus ak 5 aE 3 aH(10.49) The fact that points along ak causes to be regarded as a "pointing" vector. Again, if we assume that E 1 z, t 2 5 Eoe2az cos 1 vt 2 bz 2 ax then and H 1 z, t 2 5 1 z, t 2 5 5 Eo 2az e cos 1 vt 2 bz 2 u h 2 ay 0h0 E2o 22az e cos 1 vt 2 bz 2 cos 1 vt 2 bz 2 u h 2 az 0h0 E2o 22az 3 cos u h 1 cos 1 2vt 2 2bz 2 u h 2 4 az e 20h0 (10.76) 2 After J. H. Poynting, "On the transfer of energy in the electromagnetic field," Philosophical Transactions, vol. 174, 1883, p. 343. 10_Sadiku_Ch10.indd 503 25/09/17 3:16 PM 504   CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION FIGURE 10.11   Illustration of power balance for EM fields. S S since cos A cos B 5 12 3 cos 1 A 2 B 2 1 cos 1 A 1 B 2 4 . To determine the time-average Poynting vector ave(z) (in W/m2), which is of more practical value than the instantaneous Poynting vector (z, t), we integrate eq. (10.76) over the period T 5 2p/v; that is, T ave 1 z 2 5 1 3 1 z, t 2 dt T 0 (10.77) It can be shown (see Problem 10.42) that this is equivalent to ave 1 z 2 5 1 Re 1 Es 3 H*s 2 2 (10.78) By substituting eq. (10.76) into eq. (10.77), we obtain ave 1 z 2 5 E2o 22az cos u h az e 20h0 (10.79) The total time-average power crossing a given surface S is given by Pave 5 3 ave # dS (10.80) S 10_Sadiku_Ch10.indd 504 25/09/17 3:16 PM 10.8 Power and the Poynting Vector   505 We should note the difference between , ave, and Pave: whereas 1 x, y, z, t 2 is the Poynting vector in watts per square meter and is time varying, ave(x, y, z), also in watts per square meter, is the time average of the Poynting vector ; it is a vector but is time invariant. Finally, Pave is a total time-average power through a surface in watts; it is a scalar. EXAMPLE 10.8 In a nonmagnetic medium E 5 4 sin 1 2p 3 107t 2 0.8x 2 az V/m Find (a) «r, h (b) The time-average power carried by the wave (c) The total power crossing 100 cm2 of plane 2x 1 y 5 5 Solution: (a) Since a 5 0 and b 2 v/c, the medium is not free space but a lossless medium: Hence b 5 0.8, v 5 2p 3 107, m 5 mo 1 nonmagnetic 2 , e 5 eoer b 5 v"me 5 v"moeoer 5 v "er c or "er 5 bc 0.8 1 3 3 108 2 12 5 5 v p 2p 3 107 er 5 14.59 h5 mo m 120p p 5 5 5 120p # 5 10p2 Åe Å eoer 12 "er 5 98.7 V (b) 5 E 3 H 5 E2o h sin2 1 vt 2 bx 2 ax ave 5 T E2o 1 16 a 5 a 3 dt 5 T 0 2h x 2 3 10p2 x 5 81ax mW/m2 (c) On plane 2x 1 y 5 5 (see Example 3.5 or 8.5), 2ax 1 ay an 5 "5 10_Sadiku_Ch10.indd 505 25/09/17 3:16 PM 506   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION Hence the total power is Pave 5 3 ave # dS 5 ave # S an 5 1 81 3 1023ax 2 # 1 100 3 1024 2 c 5 162 3 1025 "5 2ax 1 ay "5 d 5 724.5 mW PRACTICE EXERCISE   10.8 In free space, H 5 0.2 cos 1 vt 2 bx 2 az A/m. Find the total power passing through: (a)   A square plate of side 10 cm on plane x 1 y 5 1 (b)   A circular disk of radius 5 cm on plane x 5 1. Answer: (a) 53.31 mW,   (b) 59.22 mW. 10.9   REFLECTION OF A PLANE WAVE AT NORMAL INCIDENCE So far, we have considered uniform plane waves traveling in unbounded, homogeneous, isotropic media. When a plane wave from one medium meets a different medium, it is partly reflected and partly transmitted.

The proportion of the incident wave that is reflected or transmitted depends on the constitutive parameters 1 e, m, s 2 of the two media involved. Here we will assume that the incident plane wave is normal to the boundary between the media; oblique incidence of plane waves will be covered in the next section after we have presented the simpler case of normal incidence. Suppose that a plane wave propagating along the 1z-direction is incident normally on the boundary z 5 0 between medium 1 1 z , 0 2 characterized by s1, «1, m1 and medium 2 1 z . 0 2 characterized by s2, «2, m2, as shown in Figure 10.12.

In Figure 10.12, subscripts i, r, and t denote incident, reflected, and transmitted waves, respectively. The incident, reflected, and transmitted waves shown in Figure 10.12 are obtained as follows. Incident Wave 1 Ei, Hi 2 is traveling along 1az in medium 1.

If we suppress the time factor e jvt and assume that 10_Sadiku_Ch10.indd 506 Eis 1 z 2 5 Eioe2g1z ax(10.81) 25/09/17 3:16 PM 10.9 Reflection of a Plane Wave at Normal Incidence   507 M 1 M I T R FIGURE 10.12   A plane wave incident normally on an interface between two different media. then His 1 z 2 5 Hioe2g1z ay 5 Eio 2g1z e ay(10.82) h1 Reflected Wave 1 Er, Hr 2 is traveling along 2az in medium 1. If then Ers 1 z 2 5 Eroeg1z ax(10.83) Hrs 1 z 2 5 Hro eg1z 1 2ay 2 5 2 Ero g1z e ay(10.84) h1 where Ers has been assumed to be along ax.

To satisfy the necessary boundary conditions at the interface, we will consistently assume that for normal incidence Ei, Er, and Et have the same polarization. Transmitted Wave 1 Et, Ht 2 is traveling along 1az in medium 2. If then 10_Sadiku_Ch10.indd 507 Ets 1 z 2 5 Eto e2g2z ax(10.85) Hts 1 z 2 5 Hto e2g2z ay 5 Eto 2g2z e ay(10.86) h2 25/09/17 3:16 PM 508   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION In eqs. (10.81) to (10.86), Eio, Ero, and Eto are, respectively, the magnitudes of the incident, reflected, and transmitted electric fields at z 5 0. Notice from Figure 10.12 that the total field in medium 1 comprises both the incident and the reflected fields, whereas medium 2 has only the transmitted field; that is, E1 5 Ei 1 Er, E2 5 Et, H1 5 Hi 1 Hr H2 5 Ht At the interface z 5 0, the boundary conditions require that the tangential components of E and H fields must be continuous. Since the waves are transverse, E and H fields are entirely tangential to the interface. Hence at z 5 0, E1tan 5 E2tan and H1tan 5 H2tan imply that Ei 1 0 2 1 Er 1 0 2 5 Et 1 0 2 Hi 1 0 2 1 Hr 1 0 2 5 Ht 1 0 2 From eqs. (10.87) and (10.88), we obtain S S Eio 1 Ero 5 Eto(10.87) E 1 1 E 2 Ero 2 5 to (10.88) h1 io h2 Ero 5 h2 2 h1 E (10.89) h2 1 h1 io Eto 5 2h2 E (10.90) h2 1 h1 io and We now define the reflection coefficient G and the transmission coefficient t from eqs.

(10.89) and (10.90) as G5 Ero h2 2 h1 (10.91a) 5 Eio h2 1 h1 or Ero 5 GEio(10.91b) and t5 Eto 2h2 (10.92a) 5 Eio h2 1 h1 or 10_Sadiku_Ch10.indd 508 Eto 5 tEio(10.92b) 25/09/17 3:16 PM 10.9 Reflection of a Plane Wave at Normal Incidence   509 Note that 1. 1 1 G 5 t 2. Both and are dimensionless and may be complex. 3. 0 # 0 G 0 # 1(10.93) The case just considered is the general case. Let us now consider the following special case: medium 1 is a perfect dielectric 1 lossless, s1 5 0 2 and medium 2 is a perfect conduc tor 1 s2 . ` 2 . For this case, h2 5 0; hence, G 5 21, and t 5 0, showing that the wave is totally reflected. This should be expected because fields in a perfect conductor must vanish, so there can be no transmitted wave 1 E2 5 0 2 . The totally reflected wave combines with the incident wave to form a standing wave. A standing wave "stands" and does not travel; it consists of two traveling waves 1 Ei and Er 2 of equal amplitudes but in opposite directions. Combining eqs. (10.81) and (10.83) gives the standing wave in medium 1 as But E1s 5 Eis 1 Ers 5 1 Eioe2g1z 1 Eroeg1z 2 ax(10.94) G5 Ero 5 21, s1 5 0, a1 5 0, g1 5 jb1 Eio Hence, or E1s 5 2Eio 1 e jb1z 2 e2jb1z 2 ax E1s 5 22jEio sin b1z ax(10.95) Thus or E1 5 Re 1 E1se jvt 2 E1 5 2Eio sin b1z sin vt ax(10.96) By taking similar steps, it can be shown that the magnetic field component of the wave is H1 5 2Eio cos b1z cos vt ay(10.97) h1 A sketch of the standing wave in eq. (10.96) is presented in Figure 10.13 for t 5 0, T/8, T/4, 3T/8, T/2, and so on, where T 5 2p/v . From Figure 10.13, we notice that the wave does not travel but oscillates. When media 1 and 2 are both lossless, we have another special case: s1 5 0 5 s2.

In this case, h1 and h2 are real and so are G and t. Let us consider two more cases: 10_Sadiku_Ch10.indd 509 25/09/17 3:16 PM 510   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION FIGURE   10.13   Standing waves E 5 2Eio sin b1z sin vt ax. The curves 0, 1, 2, 3, 4, . . .

, are, respectively, at times t 5 0, T/8, T/4, 3T/8, T/2, . . . ; l 5 2p/b1. CASE 1. If h2 . h1, G . 0. Again there is a standing wave in medium 1, but there is also a transmit ted wave in medium 2. However, the incident and reflected waves have amplitudes that are not equal in magnitude. It can be shown that a relative maximum of 0 E1 0 occurs at 2b1zmax 5 np or zmax 5 2 nl1 np 52 , b1 2 n 5 0, 1, 2, . . .(10.98) and the minimum values of 0 E1 0 occur at 2b1zmin 5 1 2n 1 1 2 or zmin 5 2 1 2n 1 1 2 p 2b1 52 1 2n 1 1 2 4 p 2 l1, n 5 0, 1, 2, . . .(10.99) CASE 2. If h2 , h1, G , 0. For this case, the locations of 0 E1 0 maximum are given by eq. (10.99), whereas those of 0 E1 0 minimum are given by eq. (10.98).

All these are illustrated in Figure 10.14.

Note that 10_Sadiku_Ch10.indd 510 25/09/17 3:17 PM 10.9 Reflection of a Plane Wave at Normal Incidence   511 4 FIGURE 10.14   Standing waves due to reflection at an interface between two lossless media; l 5 2p/b1. 1. 0 H1 0 minimum occurs whenever there is 0 E1 0 maximum, and vice versa. 2. The transmitted wave (not shown in Figure 10.14) in medium 2 is a purely travel ing wave, and consequently there are no maxima or minima in this region. The ratio of 0 E1 0 max to 0 E1 0 min 1 or 0 H1 0 max to 0 H1 0 min 2 is called the standing wave ratio s; that is, or s5 0 E1 0 max 0 H1 0 max 1 1 0G0 (10.100) 5 5 0 E1 0 min 0 H1 0 min 1 2 0G0 0G0 5 s21 (10.101) s11 Since 0 G 0 # 1, it follows that 1 # s # ` . The standing wave ratio is dimensionless, and it is customarily expressed in decibels (dB) as EXAMPLE 10.9 s dB 5 20 log10 s(10.102) In free space 1 z # 0 2 , a plane wave with Hi 5 10 cos 1 108t 2 bz 2 ax mA/m is incident normally on a lossless medium 1 e 5 2eo, m 5 8mo 2 in region z $ 0.

Determine the reflected wave Hr, Er and the transmitted wave Ht, Et. 10_Sadiku_Ch10.indd 511 25/09/17 3:17 PM 512   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION Solution: This problem can be solved in two different ways. Method 1:   Consider the problem as illustrated in Figure 10.15. For free space, b1 5 108 v 1 5 5 c 3 3 3 108 h1 5 ho 5 120p For the lossless dielectric medium, b2 5 v"me 5 v"moeo"mr er 5 h2 5 v# 4 1 4 2 5 4b1 5 c 3 mo mr m 5 5 2 ho Åe Å eo Å er Given that Hi 5 10 cos 1 108t 2 b1z 2 ax mA/m, we expect that Ei 5 Eio cos 1 108t 2 b1z 2 aEi where aEi 5 aHi 3 aki 5 ax 3 az 5 2ay and Eio 5 h1Hio 5 10 ho Hence, Ei 5 210ho cos 1 108t 2 b1z 2 ay mV/m FIGURE 10.15 For Example 10.9. F 10_Sadiku_Ch10.indd 512 L 25/09/17 3:17 PM 10.9 Reflection of a Plane Wave at Normal Incidence   513 Now Ero h2 2 h1 2ho 2 ho 1 5G5 5 5 Eio h2 1 h1 2ho 1 ho 3 Ero 5 1 E 3 io Thus Er 5 2 10 1 ho cosa108t 1 zbay mV/m 3 3 from which we easily obtain Hr as Hr 5 2 10 1 cosa108t 1 zbax mA/m 3 3 Similarly, Eto 4 5t511G5 Eio 3 or Eto 5 4 E 3 io Thus where aEt 5 aEi 5 2ay. Hence, Et 5 Eto cos 1 108t 2 b2z 2 aEt Et 5 2 40 4 h cosa108t 2 zbay mV/m 3 o 3 from which we obtain Ht 5 20 4 cosa108t 2 zbax mA/m 3 3 Method 2:   Alternatively, we can obtain Hr and Ht directly from Hi by using Hro 5 2G and Hio h1 Hto 5t h2 Hio Thus 1 10 Hro 5 2 Hio 5 2 3 3 Hto 5 10_Sadiku_Ch10.indd 513 4 ho # 2 20 Hio 5 Hio 5 3 2ho 3 3 25/09/17 3:17 PM 514   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION and Hr 5 2 Ht 5 10 cos 1 108t 1 b1z 2 ax mA/m 3 20 cos 1 108t 2 b2z 2 ax mA/m 3 as obtained by Method 1. Notice that the boundary conditions at z 5 0, namely, and Ei 1 0 2 1 Er 1 0 2 5 Et 1 0 2 5 2 40 h cos 1 108t 2 ay 3 o Hi 1 0 2 1 Hr 1 0 2 5 Ht 1 0 2 5 20 cos 1 108t 2 ax 3 are satisfied. The boundary conditions can always be used to cross-check E and H.   PRACTICE EXERCISE   10.9 A 5 GHz uniform plane wave Eis 5 10 e2jbz ax V/m in free space is incident normally on a large, plane, lossless dielectric slab 1 z . 0 2 having e 5 4eo, m 5 mo. Find the reflected wave Ers and the transmitted wave Ets. Answer: 23.333 exp 1 jb1z 2 ax V/m, 6.667 exp 1 2jb2z 2 ax V/m, where b2 5 2b1 5 200p/3. EXAMPLE 10.10 Given a uniform plane wave in air as Ei 5 40 cos 1 vt 2 bz 2 ax 1 30 sin 1 vt 2 bz 2 ay V/m (a) Find Hi. (b) If the wave encounters a perfectly conducting plate normal to the z-axis at z 5 0, find the reflected wave Er and Hr. (c) What are the total E and H fields for z # 0? (d) Calculate the time-average Poynting vectors for z # 0 and z $ 0. Solution: (a) This is similar to the problem in Example 10.3. We may treat the wave as consisting of two waves Ei1 and Ei2, where Ei1 5 40 cos 1 vt 2 bz 2 ax, 10_Sadiku_Ch10.indd 514 Ei2 5 30 sin 1 vt 2 bz 2 ay 25/09/17 3:17 PM 10.9 Reflection of a Plane Wave at Normal Incidence   515 At atmospheric pressure, air has er 5 1.0006 . 1. Thus air may be regarded as free space. Let Hi 5 Hi1 1 Hi2. where Hi1 5 Hi1o cos 1 vt 2 bz 2 aH1 Hi1o 5 Ei1o 40 1 5 5 ho 120p 3p aH 1 5 ak 3 aE 5 az 3 ax 5 ay Hence Hi1 5 Similarly, where 1 cos 1 vt 2 bz 2 ay 3p Hi2 5 Hi2o sin 1 vt 2 bz 2 aH2 Hi2o 5 Ei2o 30 1 5 5 ho 120p 4p aH2 5 ak 3 aE 5 az 3 ay 5 2ax Hence Hi2 5 2 and 1 sin 1 vt 2 bz 2 ax 4p Hi 5 Hi1 1 Hi2 1 1 52 sin 1 vt 2 bz 2 ax 1 cos 1 vt 2 bz 2 ay mA/m 4p 3p This problem can also be solved using Method 2 of Example 10.3. (b) Since medium 2 is perfectly conducting, s2 W 1 S h2 V h1 ve2 that is, G . 21, t50 showing that the incident E and H fields are totally reflected: Ero 5 G Eio 5 2Eio Hence, Er 5 240 cos 1 vt 1 bz 2 ax 2 30 sin 1 vt 1 bz 2 ay V/m 10_Sadiku_Ch10.indd 515 25/09/17 3:17 PM 516   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION We can find Hr from Er just as we did in part (a) of this example or by using Method 2 of Example 10.9, starting with Hi. Whichever approach is taken, we obtain Hr 5 1 1 cos 1 vt 1 bz 2 ay 2 sin 1 vt 1 bz 2 ax A/m 3p 4p (c) The total fields in air E1 5 Ei 1 Er and H1 5 Hi 1 Hr can be shown to be standing waves.

The total fields in the conductor are E2 5 Et 5 0, H2 5 Ht 5 0 (d) For z # 0, 1ave 5 0 E1s 0 2 2h1 ak 5 1 3 E2ioaz 2 E2roaz 4 2ho 1 3 1 402 1 302 2 az 2 1 402 1 302 2 az 4 240p 0 5 For z $ 0, 2ave 5 0 E2S 0 2 2h2 ak 5 E2to a 50 2h2 z PRACTICE EXERCISE   10.10 The plane wave E 5 50 sin 1 vt 2 5x 2 ay V/m in a lossless medium 1 m 5 4mo, e 5 eo 2 encounters a lossy medium 1 m 5 mo, e 5 4eo, s 5 0.1 S/m 2 normal to the x-axis at x 5 0. Find (a)   G, t, and s (b)   Er and Hr (c)   Et and Ht (d)   The time-average Poynting vectors in both regions Answer: (a) 0.8186 l171.1°, 0.2295 l 33.56°, 10.025, (b) 40.93 sin 1 vt 1 5x 1 ii ii 171.9° 2 ay V/m, 254.3 sin 1 vt 1 5x 1 171.9° 2 az mA/m, (c) 11.47 e26.021xsin 1 vt 27.826x 1 33.56 º 2 ay V/m, 120.2 e26.021x sin 1 vt 2 7.826x 2 4.01º2 az mA/m, (d) 0.5469 ax W/m2, 0.5469 exp 1 212.04x 2 ax W/m2. 10_Sadiku_Ch10.indd 516 25/09/17 3:17 PM 10.10 Reflection of a Plane Wave at Oblique Incidence   517 † 10.10   REFLECTION OF A PLANE WAVE AT OBLIQUE INCIDENCE We now consider a more general situation than that in Section 10.9. To simplify the analy sis, we will assume that we are dealing with lossless media.

(We may extend our analysis to that of lossy media by merely replacing « by «c.) It can be shown (see Problems 10.69 and 10.72) that a uniform plane wave takes the general form of E 1 r, t 2 5 Eo cos 1 k # r 2 vt 2 5 Re 3 Eoe j1k # r2vt2 4 (10.103) where r 5 xax 1 yay 1 zaz is the radius or position vector and k 5 kxax 1 kyay 1 kzaz is the wave number vector or the propagation vector; k is always in the direction of wave propagation. The magnitude of k is related to v according to the dispersion relation:3 k2 5 k2x 1 k2y 1 k2z 5 v 2me(10.104) Thus, for lossless media, k is essentially the same as b in the preceding sections. With the general form of E as in eq. (10.103), Maxwell's equations for a source-free region reduce to k 3 E 5 vmH(10.105a) k 3 H 5 2veE(10.105b) k # H 5 0(10.105c) k # E 5 0(10.105d) showing that (i) E, H, and k are mutually orthogonal, and (ii) E and H lie on the plane k # r 5 kxx 1 kyy 1 kzz 5 constant From eq. (10.105a), the H field corresponding to the E field in eq.

(10.103) is H5 ak 3 E 1 (10.106) k3E5 vm h Having expressed E and H in the general form, we can now consider the oblique inci dence of a uniform plane wave at a plane boundary as illustrated in Figure 10.16(a). The plane defined by the propagation vector k and a unit normal vector an to the boundary is called the plane of incidence. The angle ui between k and an is the angle of incidence. 3 The phenomenon of signal distortion due to a dependence of the phase velocity on frequency is known as dispersion. 10_Sadiku_Ch10.indd 517 25/09/17 3:17 PM 518   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION Again, both the incident and the reflected waves are in medium 1, while the transmit ted (or refracted) wave is in medium 2. Let Ei 5 Eio cos 1 kixx 1 kiyy 1 kizz 2 v it 2 (10.107a) Er 5 Ero cos 1 krxx 1 kryy 1 krzz 2 v rt 2 (10.107b) Et 5 Eto cos 1 ktxx 1 ktyy 1 ktzz 2 v tt 2 (10.107c) where ki, kr, and kt with their normal and tangential components are shown in Figure 10.16(b).

Since the tangential component of E must be continuous across the boundary z 5 0, Ei 1 z 5 0 2 1 Er 1 z 5 0 2 5 Et 1 z 5 0 2 (10.108) This boundary condition can be satisfied by the waves in eq.

(10.107) for all x and y only if 1. v i 5 v r 5 v t 5 v 2. kix 5 krx 5 ktx 5 kx 3. kiy 5 kry 5 kty 5 ky M M FIGURE 10.16   Oblique incidence of a plane wave: (a) illustration of i, r, and t; (b) illustration of the normal and tangential components of k. 10_Sadiku_Ch10.indd 518 25/09/17 3:17 PM 10.10 Reflection of a Plane Wave at Oblique Incidence   519 Condition 1 implies that the frequency is unchanged. Conditions 2 and 3 require that the tangential components of the propagation vectors be continuous (called the phase-matching conditions). This means that the propagation vectors ki, kt, and kr must all lie in the plane of incidence. Thus, by conditions 2 and 3, ki sin u i 5 kr sin u r(10.109) ki sin u i 5 kt sin u t(10.110) where ur is the angle of reflection and ut is the angle of transmission. But for lossless media, ki 5 kr 5 b1 5 v"m1e1(10.111a) kt 5 b2 5 v"m2e2(10.111b) From eqs. (10.109) and (10.111a), it is clear that u r 5 u i(10.112) so that the angle of reflection ur equals the angle of incidence ui, as in optics. Also from eqs. (10.110) and (10.111), ki sin u t m1e1 u2 5 5 5 u1 Å m2e2 sin u i kt (10.113) where u 5 v/k is the phase velocity. Equation (10.113) is the well-known Snell's law, which can be written as n1 sin u i 5 n2 sin u t (10.114) where n1 5 c"m1e1 5 c /u1 and n2 5 c"m2e2 5 c /u2 are the refractive indices of the media.

Based on these general preliminaries on oblique incidence, we will now consider two special cases: one with the E field perpendicular to the plane of incidence and the other with the E field parallel to it. Any other polarization may be considered as a linear combi nation of these two cases. A. Parallel Polarization Figure 10.17, where the E field lies in the xz-plane, the plane of incidence, illustrates the case of parallel polarization.

In medium 1, we have both incident and reflected fields given by 10_Sadiku_Ch10.indd 519 Eis 5 Eio 1 cos u i ax 2 sin u i az 2 e2jb11x sin ui1z cos ui2 (10.115a) 25/09/17 3:17 PM 520   CHAPTER 10   ELECTROMAGNETIC WAVE PROPAGATION FIGURE 10.17   Oblique incidence with E parallel to the plane of incidence. Er kr ur ui Ei Hi ki M M Eio

2jb11x sin ui1z cos ui2 e ay h1 His 5 Ers 5 Ero 1 cos u r ax 1 sin u r az 2 e2jb11x sin ur2z cos ur2(10.116a) Hrs 5 2 Ero 2jb11x sin ur2z cos ur2 e ay h1 where b1 5 v"m1e1. Notice carefully trick in deriving the components is to Figure 10.16(b) for incident, reflected, define Es such that = # Es 5 0 or k # k E 3 Es 5 ak 3 . vm h (10.115b) (10.116b) how we arrive at each field component. The first get the propagation vector k as shown in and transmitted waves. Once k is known, we Es 5 0 and then Hs is obtained from Hs 5 The transmitted fields exist in medium 2 and are given by Ets 5 Eto 1 cos u t ax 2 sin u t az 2 e2jb21x sin ut1z cos ut2(10.117a) Hts 5 Eto 2jb21x sin ut1z cos ut2 e ay(10.117b) h2 where b2 5 v"m2e2.

Should our assumption about the relative directions in eqs. (10.115) to (10.117) be wrong, the final result will show us this by means of its sign. Requiring that u r 5 u i and that the tangential components of E and H be continuous at the boundary z 5 0, we obtain 1 Eio 1 Ero 2 cos u i 5 Eto cos u t (10.118a) 1 1 1 E 2 Ero 2 5 E (10.118b) h1 io h2 to Expressing Ero and Eto in terms of Eio, we obtain 10_Sadiku_Ch10.indd 520 25/09/17 3:17 PM 10.10 Reflection of a Plane Wave at Oblique Incidence 521 Gy 5 Ero h2 cos u t 2 h1 cos u i (10.119a) 5 Eio h2 cos u t 1 h1 cos u i or Ero 5 G y Eio(10.119b) and ty 5 Eto 2h2 cos u i (10.120a) 5 Eio h2 cos u t 1 h1 cos u i or Eto 5 t y Eio(10.120b) Equations (10.119) and (10.120) are called Fresnel's equations. G y and t y are known as Fresnel coefficients. Note that the equations reduce to eqs. (10.91) and (10.92) when u i 5 u t 5 0 as expected. Since ui and ut are related according to Snell's law of eq. (10.113), eqs. (10.119) and (10.120) can be written in terms of ui by substituting cos u t 5 "1 2 sin2 u t 5 "1 2 1 u2 /u1 2 2sin2 u i (10.121) From eqs. (10.119) and (10.120), it is easily shown that 1 1 Gy 5 ty a cos u t b(10.122) cos u i From eq. (10.119a), it is evident that it is possible that G y 5 0 because the numerator is the difference of two terms. Under this condition, there is no reflection 1 Ero 5 0 2 , and the incident angle at which this takes place is called the Brewster angle u B y . The Brewster angle is also known as the polarizing angle because an arbitrarily polarized incident wave will be reflected with only the component of E perpendicular to the plane of incidence.

The Brewster effect is utilized in a laser tube where quartz windows are set at the Brewster angle to control polarization of emitted light. The Brewster angle is obtained by setting u i 5 u B y when G y 5 0 in eqs.

(10.119); that is, h2 cos u t 5 h1 cos u B y or h22 1 1 2 sin2 u t 2 5 h21 1 1 2 sin2 u B y 2 Introducing eq. (10.113) or (10.114) gives 10_Sadiku_Ch10.indd 521 sin2 u B y 5 1 2 m2e1 /m1e2 (10.123) 1 2 1 e1 /e2 2 2 25/09/17 3:17 PM 522 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION It is of practical value to consider the case when the dielectric media are not only lossless but nonmagnetic as well—that is, m1 5 m2 5 mo. For this situation, eq. (10.123) becomes sin2 u B y 5 Å e 1 e2 1 1 e1 /e2 1 or tan u B y 5 e2 n2 5 e n1 Å 1 (10.124) showing that there is a Brewster angle for any combination of «1 and «2.

B. Perpendicular Polarization When the E field is perpendicular to the plane of incidence (the xz-plane) as shown in Figure 10.18, we have perpendicular polarization. This may also be viewed as the case in which the H field is parallel to the plane of incidence. The incident and reflected fields in medium 1 are given by Eis 5 Eioe2jb11x sin ui1z cos ui2ay His 5 Ers 5 Eroe2jb11x sin ur2z cos ur2 ay Hrs 5 (10.125a) Eio 1 2cos u i ax 1 sin u i az 2 e2jb11x sin ui1z cos ui2(10.125b) h1 (10.126a) Ero 1 cos u r ax 1 sin u r az 2 e2jb11x sin ur2z cos ur2 (10.126b) h1 while the transmitted fields in medium 2 are given by Ets 5 Etoe2jb21x sin ut1z cos ut2 ay Hts 5 (10.127a) Eto 1 2cos u t ax 1 sin u t az 2 e2jb21x sin ut1z cos ut2(10.127b) h2 Notice that in defining the field components in eqs. (10.125) to (10.127), Maxwell's equa tions (10.105) are always satisfied.

Again, requiring that the tangential components of E and H be continuous at z 5 0 and setting ur equal to ui, we get 10_Sadiku_Ch10.indd 522 Eio 1 Ero 5 Eto(10.128a) 1 1 1 E 2 Ero 2 cos u i 5 E cos u t(10.128b) h1 io h2 to 25/09/17 3:17 PM 10.10 Reflection of a Plane Wave at Oblique Incidence 523 FIGURE 10.18 Oblique incidence with E perpendicular to the plane of incidence. M M Expressing Ero and Eto in terms of Eio leads to G' 5 h2 cos u i 2 h1 cos u t Ero (10.129a) 5 Eio h2 cos u i 1 h1 cos u t or Ero 5 G' Eio(10.129b) and t' 5 Eto 2h2 cos u i (10.130a) 5 Eio h2 cos u i 1 h1 cos u t or Eto 5 t'Eio(10.130b) which are the Fresnel's equations for perpendicular polarization G y and t y are known as Fresnel coefficients. From eqs. (10.129) and (10.130), it is easy to show that 1 1 G' 5 t'(10.131) which is similar to eq. (10.93) for normal incidence. Also, when u i 5 u t 5 0, eqs. (10.129) and (10.130) become eqs. (10.91) and (10.92), as they should. For no reflection, G' 5 0 1 or Er 5 0 2 . This is the same as the case of total transmis sion 1 t' 5 1 2 .

By replacing ui with the corresponding Brewster angle u B', we obtain h2 cos u B' 5 h1 cos u t or h22 1 1 2 sin2 u B' 2 5 h21 1 1 2 sin2 u t 2 10_Sadiku_Ch10.indd 523 25/09/17 3:17 PM 524 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION Incorporating eq. (10.114) yields sin2 u B' 5 1 2 m1e2 /m2e1 (10.132) 1 2 1 m1 /m2 2 2 Note that for nonmagnetic media 1 m1 5 m2 5 mo 2 , sin2 u B' S ` in eq. (10.132), so u B' does not exist because the sine of an angle is never greater than unity. Also if m1 2 m2 and e1 5 e2, eq. (10.132) reduces to sin u B' 5 m2 Å m1 1 m2 or tan u B' 5 m2 Å m1 (10.133) Although this situation is theoretically possible, it rarely occurs in practice. EXAMPLE 10.11 An EM wave travels in free space with the electric field component Es 5 100 e j 10.866y10.5z2ax V/m Determine (a) v and l (b) The magnetic field component (c) The time average power in the wave Solution: (a) Comparing the given E with # Es 5 Eo e jk r 5 Eoe j 1kxx1kyy1kzz2ax it is clear that kx 5 0, ky 5 0.866, kz 5 0.5 Thus k 5 "k2x 1 k2y 1 k2z 5 " 1 0.866 2 2 1 1 0.5 2 2 5 1 But in free space, k 5 b 5 v"moeo 5 v 2p 5 c l Hence, v 5 kc 5 3 3 108 rad/s 10_Sadiku_Ch10.indd 524 25/09/17 3:17 PM 10.10 Reflection of a Plane Wave at Oblique Incidence 525 l5 2p 5 2p 5 6.283 m k (b) From eq.

(10.106), the corresponding magnetic field is given by Hs 5 5 1 k 3 Es mv 1 0.866ay 1 0.5az 2 4p 3 1027 3 3 3 108 3 100 axe jk #r or Hs 5 (132.63ay 2 229.7az) e j(0.866y10.5z) mA/m (c) The time-average power is E2o 1 ave 5 Re 1 Es 3 H*s 2 5 a 2 2h k 5 1 100 2 2 1 0.866ay 1 0.5az 2 2 1 120p 2 5 11.49ay 1 6.631az W/m2 PRACTICE EXERCISE 10.11 Rework Example 10.11 if in free space. E 5 1 10ay 1 5az 2 cos 1 vt 1 2y 2 4z 2 V/m Answer: (a) 1.342 3 109 rad/s, 1.405 m, (b) 229.66 cos 1 1.342 3 109t 1 2y 2 4z 2 ax mA/m, (c) 20.07415ay 1 0.489az W/m2.

EXAMPLE 10.12 A uniform plane wave in air with E 5 8 cos 1 vt 2 4x 2 3z 2 ay V/m is incident on a dielectric slab 1 z $ 0 2 with mr 5 1.0, er 5 2.5, s 5 0. Find (a) (b) (c) (d) 10_Sadiku_Ch10.indd 525 The polarization of the wave The angle of incidence The reflected E field The transmitted H field 25/09/17 3:17 PM 526 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION Solution: (a) From the incident E field, it is evident that the propagation vector is ki 5 4ax 1 3az S ki 5 5 5 v"moeo 5 v c Hence, v 5 5c 5 15 3 108 rad/s A unit vector normal to the interface 1 z 5 0 2 is az. The plane containing k and az is y 5 constant, which is the xz-plane, the plane of incidence. Since Ei is normal to this plane, we have perpendicular polarization (similar to Figure 10.18). (b) The propagation vectors are illustrated in Figure 10.19 where it is clear that tan u i 5 kix 4 5 S u i 5 53.13° kiz 3 Alternatively, without Figure 10.19, we can obtain ui from the fact that ui is the angle between k and an; that is, or 4ax 1 3az # 3 cos u i 5 ak # an 5 a b az 5 5 5 u i 5 53.13° (c) An easy way to find Er is to use eq. (10.126a) because we have noticed that this prob lem is similar to that considered in Section 10.10B. Suppose we are not aware of this. Let Er 5 Ero cos 1 vt 2 kr # r 2 ay FIGURE 10.19 Propagation vectors of Example 10.12. 10_Sadiku_Ch10.indd 526 25/09/17 3:17 PM 10.10 Reflection of a Plane Wave at Oblique Incidence 527 which is similar to the form of the given Ei. The unit vector ay is chosen in view of the fact that the tangential component of E must be continuous at the interface. From Figure 10.19, kr 5 krx ax 2 krz az where krx 5 kr sin u r, krz 5 kr cos u r But u r 5 u i and kr 5 ki 5 5 because both kr and ki are in the same medium. Hence, kr 5 4ax 2 3az To find Ero, we need t. From Snell's law sin u t 5 5 c"m1e1 n1 sin u i 5 sin u i n2 c"m2e2 sin 53.13º "2.5 or u t 5 30.39º G' 5 5 where h1 5 ho 5 377 V, h2 5 G' 5 Ero Eio h2 cos u i 2 h1 cos u t h2 cos u i 1 h1 cos u t momr2 377 5 5 238.4 V Å eoer2 "2.5 238.4 cos 53.13° 2 377 cos 30.39° 5 20.389 238.4 cos 53.13° 1 377 cos 30.39° Hence, and Ero 5 G' Eio 5 20.389 1 8 2 5 23.112 Er 5 23.112 cos 1 15 3 108t 2 4x 1 3z 2 ay V/m (d) Similarly, let the transmitted electric field be Et 5 Eto cos 1 vt 2 kt # r 2 ay 10_Sadiku_Ch10.indd 527 25/09/17 3:17 PM 528 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION where kt 5 b2 5 v"m2e2 5 5 From Figure 10.19, v "mr2er2 c 15 3 108 "1 3 2.5 5 7.906 3 3 108 ktx 5 kt sin u t 5 4 ktz 5 kt cos u t 5 6.819 or kt 5 4ax 1 6.819 az Notice that kix 5 krx 5 ktx as expected.

t' 5 Eto 2 h2 cos u i 5 Eio h2 cos u i 1 h1 cos u t 5 2 3 238.4 cos 53.13º 238.4 cos 53.13º 1 377 cos 30.39º 5 0.611 The same result could be obtained from the relation t' 5 1 1 G'. Hence, Eto 5 t' Eio 5 0.611 3 8 5 4.888 Et 5 4.888 cos(15 3 108t 2 4x 2 6.819z)ay V/m From Et, Ht is easily obtained as Hi 5 5 ak t 3 Et 1 kt 3 Et 5 m2v h2 4ax 1 6.819 az 3 4.888 ay cos 1 vt 2 k # r 2 7.906 1 238.4 2 Ht 5 1 217.69ax 1 10.37az 2 cos 1 15 3 108t 2 4x 2 6.819z 2 mA/m PRACTICE EXERCISE 10.12 If the plane wave of Practice Exercise 10.11 is incident on a dielectric medium having s 5 0, e 5 4eo, m 5 mo and occupying z $ 0, calculate (a) The angles of incidence, reflection, and transmission (b) The reflection and transmission coefficients 10_Sadiku_Ch10.indd 528 25/09/17 3:17 PM 10.11 Application Note—Microwaves 529 (c) The total E field in free space (d) The total E field in the dielectric (e) The Brewster angle Answer: (a) 26.56°, 26.56°, 12.92°, (b) 20.295, 0.647, (c) 1 10ay 1 5az 2 3 cos 1 vt 1 2y 2 4z 2 1 1 22.946ay 1 1.473az 2 cos 1 vt 1 2y 1 4z 2 V/m, (d) 1 7.055ay 1 1.618az 2 cos 1 vt 1 2y 2 8.718z 2 V/m, (e) 63.43°. † 10.11 APPLICATION NOTE—MICROWAVES At the moment, there are three means for carrying thousands of channels over long dis tances: (a) microwave links, (b) coaxial cables, and (c) fiber optic, a technology to be cov ered later in Section 12.9. Microwaves are EM waves whose frequencies range from approximately 300 MHz to 1000 GHz. For comparison, the signal from an AM radio station is about 1 MHz, while that from an FM station is about 100 MHz. The higher-frequency edge of microwaves borders on the optical spectrum. This accounts for why microwaves behave more like rays of light than ordinary radio waves.

You may be familiar with microwave appliances such as the micro wave oven, which operates at 2.4 GHz, the satellite television receiver, which operates at about 4 GHz, and the police radar, which works at about 22 GHz. Features that make microwaves attractive for communications include wide available bandwidths (capacities to carry information) and directive properties of short wavelengths. Since the amount of information that can be transmitted is limited by the available band width, the microwave spectrum provides more communication channels than the radio and TV bands. With the ever-increasing demand for channel allocation, microwave com munications has become more common. A microwave system4 normally consists of a transmitter (including a microwave oscillator, waveguides, and a transmitting antenna) and a receiver subsystem (including a receiving antenna, transmission line or waveguide, microwave amplifiers, and a receiver). A microwave network is usually an interconnection of various microwave components and devices. There are several microwave components and variations of these components.

Common microwave components include the following: • Coaxial cables, which are transmission lines for interconnecting microwave components • Resonators, which are usually cavities in which EM waves are stored 4 For a comprehensive treatment of microwaves, see D. M. Pozar, Microwave Engineering, 3rd ed. Hoboken, NJ: John Wiley & Sons, 2004. 10_Sadiku_Ch10.indd 529 25/09/17 3:17 PM 530 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION Satellite (a) Point-to-point link via satellite microwave Satellite Multiple receivers Transmitter Multiple receivers (b) Broadcast link via satellite microwave FIGURE 10.20 Satellite communications configurations. (From W. Stallings, Data and Computer Communications, 5th ed. Upper Saddle River, NJ: Prentice Hall, 1997, p. 90.) 10_Sadiku_Ch10.indd 530 25/09/17 3:17 PM 10.11 Application Note—Microwaves 531 • Waveguide sections, which may be straight, curved, or twisted • Antennas, which transmit or receive EM waves efficiently • Terminators, which are designed to absorb the input power and therefore act as one-port networks • Attenuators, which are designed to absorb some of the EM power passing through the device, thereby decreasing the power level of the microwave signal • Directional couplers, which consist of two waveguides and a mechanism for cou pling signals between them • Isolators, which allow energy flow in only one direction • Circulators, which are designed to establish various entry/exit points where power can be either fed or extracted • Filters, which suppress unwanted signals and/or separate signals of different frequencies. The use of microwaves has greatly expanded. Examples include telecommunications, radio astronomy, land surveying, radar, meteorology, UHF television, terrestrial microwave links, solid-state devices, heating devices, medical therapeutic and diagnostic equipment, and identification systems. We will c onsider only three of these. 1. Telecommunications (the transmission of analog or digital information from one point to another): This is the largest application of microwave frequencies.

Microwaves propagate along a straight line like light rays and are not bent by the iono sphere as are signals of lower frequency. This makes communication satellites possible. In essence, a communication satellite is a microwave relay station that is used to link two or more ground-based transmitters and receivers. The satellite receives signals at one frequency, repeats or amplifies them, and transmits at another frequency. Two common modes of o peration for satellite communication are portrayed in Figure 10.20.

The satel lite provides a point-to-point link in Figure 10.20(a), while it is being used to provide multiple links between one ground-based transmitter and several ground-based receiv ers in Figure 10.20(b). 2. Radar systems: Radar systems provided the major incentive for the development of microwave technology because they give better resolution for radar instruments at higher frequencies. Only the microwave region of the spectrum could provide the required resolution with antennas of reasonable size. The ability to focus a radiated wave sharply is what makes microwaves so useful in radar applications. Radar is used to detect aircraft, guide supersonic missiles, observe and track weather patterns, and control flight traffic at airports. It is also used in burglar alarms, garage-door openers, and police speed detectors. 3. Heating: Microwave energy is more easily directed, controlled, and concentrated than low-frequency EM waves. Also, various atomic and molecular resonances occur at microwave frequencies, creating diverse application areas in basic science, remote sensing, and heating methods. The heating properties of microwave power are useful in a wide vari ety of commercial and industrial applications. The microwave oven, shown in Figure 10.21, is a typical example. When the magnetron oscillates, microwave energy is extracted from the resonant cavities. The reflections from the stationary walls and the motion of the stir ring fan cause the microwave energy to be well distributed. Thus the microwave enables the 10_Sadiku_Ch10.indd 531 25/09/17 3:17 PM 532 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION Stirrer fan Waveguide Magnetron tube Front panel Metal cavity Power supply FIGURE 10.21 Microwave oven. (From N. Schlager, ed., How Products Are Made. Detroit: Gale Research, 1994, p. 289.) cooking process to be fast and even. Microwave heating properties also are used in physical diathermy and in drying potato chips, paper, cloth, and so on. A microwave circuit consists of microwave components such as sources, transmis sion lines, waveguides, attenuators, resonators, circulators, and filters. One way of analyz ing, for example, a circuit, is to relate the input and output variables of each component. Several sets of parameters can be used for relating input and output variables; but at high frequencies, such as microwave frequencies, where voltage and current are not well defined, S-parameters are often used to analyze microwave circuits. The scattering or S-parameters are defined in terms of wave variables, which are more easily measured at microwave fre quencies than voltage and current. Consider the two-port network shown in Figure 10.22.

The traveling waves are related to the scattering parameters according to b1 5 S11a1 1 S12a2 b2 5 S21a1 1 S22a2(10.134) or in matrix form c b1 S11 d 5 c b2 S21 S12 a1 d c d (10.135) S22 a2 where a1 and a2 represent the incident waves at ports 1 and 2, respectively, while b1 and b2 represent the reflected waves, as shown in Figure 10.22. For the S matrix, the off-diagonal terms represent voltage wave transmission coefficients, while the diagonal terms represent reflection coefficients. If the network is reciprocal, it will have the same transmission char acteristics in either direction; that is, 10_Sadiku_Ch10.indd 532 S12 5 S21(10.136) 25/09/17 3:17 PM 10.11 Application Note—Microwaves 533 a2 a1 Zg S l1 S l2 ZL Vg S21 S22 b2 b1 FIGURE 10.22 A two-port network. If the network is symmetrical, then S11 5 S22(10.137) For a matched two-port network, the reflection coefficients are zero and S11 5 S22 5 0(10.138) The input reflection coefficient can be expressed in terms of the S-parameters and the load ZL as Gi 5 b1 S12S21GL (10.139) 5 S11 1 a1 1 2 S22GL where GL 5 ZL 2 Zo (10.140) ZL 1 Zo Similarly, the output reflection coefficient (with Vg 5 0) can be expressed in terms of the generator impedance Zg and the S-parameters as where EXAMPLE 10.13 10_Sadiku_Ch10.indd 533 Go 5 S12S21Gg b2 (10.141) 5 S22 1 ` a2 Vg 50 1 2 S11Gg Gg 5 Zg 2 Zo Zg 1 Zo (10.142) The following S-parameters are obtained for a microwave transistor operating at 2.5 GHz: S11 5 0.85l230°, S12 5 0.07l56°, S21 5 1.68l120° , S22 5 0.85l240 . Determine the ii i ii ii input reflection coefficient when ZL 5 Zo 5 75 V. 25/09/17 3:17 PM 534 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION Solution: From eq. (10.140), GL 5 ZL 2 Zo 50 ZL 1 Zo Hence, using eq. (10.139) leads to Gi 5 S11 5 0.85l230° ii PRACTICE EXERCISE 10.13 For an hybrid coupler, the voltage standing wave ratios (VSWRs) for the input and output ports are, respectively, given as si 5 so 5 1 1 0 S11 0 1 2 0 S11 0 1 1 0 S22 0 1 2 0 S22 0 Calculate si and so for the following scattering matrix: Answer: 2.333, 1.5. S5 c 0.4 j0.6 d j0.6 0.2 10.12 APPLICATION NOTE—60 GHz TECHNOLOGY The next-generation wireless technology—namely, 60 GHz—can provide wireless connectivity for short distances between electronic devices at speeds in the multigigabytepersecond range, as typically shown in Figure 10.23. At such a high frequency, the wavelength is nearly 5 mm. Although millimeter wave (mmWave) technology has been known for a long time, it was used initially only for military applications. With the strides in process technologies and low-cost integration solutions, academia, indus try, and standardization bodies also turned to mmWave technology. Broadly speaking, mmWave refers to the electromagnetic spectrum that spans 30 to 300 GHz, correspond ing to wavelengths from 10 mm down to 1 mm.

At these smaller wavelengths, the data rates are expected to be 40 to 100 times faster than current wireless technologies for local-area networks.

Also, IBM's high-speed signal processing and coding techniques, 10_Sadiku_Ch10.indd 534 25/09/17 3:17 PM 10.12 Application Note—60 GHz Technology 535 FIGURE 10.23 Predicting the use of 60 GHz Wi-Fi. 60 GHz such as error and packet recovery, enable efficient and robust digital data transmission for small additional cost and low power consumption. Some of the factors that make this technology attractive are the following. • There is a large bandwidth (up to 7 GHz) available worldwide.

• The 60 GHz technology is less restricted in terms of power limits. • At 60 GHz, the path loss is higher, but higher transmitting power overcomes this, especially when the operation is restricted to indoor environments. The effective interference levels for 60 GHz are less severe than those systems located in the congested 2–2.5 and 5–5.8 GHz bands. In addition, higher frequency reuse can also be achieved in indoor environments, allowing a very high throughput. • The compact size of the 60 GHz radio band permits the use of multiple antenna arrays, which can be conveniently integrated into consumer electronic products. • Operators at these bands are exempt from license fees.

• Narrow beamwidth is possible. • For example, this technology can be applied to an in-flight entertainment distribution system without causing interference with flight controls or navigation equipment. • Oxygen absorption does not pose a problem when a 60 GHz system is used between satellites. While the advantages of 60 GHz technology are very attractive, there are also some challenges to be met. At the data rates and range of communication offered by 60 GHz technology, ensuring a reliable communication link with sufficient power margin is not a trivial task. Delay spread of the channel under study, another limiting factor for highspeed transmissions, necessitates sophisticated coding techniques. Large delay spread values can easily increase the complexity of the system beyond the practical limit for channel equalization. The technology permits instant wireless downloading of multimedia content. Transmitting a large amount of data across remotely controlled miniature robots or vehicles without cable connections would be possible. Rescuing people in case of emergency or accidents becomes less problematic. Applications of mmWave technology to automobiles are also attractive. Overall, this technol ogy could significantly affect the way computers and electronic devices communicate with each other. Intense efforts are under way to expedite the commercialization of this fascinating technology. For example, industrial alliances and regulatory bodies are working to draft standards for mmWave. 10_Sadiku_Ch10.indd 535 25/09/17 3:17 PM 536 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION MATLAB 10.1 % This script assists with the solution and graphing of Example 10.1 % We use symbolic variables in the creation of the waveform equation % that describes the expression for the electric field clear syms E omega Beta t x % symbolic variables for fields, % time, and frequency % Enter the frequency (in rad/s) w = input('Enter the angular frequency > '); % The expression for the y-component of the electric field E = 50*cos(w*t+Beta*x); % % B E part (b) solve for Beta = w/3e8; % B is the numeric variable for Beta, % with value as calculated here = subs(E,Beta,B); % substitute the value B in for % variable Beta % Generate numerical sequence xfinal=ceil(6*2*pi/B); % we will compute spatial values % out to 3 wavelengths dx=xfinal/1000; % the discrete distance space=0:dx:xfinal; % create a vector of 1000 discrete space segments unityvec=ones(1,length(space)); % create a vector of 1s that is % the same length as the spatial vector discrete space segments % Plot figure f = w/(2*pi); % determine the frequency for time=0:1/(20*f):1/f, % time loop - each interation of this loop % will plot the e-field waveform for % a different increment of time we will loop % through exactly one wavelength En = subs(E,{x,t},{space,unityvec*time}); % substitute % the time and space vector into the variable % to get a vector of the field as a function of plot(space, En) axis([0 6*2*pi/B min(En)-10 max(En)+10]) % add buffer space of 5 units to graph xlabel('x-axis (m)') ylabel('y-axis (m)') str=strcat('time = ', num2str(time), ' (s)'); % concatenate string "time = " to the actual time text(1.5, max(En)+5, str) % put annotation on figure % to show time pause(0.5) % pause for half a second then re-draw hold off end 10_Sadiku_Ch10.indd 536 25/09/17 3:17 PM Summary 537 SUMMARY 1. The wave equation is of the form 2 '2 2 ' 2 u 50 't2 'z2 with the solution 5 A sin 1 vt 2 bz 2 where u 5 wave velocity, A 5 wave amplitude, v 5 angular frequency 1 52pf 2 , and b 5 phase constant. Also, b 5 v/u 5 2p/l or u 5 f l 5 l/T, where l 5 wavelength and T 5 period. 2. In a lossy, charge-free medium, the wave equation based on Maxwell's equations is of the form =2As 2 g2As 5 0 where As is either Es or Hs and g 5 a 1 jb is the propagation constant. If we assume Es 5 Exs 1 z 2 ax, we obtain EM waves of the form E 1 z, t 2 5 Eoe2az cos 1 vt 2 bz 2 ax H 1 z, t 2 5 Hoe2az cos 1 vt 2 bz 2 u h 2 ay where a 5 attenuation constant, b 5 phase constant, h 5 0 h 0 lu h 5 intrinsic imped ance of the medium, also called wave impedance. The reciprocal of a is the skin depth 1 d 5 1/a 2 . The relations between b, v, and l as stated in item 1 remain valid for EM waves. 3. Wave propagation in media of other types can be derived from that for lossy media as special cases. For free space, set s 5 0, e 5 eo, m 5 mo; for lossless dielectric media, set s 5 0, e 5 eoer, and m 5 momr; and for good conductors, set s .

`, e 5 eo, m 5 mo, or /« → . 4. A medium is classified as a lossy dielectric, a lossless dielectric, or a good conductor depending on its loss tangent, given by tan u 5 0 Js 0 0 Jds 0 5 s es 5 ve er where ec 5 er 2 jes is the complex permittivity of the medium. For lossless dielectrics tan u V 1, for good conductors tan u W 1, and for lossy dielectrics tan u is of the order of unity. 5.

In a good conductor, the fields tend to concentrate within the initial distance d from the conductor surface. This phenomenon is called the skin effect.

For a conductor of width w and length ,, the effective or ac resistance is Rac 5 , s wd where d is the skin depth. 10_Sadiku_Ch10.indd 537 25/09/17 3:17 PM 538 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION 6. The Poynting vector is the power-flow vector whose direction is the same as the direction of wave propagation; its magnitude is the same as the amount of power flowing through a unit area normal to the propagation direction. 5 E 3 H, ave 5 1/2 Re 1 Es 3 H*s 2 7. If a plane wave is incident normally from medium 1 to medium 2, the reflection coef ficient G and transmission coefficient t are given by G5 Ero h2 2 h1 5 , Eio h2 1 h1 t5 Eto 511G Eio The standing wave ratio, s, is defined as s5 1 1 0G0 1 2 0G0 8. For oblique incidence from lossless medium 1 to lossless medium 2, we have the Fresnel coefficients as Gy 5 h2 cos u t 2 h1 cos u i , h2 cos u t 1 h1 cos u i ty 5 2h2 cos u i h2 cos u t 1 h1 cos u i t' 5 2h2 cos u i h2 cos u i 1 h1 cos u t for parallel polarization and G' 5 h2 cos u i 2 h1 cos u t h2 cos u i 1 h1 cos u t for perpendicular polarization. As in optics, ur 5 ui sin u t m1e1 b1 5 5 sin u i b2 Å m2e2 Total transmission or no reflection 1 G 5 0 2 occurs when the angle of incidence ui is equal to the Brewster angle. 9. Microwaves are EM waves of very short wavelengths. They propagate along a straight line like light rays and can therefore be focused easily in one direction by antennas. They are used in radar, guidance, navigation, and heating. REVIEW QUESTIONS 10.1 Which of these is not a correct form of the wave Ex 5 cos 1 vt 2 bz 2 ? (a) cos 1 bz 2 vt 2 (b) sin 1 bz 2 vt 2 p/2 2 (c) cosa 10_Sadiku_Ch10.indd 538 2pt 2pz 2 b T l 25/09/17 3:17 PM Review Questions 539 (d) Re(e j(t2bz)) (e) cos b 1 z 2 ut 2 10.2 Which of these functions does not satisfy the wave equation? (a) 50e jv1t23z2 (b) sin v 1 10z 1 5t 2 (c) 1 x 1 2t 2 2 (d) cos2 1 y 1 5t 2 (e) sin x cos t (f) cos 1 5y 1 2x 2 10.3 Which of the following statements is not true of waves in general?

(a) The phenomenon may be a function of time only. (b) The phenomenon may be sinusoidal or cosinusoidal. (c) The phenomenon must be a function of time and space. (d) For practical reasons, it must be finite in extent.

10.4 The electric field component of a wave in free space is given by E 5 10 cos 1 107t 1 kz 2 ay V/m. It can be inferred that (a) The wave propagates along ay. (b) The wavelength l 5 188.5 m. (c) The wave amplitude is 10 V/m. (d) The wave number k 5 0.33 rad/m. (e) The wave attenuates as it travels.

10.5 Given that H 5 0.5 e20.1x sin 1 106t 2 2x 2 az A/m, which of these statements are incorrect? (a) a 5 0.1 Np/m (b) b 5 22 rad/m (c) v 5 106 rad/s (d) The wave travels along ax.

(e) The wave is polarized in the z-direction.

(f) The period of the wave is 1 ms. 10.6 What is the major factor for determining whether a medium is free space, a lossless dielec tric, a lossy dielectric, or a good conductor?

(a) Attenuation constant (b) Constitutive parameters (s, «, m) (c) Loss tangent (d) Reflection coefficient 10.7 In a certain medium, E 5 10 cos 1 108t 2 3y 2 ax V/m. What type of medium is it? (a) Free space (b) Lossy dielectric (c) Lossless dielectric (d) Perfect conductor 10.8 Electromagnetic waves travel faster in conductors than in dielectrics.

(a) True (b) False 10_Sadiku_Ch10.indd 539 25/09/17 3:17 PM 540 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION 10.9 In a good conductor, E and H are in time phase. (a) True (b) False 10.10 The Poynting vector physically denotes the power density leaving or entering a given volume in a time-varying field. (a) True (b) False

Answers: 10.1b, 10.2f, 10.3a, 10.4b,c 10.5b,e,f, 10.6c, 10.7c, 10.8b, 10.9b, 10.10a. PROBLEMS Section 10.2—Waves in General 10.1 An EM wave propagating in a certain medium is described by E 5 25 sin 1 2p 3 106t 2 6x 2 az V/m (a) Determine the direction of wave propagation.

(b) Compute the period T, the wavelength l, and the velocity u. (c) Sketch the wave at t 5 0, T/8, T/4, T/2. 10.2 Calculate the wavelength for plane waves in vacuum at the following frequencies: (a) 60 Hz (power line) (b) 2 MHz (AM radio) (c) 120 MHz (FM radio) (d) 2.4 GHz (microwave oven) 10.3 An EM wave in free space is described by H 5 0.4 cos 1 108t 1 2 A/M Determine (a) the angular frequency , (b) the wave number b, (c) the wavelength l, (d) the direction of wave propagation, (e) the value of H(2, 3, 4, 10 ns). 10.4 (a) Show that E 1 x, t 2 5 cos 1 x 1 vt 2 1 cos 1 x 2 vt 2 satisfies the scalar wave equation. (b) Determine the velocity of wave propagation.

Section 10.3—Wave Propagation in Lossy Dielectrics 10.5 (a) Derive eqs.

(10.23) and (10.24) from eqs. (10.18) and (10.20).

(b) Using eq. (10.29) in conjunction with Maxwell's equations, show that h5 jvm g (c) From part (b), derive eqs.

(10.32) and (10.33). 10_Sadiku_Ch10.indd 540 11/11/17 2:48 PM Problems 541 10.6 Show that the phase constant in eq. (10.24) can be approximated as b 5 v"me a1 1 for s 5 1. ve s b 8v 2e2 10.7 At 50 MHz, a lossy dielectric material is characterized by e 5 3.6eo, m 5 2.1mo, and s 5 0.08 S/m. If Es 5 6e2gx az V/m, compute (a) g, (b) l, (c) u, (d) h, (e) Hs. 10.8 Determine the loss tangent for each of the following nonmagnetic media at 12 MHz. (a) wet earth 1 e 5 10eo , s 5 10 22 S/m 2 (b) dry earth 1 e 5 4eo , s 5 10 24 S/m 2 (c) seawater 1 e 5 81eo , s 5 4 S/m 2 10.9 Alumina is a ceramic material used in making printed circuit boards. At 15 GHz, e 5 9.6eo, 5 o, tan u 5 3 3 1024. Calculate (a) the penetration depth, (b) the total attenuation over a thickness of 5 mm. 10.10 The properties of a medium are s 5 ev, e 5 4eo, m 5 mo.

If the wavelength in free space is 12 cm, find a, b, and u in the medium. 10.11 At f 5 100 MHz, show that silver 1 s 5 6.1 3 107 S/m, mr 5 1, er 5 1 2 is a good con ductor, while rubber 1 s 5 10 215 S/m, mr 5 1, er 5 3.1 2 is a good insulator. 10.12 Seawater plays a vital role in the study of submarine communications. Assuming that for seawater, s 5 4 S/m, er 5 80, mr 5 1, and f 5 100 kHz, calculate (a) the phase velocity, (b) the wavelength, (c) the skin depth, (d) the intrinsic impedance. 10.13 In a certain medium with m 5 mo, e 5 4eo, H 5 12e20.1y sin 1 p 3 108t 2 by 2 ax A/m Find (a) the wave period T, (b) the wavelength l, (c) the electric field E, (d) the phase difference between E and H. 10.14 In a nonmagnetic medium, Find E.

H 5 50e 2100x cos 1 2p 3 109t 2 200x 2 ay mA/m 10.15 A certain medium has 1 S/m, « 4«o, and 9 o at a frequency of 1 GHz. Determine the (a) attenuation constant, (b) phase constant, (c) intrinsic impedance, and (d) wave velocity. 10_Sadiku_Ch10.indd 541 11/11/17 2:48 PM 542 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION Sections 10.4 and 10.5—Waves in Lossless Dielectrics and Free Space 10.16 The electric field of a TV broadcast signal progagating in air is given by E (z,t) 0.2 cos(t – 6.5z)ax V/m (a) Determine the wave frequency v and the wavelength . (b) Sketch Ex as a function of t at z 0 and z /2. (c) Find the corresponding H(z, t). 10.17 A uniform plane wave has a wavelength of 6.4 cm in free space and 2.8 cm in a dielectric (s 5 0, e 5 eoer, m 5 mo). Find «r . 10.18 The magnetic field component of an EM wave propagating through a nonmagnetic medium 1 m 5 mo 2 is H 5 25 sin 1 2 3 108t 1 6x 2 ay mA/m Determine: (a) The direction of wave propagation (b) The permittivity of the medium (c) The electric field intensity 10.19 A manufacturer produces a ferrite material with m 5 750mo, e 5 5eo, and s 51026 S/m at 10 MHz. (a) Would you classify the material as lossless, lossy, or conducting? (b) Calculate b and l. (c) Determine the phase difference between two points separated by 2 m. (d) Find the intrinsic impedance. 10.20 The electric field intensity of a uniform plane wave in air is given by E 5 50 sin 1 108pt 2 bx 2 az mV/m (a) Calculate b. (b) Determine the location(s) where E vanishes at t 5 50 ns. (c) Find H. 10.21 For a uniform plane wave at 4 GHz, the intrinsic impedance and phase velocity of an unknown material are measured as 105 and 7.6 3 107 m/s, respectively. Find «r and r of the material. 10.22 In a lossless medium (er 4.5, mr 1), a uniform plane wave E 8 cos(vt bz) ax 6 sin(vt bz)ay V/m propagates at 40 MHz. (a) Find H. (b) Determine b, l, h, and u. 10.23 A uniform plane wave in a lossy nonmagnetic medium has Es 5 1 5 ax 1 12 ay 2 e2gz, g 5 1 0.2 1 j3.4 2 /m (a) Compute the magnitude of the wave at z 5 4 m, t 5 T/8. (b) Find the loss in decibels suffered by the wave in the interval 0 , z , 3 m. (c) Calculate the intrinsic impedance.

10_Sadiku_Ch10.indd 542 11/11/17 2:48 PM Problems 543 Section 10.6—Plane Waves in Good Conductors 10.24 The magnet field intensity of a uniform plane wave in a good conductor (« 5 «o, 5 o) is H 5 20e 212z cos 1 2p 3 106t 1 12z 2 ay mA/m Find the conductivity and the corresponding E field. 10.25 Which of the following media may be treated as conducting at 8 MHz? (a) Wet marshy soil 1 e 5 15eo, m 5 mo, s 5 1022 S/m 2 (b) Intrinsic germanium 1 e 5 16eo, m 5 mo, s 5 0.025 S/m 2 (c) Seawater 1 e 5 81eo, m 5 mo, s 5 25 S/m 2 10.26 A uniform plane wave impinges normally on a conducting medium. If the frequency is 100 MHz and the skin depth is 0.02 mm, determine the velocity of the wave in the conducting medium. 10.27 (a) Determine the dc resistance of a round copper wire 1 s 5 5.8 3 107 S/m, mr 5 1, er 5 1 2 of radius 1.2 mm and length 600 m. (b) Find the ac resistance at 100 MHz. (c) Calculate the approximate frequency at which dc and ac resistances are equal. 10.28 A 10 GHz wave passes through a medium made of copper (e 5 eo, m 5 mo, s 5 5.8 3 107 S/m); find: (a) attenuation constant, (b) the skin depth, (c) the intrinsic impedance. 10.29 For silver, 5 6.1 3 107 S/m, r 5 1, «r 5 1, determine the frequency at which the penetration depth is 2 mm. 10.30 Compute the penetration depth of copper at the power frequency of 60 Hz. 10.31 By measurements conducted at 12 MHz on a certain material, it is found that the intrin sic impedance is 24.6/45° V with m 5 m. Find a, b, l, and u. 10.32 Fat tissue at 2.42 GHz has the following properties: s 5 0.12 S/m, e 5 5.5 eo, and m mo. Find the penetration depth. 10.33 Brass waveguides are often silver plated to reduce losses. If the thickness of - silver 1 m 5 mo, e 5 eo, s 5 6.1 3 107 S/m 2 must be 5d, find the minimum thickness required for a waveguide operating at 12 GHz. 10.34 How deep does a radar wave at 2 GHz travel in seawater before its amplitude is reduced to 1025 of its amplitude just below the surface? Assume that 5 o, « 5 24«o, 5 4 S/m. Section 10.7—Wave Polarization 10.35 The electric field intensity of a uniform plane wave in a medium (5 0, 5 o, « 5 «o«r) is 10_Sadiku_Ch10.indd 543 E 5 12 sin 1 2p 3 107t 2 3y 2 az V/m (a) Determine the polarization of the wave. (b) Find the frequency. (c) Calculate «r. (d) Obtain the magnetic field intensity H. 11/11/17 2:48 PM 544 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION 10.36 Let E 2 sin(t x)ay 2 5 sin(t x)az V/m. What is the wave polarization? 10.37 Determine the wave polarization of each of the following waves: (a) Eo cos(t 1 y)ax Eo sin(t 1 y)az V/m (b) Eo cos(t 2 y)ax 2 3Eo sin(t 1 y)ax V/m 10.38 Determine the polarization of the following waves: (a) Es 5 40e j10zax 1 60e j10zay V/m (b) Es 5 12e j/3e2j10xay 1 5e2j/3e2j10xaz V/m 10.39 The electric field intensity of a uniform plane wave in free space is given by E 40 cos(t z)ax 1 60 sin(t z)ay V/m (a) What is the wave polarization? (b) Determine the magnetic field intensity. 10.40 Show that a linearly polarized plane wave of the form Es 5 Eoe 2jbz ax can be expressed as the sum of two circularly polarized waves. 10.41 Suppose E(y,t) 5 Eo1 cos(t 2 y)ax Eo2 cos(t 2 y 1)az V/m. Determine the polarization when (a) f 5 0, (b) f 5 p/2, (c) f 5 p. Section 10.8—Power and the Poynting Vector 10.42 Show that eqs. (10.77) and (10.78) are equivalent. 10.43 The electric field intensity in a dielectric medium (5 0, « 5 «o«r) is given by E 5 150 cos(109t 1 8x)az V/m Calculate (a) The dielectric constant «r (b) The intrinsic impedance (c) The velocity of propagation (d) The magnetic field intensity (e) The Poynting vector 10.44 In free space, E 5 40 cos(vt 2 10z)ay V/m. Find the total average power passing through a circular disk of radius 1.5 m in the z = 0 plane. 10.45 The electric field due a short dipole antenna located in free space is Es 5 10_Sadiku_Ch10.indd 544 10 sin ue 2j3rau V/m r Find (a) Hs , (b) the average power crossing the surface r = 2, 0 , , p/6, 0 , f , p. 11/11/17 2:48 PM Problems 545 10.46 The electric field component of a uniform plane wave traveling in seawater 1 s 5 4 S/m, e 5 81 eo, m 5 mo 2 is E 5 8e 20.1z cos 1 vt 2 0.3z 2 ax V/m (a) Determine the average power density. (b) Find the depth at which the power density is reduced by 20 dB.

10.47 In a coaxial transmission line filled with a lossless dielectric 1 e 5 4.5eo, m 5 mo 2 , E5 40 sin 1 vt 2 2z 2 ar V/m r Find (a) v and H, (b) the Poynting vector, (c) the total time-average power crossing the surface z 5 1 m, 2 mm , r , 3 mm, 0 , f , 2p. 10.48 An antenna is located at the origin of a spherical coordinate system. The fields produced by the antenna in free space are Eo sin u sin v 1 t 2 r/c 2 au r Eo sin u sin v 1 t 2 r/c 2 af H5 hr E5 where c 5 antenna. 1 "moeo and h 5 mo .

Determine the time-average power radiated by the Å mo 10.49 A plane wave in free space has H(x, t) (10ay 20az) sin(t 40x) A/m Find , E, and ave.

10.50 Human exposure to the electromagnetic radiation in air is regarded as safe if the power density is less than 10 mW/m2. What is the corresponding electric field intensity? 10.51 Given that E 5 cos(t 2 z)ax sin(t 2 z)ay V/m, show that the Poynting vector is constant everywhere. 10.52 At the bottom of a microwave oven, E 5 2.4 kV/m. If this value is found uniformly over the entire area of the oven, which is 450 cm2, determine the power delivered by the oven. Assume 5 o, « 5 «o. 10.53 A coaxial cable consists of two conducting cylinders of radii a and b. The electric and magnetic fields in the cable are E5 10_Sadiku_Ch10.indd 545 Vo sin 1 vt 2 bz 2 ar, a , r , b r1nAb/aB 11/11/17 2:48 PM 546 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION H5 Io sin 1 vt 2 bz 2 af, a , r , b 2pr where Vo and Io are constants. (a) Determine the time-average Poynting vector. (b) Find the time-average power flowing through the cable. Section 10.9—Reflection at Normal Incidence 10.54 (a) For a normal incidence upon the dielectric–dielectric interface for which m1 5 m2 5 mo, we define R and T as the reflection and transmission coefficients for average powers; that is, Pr,ave 5 RPi,ave and Pt,ave 5 TPi,ave. Prove that R5a n1 2 n2 2 b n1 1 n2 and T5 4n1n2 1 n1 1 n2 2 2 where n1 and n2 are the refractive indices of the media. (b) Determine the ratio n1/n2 so that the reflected and the transmitted waves have the same average power. 10.55 A plane wave in a lossless medium (s 5 0, m 5 2mo, e 5 8eo) is given as E 5 60sin(vt 2 10z)ax 1 30sin(vt 2 10z 1 p/6)ay V/m. This wave is incident on a lossless medium (s 5 0, m 5 mo, e 5 16eo). Determine the reflected and transmitted electric fields. 10.56 A uniform plane wave in free space impinges perpendicularly on a lossless nonmagnetic material with e 5 9eo. Calculate the fraction of the incident average power that is trans mitted. 10.57 A uniform plane wave in air with H 5 4 sin 1 vt 2 5x 2 ay A/m is normally incident on a plastic region 1 x $ 0 2 with the parameters m 5 mo, e 5 4eo, and s 5 0. (a) Obtain the total electric field in air. (b) Calculate the time-average power density in the plastic region. (c) Find the standing wave ratio. 10.58 R egion 1 is a lossless medium for which y $ 0, m 5 mo, e 5 4eo, whereas region 2 is free space, y # 0. If a plane wave Ei 5 5 cos 1 108t 1 by 2 az V/m exists in region 1, find (a) the total electric field component of the wave in region 1, (b) the timeaverage Poynting vector in region 1, (c) the time-average Poynting vector in region 2. 10.59 A uniform plane wave propagates in a medium for which s 5 0, e 5 16eo m 5 mo. The electric field in the medium is E 5 60cos(vt 2 bz)ax V/m where v 5 90 Mrad/s. If the wave hits free space normally at z = 0, determine the power densities in both media. 10_Sadiku_Ch10.indd 546 11/11/17 2:48 PM Problems 547 10.60 A uniform plane wave in air is normally incident on an infinite lossless dielectric material occupying z 0 and having e 5 3eo and m 5 mo. If the incident wave is Ei 5 10 cos 1 vt 2 z 2 ay V/m, find (a) l and v of the wave in air and the transmitted wave in the dielectric medium (b) The incident Hi field (c) G and t (d) The total electric field and the time-average power in both regions 10.61 A 100 MHz plane wave is normally incident from air to the sea surface, which may be assumed to be calm and smooth. If 5 4 S/m, r 5 1, and «r 5 81 for seawater, calculate the fractions of the incident power that are transmitted and reflected.

10.62 A uniform plane wave in a certain medium (o, « 4«o) is given by E 12 cos(t 40x)az V/m (a) Find . (b) If the wave is normally incident on a dielectric (o, « 3.2«o), determine Er and Et. *10.63 A signal in air 1 z $ 0 2 with the electric field component E 5 10 sin 1 vt 1 3z 2 ax V/m hits normally the ocean surface at z 5 0 as in Figure 10.24. Assuming that the ocean surface is smooth and that e 5 80eo, m 5 mo, s 5 4 S/m in ocean, determine (a) v (b) The wavelength of the signal in air (c) The loss tangent and intrinsic impedance of the ocean (d) The reflected and transmitted E field 10.64 Sketch the standing wave in eq. (10.97) at t 5 0, T/8, T/4, 3T/8, T/2, and so on, where T 5 2p/v .

10.65 A uniform plane wave is incident at an angle u i 5 45° on a pair of dielectric slabs joined together as shown in Figure 10.25. Determine the angles of transmission t1 and t2 in the slabs. FIGURE 10.24 For Problem 10.63. Ocean 10_Sadiku_Ch10.indd 547 11/11/17 2:48 PM 548 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION F F Air FIGURE 10.25 For Problem 10.65. FIGURE 10.26 For Problem 10.67. Tissue Ei z Hi 10.66 Show that the field Es 5 20 sin 1 kxx 2 cos 1 kyy 2 az where k2x 1 k2y 5 v 2moeo, can be represented as the superposition of four propagating plane waves. Find the corresponding Hs. 10.67 Electromagnetic radiation can be used to heat cancerous tumors. If a plane wave is nor mally incident on the tissue surface at 1.2 GHz as shown in Figure. 10.26, determine the refection coefficient. At 1.2 GHz, the electrical properties of the tissue are «r 50, r 1, 4 S/m. 10.68 An EM plane wave in a lossless medium impinges normally on a lossy m edium. (a) Determine the ratio of transmitted to incident power in terms of the standing wave ratio s. (b) Express the ratio of reflected to incident power in terms of s.

Section 10.10—Reflection at Oblique Incidence # # *10.69 By assuming the time-dependent fields E 5 Eoe j1k r2vt2 and H 5 Hoe j1k r2vt2 where k 5 kxax 1 kyay 1 kzaz is the wave number vector and r 5 xax 1 yay 1 zaz is the radius vector, show that = 3 E 5 2'B/'t can be expressed as k 3 E 5 mvH and d educe ak 3 aE 5 aH. 10_Sadiku_Ch10.indd 548 11/11/17 2:48 PM Problems 549 10.70 A plane wave in free space has a propagation vector k 124ax 124ay 263az Find the wavelength, frequency, and angles k makes with the x-, y-, and z-axes. 10.71 In free space, Es 5 [Eoax 1 ay 1(3 1 j4)az]e 2j(3.4x24.2y) V/m Determine Eo, Hs, and frequency. 10.72 Assume the same fields as in Problem 10.69 and show that Maxwell's equations in a source-free region can be written as k#E50 k#H50 k 3 E 5 vmH k 3 H 5 2veE From these equations deduce ak 3 aE 5 aH and ak 3 aH 5 2aE 10.73 Show that for nonmagnetic dielectric media, the reflection and transmission coefficients for oblique incidence become Gy 5 G' 5 tan 1 u t 2 u i 2 , tan 1 u t 1 u i 2 sin 1 u t 2 u i 2 , sin 1 u t 1 u i 2 ty 5 t' 5 2 cos u i sin u t sin 1 u t 1 u i 2 cos 1 u t 2 u i 2 2 cos u i sin u t sin 1 u t 1 u i 2 10.74 If region 1 is in free space, while region 2 is a nonmagnetic dielectric medium 1 s2 5 0, er2 5 6.4 2 , compute Ero/Eio and Eto/Eio for oblique incidence at u i 5 12º. Assume parallel polarization. 10.75 A parallel-polarized wave in air with E 5 1 8ay 2 6az 2 sin 1 vt 2 4y 2 3z 2 V/m impinges a dielectric half-space as shown in Figure 10.27.

Find (a) the incidence angle i, (b) the time-average power in air 1 m 5 mo, e 5 eo 2 , (c) the reflected and transmitted E fields. 10_Sadiku_Ch10.indd 549 11/11/17 2:48 PM 550 CHAPTER 10 ELECTROMAGNETIC WAVE PROPAGATION z FIGURE 10.27 For Problem 10.75. Air Dielectric (e e o, o) (e 4e 0, 0) y i Ei ki 10.76 In a dielectric medium 1 e 5 9eo, m 5 mo 2 , a plane wave with H 5 0.2 cos 1 109t 2 kx 2 k"8z 2 ay A/m is incident on an air boundary at z 5 0. Find (a) r and t (b) k (c) The wavelength in the dielectric and in air (d) The incident E (e) The transmitted and reflected E (f) The Brewster angle 10.77 A plane wave in free space has Ei 5 (4ax 1 5ay 2 3az)cos(v t 2 0.5px 2 0.866pz) Determine: (a) the perpendicular- and parallel-polarized components of the wave, (b) the angle of incidence. Assume that the xy-plane is the boundary between the two media. 10.78 A parallel-polarized wave in free space impinges on a dielectric medium (s 5 0, e 5 eoer, m 5 mo). If the Brewster angle is 688, find «r. 10.79 If u is the phase velocity of an EM wave in a given medium, the index of refraction of the medium is n 5 c/u, where c is the speed of light in vacuum. (a) Paraffin has r 5 1, «r 5 2.1. Determine n for unbounded medium of paraffin. (b) Distilled water has r 5 1, «r 5 81. Find n. (c) Polystyrene has r 5 1, «r 5 2.7. Calculate n.

Section 10.11—Application Note—Microwaves 10.80 Discuss briefly some applications of microwaves other than those discussed in the text. 10_Sadiku_Ch10.indd 550 11/11/17 2:48 PM Problems 551 10.81 A useful set of parameters, known as the scattering transfer parameters, is related to the incident and reflected waves as c a1 T11 d 5 c b1 T21 T12 b2 d c d T22 a2 (a) Express the T-parameters in terms of the S-parameters. (b) Find T when S5 c 0.2 0.4 0.4 d 0.2 10.82 The S-parameters of a two-port network are: S11 5 0.33 2 j0.16, S12 5 S21 5 0.56, S22 5 0.44 2 j0.62 Find the input and output reflection coefficients when ZL 5 Zo 5 50 V and Zg 5 2Zo. 10.83 Why can't regular lumped circuit components such as resistors, inductors, and capacitors be used at microwave frequencies? 10.84 In free space, a microwave signal has a frequency of 8.4 GHz. Calculate the wavelength of the signal. 10_Sadiku_Ch10.indd 551 11/11/17 2:48 PM BIOELECTROMAGNETICS Is it safe to live close to power transmission lines? Are cellular phones safe? Can video d isplay terminals cause problems for pregnant women? These and other questions are a ddressed by bioelectromagnetics (BEM), which is the branch of electromagnetics that deals with the biological effects of man-made EM fields with respect to humans and the environment. BEM may also be regarded as the emerging science that studies how living organisms interact with EM fields. It combines the investigative efforts of scientists from various disciplines. A major challenge the industry faces as a result of the immense spread of wireless technology is the growth of health concerns on the part of the public and health agencies alike. The recent focus has been on the design criteria for transmitters operating in the closest proximity of the human body. When a person is exposed to an EM field, energy incident on the person may be scattered, reflected, transmitted, or absorbed into the body depending on the field strength, the frequency, the dimensions of the body, and the electrical properties of the tissue. The heat produced by radiation may affect live tissue. If the body cannot dissipate this heat energy as fast as it is produced, the internal temperature of the body will rise. This may result in damage to tissues and organs and in death if the rise is s ufficiently high. An EM field is classified as ionizing if its energy is high enough to dislodge electrons from an atom or molecule. High-energy forms of EM radiation, such as gamma rays and X-rays, are strongly ionizing in biological matter. For this reason, prolonged exposure to such rays is harmful. Radiation in the middle portion of the frequency and energy spectrum—such as visible, especially ultraviolet, light—is weakly ionizing. Although it has long been known that exposure to strongly ionizing EM radiation can cause extreme damage to biological tissues, only recently has evidence implicated long-term exposure to nonionizing EM fields, such as those emitted by power lines, in increased health hazards. Researchers have reported that to prevent deep-tissue burning, individuals wearing or carrying metal objects such as hairpins, metal implants, buckles, coins, or metal-framed eyeglasses, any of which may concentrate the EM field and cause burning, should not be exposed to radio-frequency (RF) radiation. Researchers have also pointed out that radiation can be absorbed deeply and is actually greater in tissue such as muscle or the brain than in regions of poorer absorption in the bone and fatty layers near the body surface. At this time, there are fewer scientists who will say that there is positively no possibility of nonthermal hazards of low-level EM fields. Ongoing research is aimed at determining whether there is a hazard and if so, at what levels. 11_Sadiku_Ch11.indd 552 15/11/17 7:50 PM CHAPTER 11 TRANSMISSION LINES Kind hearts are the garden. Kind thoughts are the roots. Kind words are the flowers.

Kind deeds are the fruits. Take care of your garden, And keep out the weeds. Fill it up with sunshine, Kind words and kind deeds. —LONGFELLOW 11.1 INTRODUC TION Our discussion in Chapter 10 was essentially on wave propagation in unbounded media, media of infinite extent. Such wave propagation is said to be unguided in that the uniform plane wave exists throughout all space, and EM energy associated with the wave spreads over a wide area. Wave propagation in unbounded media is used in radio or TV broadcasting, where the information being transmitted is meant for everyone who may be interested. Another means of transmitting power or information is by guided structures. Guided structures serve to guide (or direct) the propagation of energy from the source to the load. Typical examples of such structures are transmission lines and waveguides. Waveguides are discussed in the next chapter; transmission lines are considered in this chapter. Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies). Transmission lines such as twisted-pair and coaxial cables (thinnet and thicknet) are used in computer networks such as the Ethernet and the Internet. A transmission line basically consists of two or more parallel conductors used to connect a source to a load. The source may be a hydroelectric generator, a transmitter, or an oscillator; the load may be a factory, an antenna, or an oscilloscope. Typical transmission lines include coaxial cable, a two-wire line, a parallel-plate or planar line, a wire above the conducting plane, and a microstrip line. These lines are portrayed in Figure 11.1. Notice that each of these lines consists of two conductors in parallel. Coaxial cables are routinely used in electrical laboratories and in connecting TV sets to antennas. Microstrip lines (similar to that in Figure 11.1e) are particularly important in integrated circuits, where metallic strips connecting electronic elements are deposited on dielectric substrates. Transmission line problems are usually solved by means of EM field theory and electric circuit theory, the two major theories on which electrical engineering is based. In this chapter, we use circuit theory because it is easier to deal with mathematically. The basic concepts of wave propagation (such as propagation constant, reflection coefficient, and standing wave ratio) covered in the preceding chapter apply here. 553 11_Sadiku_Ch11.indd 553 15/11/17 7:50 PM 554 CHAPTER 11 TRANSMISSION LINES (b) (a) (c) (d) (e) FIGURE 11.1 Typical transmission lines in cross-sectional view: (a) coaxial line, (b) two-wire line, (c) planar line, (d) wire above conducting plane, (e) microstrip line. Our analysis of transmission lines will include the derivation of the transmission line equations and characteristic quantities, the use of the Smith chart, various practical applications of transmission lines, and transients on transmission lines. 11.2 TRANSMISSION LINE PARAMETERS It is customary and convenient to describe a transmission line in terms of its line parameters, which are its resistance per unit length R, inductance per unit length L, conductance per unit length G, and capacitance per unit length C. Each of the lines shown in Figure 11.1 has specific formulas for finding R, L, G, and C. For coaxial, two-wire, and planar lines, the formulas for calculating the values of R, L, G, and C are provided in Table 11.1. The dimensions of the lines are as shown in Figure 11.2. Some of the formulas1 in Table 11.1 were derived in Chapters 6 and 8. It should be noted that 1. The line parameters R, L, G, and C are not discrete or lumped. Rather, they are distributed as shown in Figure 11.3. By this we mean that the parameters are uniformly distributed along the entire length of the line. 1 Similar formulas for other transmission lines can be obtained from engineering handbooks or data books—for example, M. A. R. Gunston, Microwave Transmission-Line Impedance Data.

London: Van Nostrand Reinhold, 1972. 11_Sadiku_Ch11.indd 554 13/11/17 2:23 PM 11.2 Transmission Line Parameters 555 TABLE 11.1 Distributed Line Parameters at High Frequencies* Parameters R 1 V/m 2 Two-Wire Line Planar Line 1 1 1 c 1 d 2pdsc a b 1 padsc 2 wdsc 1d V a2 1d V t2 1 d V a, c 2 b 2 L 1 H/m 2 m 2p ln 2ps b ln a 2pe b ln a G 1 S/m 2 C 1 F/m 2 * d5 Coaxial Line 1 "pfmcsc b a m p cosh21 d 2a md w sw d ps cosh21 d 2a pe ew d 1w W d2 d cosh21 2a 5 skin depth of the conductor; cosh21 d d d 2 .

ln if c d W 1. 2a a 2a 2.

For each line, the conductors are characterized by sc, mc, ec 5 eo, and the homogeneous dielectric separating the conductors is characterized by s, m, .

3.

G 2 1/R; R is the ac resistance per unit length of the conductors comprising the line, and G is the conductance per unit length due to the dielectric medium s eparating the conductors. 4. The value of L shown in Table 11.1 is the external inductance per unit length, that is, L 5 Lext. The effects of internal inductance Lin 1 5 R /v 2 are negligible at the high frequencies at which most communication systems operate. 5.

For each line, LC 5 me and G s 5 (11.1) e C As a way of preparing for the next section, let us consider how an EM wave propagates through a two-conductor transmission line. For example, consider the coaxial line connecting the generator or source to the load as in Figure 11.4(a). When switch S is closed, the inner conductor is made positive with respect to the outer one so that FIGURE 11.2 Common transmission lines: (a) coaxial line, (b) two-wire line, (c) planar line. 11_Sadiku_Ch11.indd 555 25/09/17 5:24 PM 556 CHAPTER 11 TRANSMISSION LINES S S FIGURE 11.3 Distributed parameters of a two-conductor transmission line.

the E field is radially outward as in Figure 11.4(b). According to Ampère's law, the H field encircles the current-carrying conductor as in Figure 11.4(b). The Poynting vector 1 E 3 H 2 points along the transmission line. Thus, closing the switch simply establishes a disturbance, which appears as a transverse electromagnetic (TEM) wave propagating along the line. This wave is a nonuniform plane wave, and by means of it, power is transmitted through the line. G C L FIGURE 11.4 (a) Coaxial line connecting the generator to the load; (b) E and H fields on the coaxial line. 11_Sadiku_Ch11.indd 556 20/11/17 7:41 PM 11.3 Transmission Line Equations 557 11.3 TRANSMISSION LINE EQUATIONS As mentioned in Section 11.2, a two-conductor transmission line supports a TEM wave; that is, the electric and magnetic fields on the line are perpendicular to each other and transverse to the direction of wave propagation. An important property of TEM waves is that the fields E and H are uniquely related to voltage V and current I, respectively: V 5 23 E # dl, I 5 C H # dl(11.2) L In view of this, we will use circuit quantities V and I in solving the transmission line problem instead of solving field quantities E and H (i.e., solving Maxwell's equations and boundary conditions). The circuit model is simpler and more convenient. Let us examine an incremental portion of length z of a two-conductor transmission line. We intend to find an equivalent circuit for this line and derive the line equations. From Figure 11.3, we expect the equivalent circuit of a portion of the line to be as in Figure 11.5. The model in Figure 11.5 is in terms of the line parameters R, L, G, and C, and may represent any of the two-conductor lines of Figure 11.2. The model is called the L-type equivalent circuit; there are other possible types. In the model of Figure 11.5, we assume that the wave propagates along the 1z-direction, from the generator to the load. By applying Kirchhoff 's voltage law to the outer loop of the circuit in Figure 11.5, we obtain or V 1 z, t 2 5 R Dz I 1 z, t 2 1 L Dz 2 'I 1 z, t 2 1 V 1 z 1 Dz, t 2 't 'I 1 z, t 2 V 1 z 1 Dz, t 2 2 V 1 z, t 2 5 RI 1 z, t 2 1 L (11.3) Dz 't FIGURE 11.5 An L-type equivalent circuit model of a two-conductor transmission line of differential length Dz. 11_Sadiku_Ch11.indd 557 25/09/17 5:24 PM 558 CHAPTER 11 TRANSMISSION LINES Taking the limit of eq. (11.3) as Dz S 0 leads to 2 'I 1 z, t 2 'V 1 z, t 2 5 RI 1 z, t 2 1 L (11.4) 'z 't Similarly, applying Kirchhoff 's current law to the main node of the circuit in Figure 11.5 gives I 1 z, t 2 5 I 1 z 1 Dz, t 2 1 DI 5 I 1 z 1 Dz, t 2 1 G Dz V 1 z 1 Dz, t 2 1 C Dz or 2 'V 1 z 1 Dz, t 2 't 'V 1 z 1 Dz, t 2 I 1 z 1 Dz, t 2 2 I 1 z, t 2 5 GV 1 z 1 Dz, t 2 1 C (11.5) Dz 't As Dz S 0, eq. (11.5) becomes 2 'V 1 z, t 2 'I 1 z, t 2 5 GV 1 z, t 2 1 C (11.6) 'z 't If we assume harmonic time dependence so that V(z, t) 5 Re[Vs(z) ejwt](11.7a) I(z, t) 5 Re[Is(z) ejwt] (11.7b) where Vs 1 z 2 and Is 1 z 2 are the phasor forms of V 1 z, t 2 and I 1 z, t 2 , respectively, eqs. (11.4) and (11.6) become 2 2 dVs 5 1 R 1 jvL 2 Is(11.8) dz dIs 5 1 G 1 jvC 2 Vs (11.9) dz The differential eqs.

(11.8) and (11.9) are coupled. To separate them, we take the second derivative of Vs in eq. (11.8) and employ eq. (11.9) so that we obtain or 11_Sadiku_Ch11.indd 558 d2Vs 5 1 R 1 jvL 2 1 G 1 jvC 2 Vs dz2 d2Vs 2 g2Vs 5 0(11.10) dz2 25/09/17 5:24 PM 11.3 Transmission Line Equations 559 where g 5 a 1 jb 5 " 1 R 1 jvL 2 1 G 1 jvC 2 (11.11) By taking the second derivative of Is in eq. (11.9) and employing eq. (11.8), we get d2Is 2 g2Is 5 0(11.12) dz2 We notice that eqs. (11.10) and (11.12) are, respectively, the wave equations for voltage and current similar in form to the wave equations obtained for plane waves in eqs. (10.17) and (10.19). Thus, in our usual notations, g in eq. (11.11) is the propagation constant (in per meter), a is the attenuation constant (in nepers per meter or decibels per meter),2 and b is the phase constant (in radians per meter). The wavelength l and wave velocity u are, respectively, given by l5 u5 2p (11.13) b v 5 f l(11.14) b The solutions of the linear homogeneous differential equations (11.10) and (11.12) are similar to Case 2 of Example 6.5, namely, 2gz gz Vs 1 z 2 5 V1 1 V2 o e o e h 1z 2z v (11.15) and 2gz gz Is 1 z 2 5 I1 1 I2 o e o e h 1z 2z v (11.16) 2 1 2 where V1 o , Vo , Io , and Io are wave amplitudes; the 1 and 2 signs, respectively, denote waves traveling along 1z- and 2z-directions, as is also indicated by the arrows. We obtain the instantaneous expression for voltage as V 1 z, t 2 5 Re 3 Vs 1 z 2 e jvt 4 2az az 5 V1 cos 1 vt 2 bz 2 1 V2 o e o e cos 1 vt 1 bz 2 (11.17) The characteristic impedance Zo of the line is the ratio of the positively traveling voltage wave to the current wave at any point on the line. Recall from eq. (10.35) that 1 Np 5 8.686 dB. 2 11_Sadiku_Ch11.indd 559 25/09/17 5:24 PM 560 CHAPTER 11 TRANSMISSION LINES The characteristic impedance Zo is analogous to h, the intrinsic impedance of the medium of wave propagation. By substituting eqs. (11.15) and (11.16) into eqs.

(11.8) and (11.9) and equating coefficients of terms egz and e2gz, we obtain Zo 5 R 1 jvL V1 V2 g o o (11.18) 5 2 5 1 2 5 g Io Io G 1 jvC or Zo 5 R 1 jvL Å G 1 jvC 5 Ro 1 jXo(11.19) where Ro and Xo are the real and imaginary parts of Zo. Do not mistake Ro for R—while R is in ohms per meter, R o is in ohms. The propagation constant g and the characteristic impedance Zo are important properties of the line because both depend on the line parameters R, L, G, and C and the frequency of operation. The reciprocal of Zo is the characteristic admittance Yo, that is, Yo 5 1/Zo. The transmission line considered thus far in this section is the lossy type in that the conductors comprising the line are imperfect 1 sc 2 ` 2 and the dielectric in which the conductors are embedded is lossy 1 s 2 0 2 .

Having considered this general case, we may now consider two special cases: the lossless transmission line and the distortionless line. A. Lossless Line (R = 0 = G) A transmission line is said to be lossless if the conductors of the line are perfect 1 sc < ` 2 and the dielectric medium separating them is lossless 1 s .

0 2 .

For such a line, it is evident from Table 11.1 that when sc . ` and s . 0, R 5 0 5 G(11.20) This is a necessary condition for a line to be lossless.

Thus for such a line, eq. (11.20) forces eqs. (11.11), (11.14), and (11.19) to become 11_Sadiku_Ch11.indd 560 g 5 jb 5 jv "LC(11.21a) a 5 0, u5 Xo 5 0, v 1 5 5 f l (11.21b) b "LC Zo 5 R o 5 L (11.21c) ÅC 25/09/17 5:24 PM 11.3 Transmission Line Equations 561 B. Distortionless Line (R/L = G/C) A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line because a is frequency dependent. Since, in general, the phase velocity of each frequency component is also f requency dependent, this will result in distortion.

A distortionless line is one in which the attenuation constant a is frequency independent while the phase constant b is linearly dependent on frequency. From the general expression for a and b [in eq. (11.11)], a distortionless line results if the line parameters are such that R G 5 (11.22) L C Thus, for a distortionless line, g 5 Å RG a1 1 5 "RG a1 1 or a 5 "RG, jvL R jvC G b a1 1 jvC G b b 5 a 1 jb b 5 v"LC(11.23a) showing that a does not depend on frequency, whereas b is a linear function of frequency. Also Zo 5 or and R 1 1 1 jvL/R 2 R L 5 5 5 Ro 1 jXo Å G 1 1 1 jvC/G 2 ÅG ÅC Ro 5 R L 5 , ÅG ÅC Xo 5 0(11.23b) v 1 5 5 f l(11.23c) b "LC Note the following important properties. u5 1. The phase velocity is independent of frequency because the phase constant b linearly depends on frequency. We have shape distortion of signals unless a and u are independent of frequency. 11_Sadiku_Ch11.indd 561 25/09/17 5:24 PM 562 CHAPTER 11 TRANSMISSION LINES TABLE 11.2 Transmission Line Characteristics Case Propagation Constant General " 1 R 1 jvL 2 1 G 1 jvC 2 g 5 a 1 jb Characteristic Impedance Zo 5 Ro 1 jXo R 1 jvL Å G 1 jvC Lossless Distortionless 0 1 jv"LC L 1 j0 ÅC "RG 1 jv"LC L 1 j0 ÅC 2. Both u and Zo remain the same as for lossless lines. 3. A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless. A summary of our discussion in this section is in Table 11.2. For the greater part of our analysis, we shall restrict our discussion to lossless transmission lines. EXAMPLE 11.1 An air line has a characteristic impedance of 70 V and a phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line. Solution: An air line can be regarded as a lossless line because s . 0 and sc S `. Hence R 5 0 5 G and Zo 5 R o 5 a50 L (11.1.1) ÅC b 5 v "LC (11.1.2) Dividing eq. (11.1.1) by eq. (11.1.2) yields Ro 1 5 b vC or C5 From eq. (11.1.1), b 3 5 5 68.2 pF/m vRo 2p 3 100 3 106 1 70 2 L 5 R2oC 5 1 70 2 2 1 68.2 3 10212 2 5 334.2 nH/m 11_Sadiku_Ch11.indd 562 25/09/17 5:24 PM 11.3 Transmission Line Equations 563 PRACTICE EXERCISE 11.1 A transmission line operating at 500 MHz has Zo 5 80 V, a 5 0.04 Np/m, b 51.5 rad/m. Find the line parameters R, L, G, and C. Answer: 3.2 V/m, 38.2 nH/m, 5 3 1024 S/m, 5.97 pF/m. EXAMPLE 11.2 A distortionless line has Zo 5 60 V, a 5 20 mNp/m, u 5 0.6c, where c is the speed of light in a vacuum. Find R, L, G, C, and l at 100 MHz. Solution: For a distortionless line, RC 5 GL or G 5 RC L and hence Zo 5 L (11.2.1) ÅC a 5 "RG 5 R or C R 5 (11.2.2a) ÅL Zo R 5 a Zo(11.2.2b) But u5 From eq. (11.2.2b), v 1 5 (11.2.3) b "LC R 5 a Zo 5 1 20 3 1023 2 1 60 2 5 1.2 V/m Dividing eq. (11.2.1) by eq. (11.2.3) results in L5 From eq. (11.2.2a), Zo 60 5 5 333 nH/m u 0.6 1 3 3 108 2 G5 11_Sadiku_Ch11.indd 563 a2 400 3 1026 5 5 333 mS/m R 1.2 25/09/17 5:24 PM 564 CHAPTER 11 TRANSMISSION LINES Multiplying eqs. (11.2.1) and (11.2.3) together gives uZo 5 1 C or C5 1 1 5 5 92.59 pF/m uZo 0.6 1 3 3 108 2 60 l5 0.6 1 3 3 108 2 u 5 5 1.8 m f 108 PRACTICE EXERCISE 11.2 A telephone line has R 5 30 V/km, L 5 100 mH/km, G 5 0, and C 5 20 mF/km.

At f 5 1 kHz, obtain: (a) The characteristic impedance of the line (b) The propagation constant (c) The phase velocity Answer: (a) 70.75l21.367° V, (b) 2.121 3 1024 1 j8.888 3 1023 /m, iii (c) 7.069 3105 m/s. 11.4 INPUT IMPEDANCE, STANDING WAVE RATIO, AND POWER Consider a transmission line of length , characterized by g and Zo, connected to a load ZL as shown in Figure 11.6(a). Looking into the line, the generator sees the line with the load as an input impedance Zin. It is our intention in this section to determine the input impedance, the standing wave ratio (SWR), and the power flow on the line. Let the transmission line extend from z 5 0 at the generator to z 5 , at the load.

First of all, we need the voltage and current waves in eqs. (11.15) and (11.16), that is, 2gz gz Vs 1 z 2 5 V1 1 V2 oe o e (11.24) Is 1 z 2 5 V1 V2 o 2gz o gz e 2 e (11.25) Zo Zo 2 where eq. (11.18) has been incorporated. To find V1 o and Vo , the terminal conditions must be given. For example, if we are given the conditions at the input, say 11_Sadiku_Ch11.indd 564 Vo 5 V 1 z 5 0 2 , Io 5 I 1 z 5 0 2 (11.26) 25/09/17 5:24 PM 11.4 Input Impedance, Standing Wave Ratio, and Power 565 o o FIGURE 11.6 (a) Input impedance due to a line terminated by a load. (b) Equivalent circuit for finding Vo and Io in terms of Zin at the input.

substituting these into eqs. (11.24) and (11.25) results in V 1o 5 V 2o 5 1 1 V 1 ZoIo 2 (11.27a) 2 o 1 1 V 2 ZoIo 2 (11.27b) 2 o If the input impedance at the input terminals is Zin, the input voltage Vo and the input current Io are easily obtained from Figure 11.6(b) as Vo 5 Zin V, Zin 1 Zg g Io 5 Vg Zin 1 Zg (11.28) On the other hand, if we are given the conditions at the load, say VL 5 V 1 z 5 , 2 , IL 5 I 1 z 5 , 2 (11.29) Substituting these into eqs.

(11.24) and (11.25) gives 11_Sadiku_Ch11.indd 565 V1 o 5 V2 o 5 1 1 V 1 ZoIL 2 eg,(11.30a) 2 L 1 1 V 2 ZoIL 2 e2g,(11.30b) 2 L 25/09/17 5:24 PM 566 CHAPTER 11 TRANSMISSION LINES Next, we determine the input impedance Zin 5 Vs 1 z 2 /Is 1 z 2 at any point on the line. At the generator, for example, eqs. (11.24) and (11.25) yield Zin 5 2 Vs 1 z 2 Zo 1 V1 o 1 Vo 2 5 2 (11.31) V1 Is 1 z 2 o 2 Vo Substituting eq. (11.30) into (11.31) and utilizing the fact that eg, 1 e2g, 5 cosh g,, 2 eg, 2 e2g, 5 sinh g,(11.32a) 2 or tanh g, 5 sinh g, eg, 2 e2g, 5 g, (11.32b) cosh g, e 1 e2g, we get Zin 5 Zo c ZL 1 Zo tanh g, d (lossy) Zo 1 ZL tanh g, (11.33) Although eq. (11.33) has been derived for the input impedance Zin at the generation end, it is a general expression for finding Zin at any point on the line. To find Zin at a distance from the load as in Figure 11.6(a), we replace by . A formula for calculating the hyperbolic tangent of a complex number, required in eq. (11.33), is found in Appendix A.3. For a lossless line, g 5 jb, tanh jb, 5 j tan b,, and Zo 5 Ro, so eq. (11.33) becomes Zin 5 Zo c ZL 1 jZo tan b, Zo 1 jZL tan b, d (lossless) (11.34) showing that the input impedance varies periodically with distance from the load. The quantity b in eq. (11.34) is usually referred to as the electrical length of the line and can be expressed in degrees or radians. We now define GL as the voltage reflection coefficient (at the load). The reflection coefficient GL is the ratio of the voltage reflection wave to the incident wave at the load; that is, GL 5 g, V2 o e 2g, V1 o e (11.35) 1 Substituting V2 o and Vo in eq. (11.30) into eq.

(11.35) and incorporating VL 5 ZLIL gives 11_Sadiku_Ch11.indd 566 GL 5 ZL 2 Zo (11.36) ZL 1 Zo 25/09/17 5:24 PM 11.4 Input Impedance, Standing Wave Ratio, and Power 567 The voltage reflection coefficient at any point on the line is the ratio of the reflected voltage wave to that of the incident wave. That is, G1z2 5 gz V2 V2 o e o 2gz 5 e 2gz V1 V1 o e o But z 5 , 2 ,r. Substituting and combining with eq. (11.35), we get G1z2 5 V2 o 2g, 22g,r e e 5 GLe22g,r(11.37) V1 o The current reflection coefficient at any point on the line is the negative of the voltage reflection coefficient at that point. g, 1 2g, Thus, the current reflection coefficient at the load is I2 5 2GL. o e /Io e Just as we did for plane waves, we define the standing wave ratio s (otherwise denoted by SWR) as s5 Vmax Imax 1 1 0 GL 0 5 5 (11.38a) Vmin Imin 1 2 0 GL 0 0 GL 0 5 S21 S11 (11.38b) It is easy to show that Imax 5 Vmax /Zo and Imin 5 Vmin /Zo. The input impedance Zin in eq. (11.34) has maxima and minima that occur, respectively, at the maxima and minima of the voltage standing wave. It can also be shown that 0 Zin 0 max 5 Vmax 5 sZo(11.39a) Imin and 0 Zin 0 min 5 Zo Vmin 5 (11.39b) s Imax As a way of demonstrating these concepts, consider a lossless line with characteristic impedance of Zo 5 50 V. For the sake of simplicity, we assume that the line is terminated in a pure resistive load ZL 5 100 V and the voltage at the load is 100 V (rms).

The conditions on the line are displayed in Figure 11.7. Note from Figure 11.7 that conditions on the line repeat themselves every half-wavelength. 11_Sadiku_Ch11.indd 567 25/09/17 5:24 PM 568 CHAPTER 11 TRANSMISSION LINES |V | |V | |I| βl (radians) λ (wavelength) FIGURE 11.7 Voltage and current standing wave patterns on a lossless line terminated by a resistive load. As mentioned at the beginning of this chapter, a transmission line is used in transferring power from the source to the load. The average input power at a distance from the load is given by an equation similar to eq. (10.78); that is, Pave 5 1 Re 3 Vs 1 , 2 I*s 1 , 2 4 2 where the factor 12 is needed because we are dealing with the peak values instead of the rms values. Assuming a lossless line, we substitute eqs.

(11.24) and (11.25) to obtain Pave 5 5 1 1 jb, Re c V1 1 Ge2jb, 2 V * 1 e2jb, 2 G*e jb, 2 d o 1e 2 Zo 2 0 V1 1 o 0 1 1 2 0 G 0 2 1 Ge22jb, 2 G*e2jb, 2 d Re c 2 Zo Since the last two terms together become purely imaginary, we have 11_Sadiku_Ch11.indd 568 Pave 5 2 0 V1 o 0 2Zo 1 1 2 0 G 0 2 2 (11.40) 25/09/17 5:24 PM 11.4 Input Impedance, Standing Wave Ratio, and Power 569 The first term is the incident power Pi, while the second term is the reflected power Pr. Thus eq. (11.40) may be written as Pt 5 Pi 2 Pr where Pt is the input or transmitted power and the negative sign is due to the negativegoing wave (since we take the reference direction as that of the voltage/current traveling toward the right). We should notice from eq. (11.40) that the power is constant and does not depend on , since it is a lossless line. Also, we should notice that maximum power is delivered to the load when G 5 0, as expected. We now consider special cases when the line is connected to load ZL 5 0, ZL 5 `, and ZL 5 Zo. These special cases can easily be derived from the general case. A. Shorted Line (ZL = 0) For this case, eq. (11.34) becomes Zsc 5 Zin ` ZL50 5 jZo tan b,(11.41a) Also, from eqs. (11.36) and (11.38) GL 5 21, s 5 `(11.41b) We notice from eq. (11.41a) that Zin is a pure reactance, which could be capacitive or inductive depending on the value of . The variation of Zin with is shown in - Figure 11.8(a). B. Open-Circuited Line (ZL = `) In this case, eq. (11.34) becomes Zo Zoc 5 lim Zin 5 5 2jZo cot b,(11.42a) j tan b, ZL S ` and from eqs. (11.36) and (11.38), GL 5 1, s 5 `(11.42b) The variation of Zin with is shown in Figure 11.8(b). Notice from eqs. (11.41a) and (11.42a) that ZscZoc 5 Z2o(11.43) C. Matched Line (ZL = Zo) The most desired case from the practical point of view is the matched line i.e., ZL = Zo. For this case, eq. (11.34) reduces to 11_Sadiku_Ch11.indd 569 Zin 5 Zo(11.44a) 25/09/17 5:24 PM 570 CHAPTER 11 TRANSMISSION LINES FIGURE 11.8 Input impedance of a lossless line: (a) when shorted, (b) when open. and from eqs. (11.36) and (11.38), GL 5 0, s 5 1 (11.44b) that is, V2 o 5 0; the whole wave is transmitted, and there is no reflection. The incident power is fully absorbed by the load. Thus maximum power transfer is possible when a transmission line is matched to the load. EXAMPLE 11.3 A certain transmission line 2 m long operating at v 5 106 rad/s has a 5 8 dB/m, b 5 1 rad/m, and Zo 5 60 1 j40 V.

If the line is connected to a source of 10l0° V, Zg 5 40 V and terminated by a load of 20 1 j50 V, determine (a) The input impedance (b) The sending-end current (c) The current at the middle of the line 11_Sadiku_Ch11.indd 570 25/09/17 5:24 PM 11.4 Input Impedance, Standing Wave Ratio, and Power 571 Solution: (a) Since 1 Np 5 8.686 dB, a5 8 5 0.921 Np/m 8.686 g 5 a 1 jb 5 0.921 1 j1 /m g, 5 2 1 0.921 1 j1 2 5 1.84 1 j2 Using the formula for tanh 1 x 1 jy 2 in Appendix A.3, we obtain tanh g, 5 1.033 2 j0.03929 Zin 5 Zo a ZL 1 Zo tanh g, b Zo 1 ZL tanh g, 5 1 60 1 j40 2 c 20 1 j50 1 1 60 1 j40 2 1 1.033 2 j0.03929 2 d 60 1 j40 1 1 20 1 j50 2 1 1.033 2 j0.03929 2 Zin 5 60.25 1 j38.79 V (b) The sending-end current is I 1 z 5 0 2 5 Io. From eq. (11.28), I1z 5 02 5 Vg Zin 1 Zg 5 10 60.25 1 j38.79 1 40 5 93.03l221.15° mA iii 2 (c) To find the current at any point, we need V1 o and Vo . But Io 5 I 1 z 5 0 2 5 93.03l221.15° mA iii Vo 5 ZinIo 5 1 71.66l 32.77° 2 1 0.09303l221.15° 2 5 6.667l 11.62° V iii iii iii From eq. (11.27), 1 1 V 1 ZoIo 2 2 o 1 5 3 6.667l 11.62° 1 1 60 1 j40 2 1 0.09303l 221.15° 2 4 5 6.687l 12.08° 2 iii iiii iii V1 o 5 V2 o 5 1 1 V 2 ZoIo 2 5 0.0518l 260° 2 o ii At the middle of the line, z 5 ,/2, gz 5 0.921 1 j1. Hence, the current at this point is 11_Sadiku_Ch11.indd 571 V1 V2 o 2gz o gz e 2 e Zo Zo 1 6.687e j12.08° 2 e20.9212j1 1 0.0518e j260° 2 e0.9211j1 5 2 60 1 j40 60 1 j40 Is 1 z 5 ,/2 2 5 25/09/17 5:24 PM 572 CHAPTER 11 TRANSMISSION LINES Note that j1 is in radians and is equivalent to j57.3°. Thus, Is 1 z 5 ,/2 2 5 6.687e j12.08°e20.921e2j57.3° 0.0518e j260°e0.921e j57.3° 2 72.1e j33.69° 72.1e33.69° 5 0.0369e2j78.91° 2 0.001805e j283.61° 5 6.673 2 j34.456 mA 5 35.10l 281° mA ii PRACTICE EXERCISE 11.3 The transmission line shown in Figure 11.9 is 40 m long and has Vg 5 15l 0° Vrms, Zo 5 30 1 j60 V, and VL 5 5l 248° Vrms. If the line is matched to the load and Zg 5 0, ii calculate: (a) The input impedance Zin (b) The sending-end current Iin and voltage Vin (c) The propagation constant g Answer: (a) 30 1 j60 V, (b) 0.2236l 263.43° A, 7.5l0° Vrms, (c) 0.0101 1 iii j0.02094 /m. FIGURE 11.9 For Practice Exercise 11.3. 11.5 THE SMITH CHART Prior to the advent of digital computers and calculators, engineers developed all sorts of aids (slide rules, tables, charts, graphs, etc.) to facilitate their calculations for design and analysis. To reduce the tedious manipulations involved in calculating the characteristics of transmission lines, graphical means were then developed. The Smith chart3 is the most commonly 3 Devised by Phillip H. Smith in 1939. See P. H.

Smith, “Transmission line calculator,” Electronics, vol. 12, pp. 29–31, 1939, and “An improved transmission line calculator,” Electronics, vol. 17, pp. 130–133, 318–325, 1944. 11_Sadiku_Ch11.indd 572 25/09/17 5:24 PM 11.5 The Smith Chart 573 FIGURE 11.10 Unit circle on which the Smith chart is constructed. used of the graphical techniques. It is basically a graphical indication of the impedance of a transmission line and of the corresponding reflection coefficient as one moves along the line. It becomes easy to use after a small amount of experience. We will first examine how the Smith chart is constructed and later employ it in our calculations of transmission line characteristics such as GL, s, and Zin. We will assume that the transmission line to which the Smith chart will be applied is lossless 1 Zo 5 Ro 2 , although this is not fundamentally required. The Smith chart is constructed within a circle of unit radius 1 0 G 0 # 1 2 as shown in Figure 11.10. The construction of the chart is based on the relation in eq. (11.36)4; that is, G5 ZL 2 Zo (11.45) ZL 1 Zo or G 5 0 G 0 lu G 5 Gr 1 jGi(11.46) i where Gr and Gi are the real and imaginary parts of the reflection coefficient G. Instead of having separate Smith charts for transmission lines with different characteristic impedances (e.g., Zo 5 60, 100, 120 V), we prefer to have just one that can be used for any line.

We achieve this by using a normalized chart in which all impedances are normalized with respect to the characteristic impedance Zo of the particular line under c onsideration. For the load impedance ZL, for example, the normalized impedance zL is given by zL 5 ZL 5 r 1 jx(11.47) Zo Substituting eq. (11.47) into eqs. (11.45) and (11.46) gives 4 11_Sadiku_Ch11.indd 573 G 5 Gr 1 jGi 5 zL 2 1 (11.48a) zL 1 1 Whenever a subscript is not attached to G, we simply mean voltage reflection coefficient at the load 1 GL 5 G 2 . 25/09/17 5:24 PM 574 CHAPTER 11 TRANSMISSION LINES or zL 5 r 1 jx 5 1 1 1 Gr 2 1 jGi (11.48b) 1 1 2 Gr 2 2 jGi Normalizing and equating real and imaginary components, we obtain r5 x5 1 2 G2r 2 G2i (11.49a) 1 1 2 Gr 2 2 1 G2i 2 Gi (11.49b) 1 1 2 Gr 2 2 1 G2i Rearranging terms in eqs. (11.49) leads to c Gr 2 2 r 1 2 d 1 G2i 5 c d (11.50) 11r 11r and 1 2 1 2 3 Gr 2 1 4 2 1 c Gi 2 d 5 c d (11.51) x x Each of eqs. (11.50) and (11.51) is similar to 1 x 2 h 2 2 1 1 y 2 k 2 2 5 a2(11.52) which is the general equation of a circle of radius a, centered at 1 h, k 2 . Thus eq. (11.50) is an r-circle (resistance circle) with center at 1 Gr, Gi 2 5 a radius 5 r , 0b(11.53a) 11r 1 (11.53b) 11r TABLE 11.3 Radii and Centers of r-Circles for Typical Values of r Normalized Resistance (r) 0 1/2 1 2 5 ` 11_Sadiku_Ch11.indd 574 Radius a 1 b 11r 1 2/3 1/2 1/3 1/6 0 Center a r , 0b 11r (0, 0) (1/3, 0) (1/2, 0) (2/3, 0) (5/6, 0) (1, 0) 25/09/17 5:24 PM 11.5 The Smith Chart 575 FIGURE 11.11 Typical r-circles for r 5 0, 0.5, 1, 2, 5, . For typical values of the normalized resistance r, the corresponding centers and radii of the r-circles are presented in Table 11.3. Typical examples of the r-circles based on the data in Table 11.3 are shown in Figure 11.11. Similarly, eq.

(11.51) is an x-circle (reactance circle) with 1 center at 1 Gr, Gi 2 5 a1, b(11.54a) x 1 radius 5 (11.54b) x Table 11.4 presents centers and radii of the x-circles for typical values of x, and Figure 11.12 shows the corresponding plots. Notice that while r is always positive, x can be positive (for inductive impedance) or negative (for capacitive impedance).

If we superpose the r-circles and x-circles, what we have is the Smith chart shown in Figure 11.13. On the chart, we locate a normalized impedance z 5 2 1 j, for example, as the point of intersection of the r 5 2 circle and the x 5 1 circle. This is point P1 in Figure 11.13. Similarly, z 5 1 2 j 0.5 is located at P2, where the r 5 1 circle and the x 5 20.5 circle intersect. Apart from the r- and x-circles (shown on the Smith chart), we can draw the s-circles or constant standing wave ratio circles (always not shown on the Smith chart), which are TABLE 11.4 Radii and Centers of x-Circles for Typical Values of x Normalized Reactance (x) 0 1/2 1 2 5 ` 11_Sadiku_Ch11.indd 575 1 Radius a b x ` 2 1 1/2 1/5 0 1 Center a1, b x (1, `) (1, 2) (1, 1) (1, 1/2) (1, 1/5) (1, 0) 25/09/17 5:24 PM 576 CHAPTER 11 TRANSMISSION LINES FIGURE 11.12 Typical x-circles for x 5 0, 0.5, 1, 2, 5, . FIGURE 11.13 Illustration of the r-, x-, and s-circles on the Smith chart.

11_Sadiku_Ch11.indd 576 25/09/17 5:24 PM 11.5 The Smith Chart 577 centered at the origin with s varying from 1 to `. The value of the standing wave ratio s is determined by locating where an s-circle crosses the Gr axis. Typical examples of s-circles for s 5 1, 2, 3, and ` are shown in Figure 11.13. Since 0 G 0 and s are related according to eq. (11.38), the s-circles are sometimes referred to as 0 G 0 -circles, with 0 G 0 varying linearly from 0 to 1 as we move away from the center O toward the periphery of the chart, while s varies nonlinearly from 1 to `. The following points should be noted about the Smith chart. 1. At point Psc on the chart r 5 0, x 5 0; that is, ZL 5 0 1 j0, showing that Psc represents a short circuit on the transmission line. At point Poc, r 5 ` and x 5 `, or ZL 5 ` 1 j`, which implies that Poc corresponds to an open circuit on the line. Also at Poc, r 5 0 and x 5 0, showing that Poc is another location of a short circuit on the line. 2.

A complete revolution (360°) around the Smith chart represents a distance of l/2 on the line. Clockwise movement on the chart is regarded as moving toward the generator (or away from the load) as shown by the arrow G in Figure 11.14(a) and (b).

Similarly, counterclockwise movement on the chart corresponds to moving toward the load (or away from the generator) as indicated by the arrow L in Figure 11.14. Notice from Figure 11.14(b) that at the load, moving toward the load does not make sense (because we are already at the load). The same can be said of the case when we are at the generator end. 3. There are three scales around the periphery of the Smith chart as illustrated in Figure 11.14(a). The three scales are included for the sake of convenience but they are actually meant to serve the same purpose; one scale should be sufficient. The scales are used in determining the distance from the load or generator in degrees or wavelengths. The outermost scale is used to determine the distance on the line from the generator end in terms of wavelengths, and the next scale determines the distance from the load end in terms of wavelengths. The innermost scale is a protractor (in degrees) and is primarily used in determining G; it can also be used to determine the distance from the load or generator. Since a l/2 distance on the line corresponds to a movement of 360° on the chart, l distance on the line corresponds to a 720° movement on the chart. → 720° (11.55) Thus we may ignore the other outer scales and use the protractor (the innermost scale) for all our G and distance calculations. 4. The voltage Vmax occurs where Zin, max is located on the chart [see eq. (11.39a)], and that is on the positive Gr-axis or on OPoc in Figure 11.14(a). The voltage Vmin is located at the same point where we have Zin, min on the chart, that is, on the negative Gr-axis or on OPsc in Figure 11.14(a). Notice that Vmax and Vmin (or Zin, max and Zin, min) are l/4 (or 180°) apart. 5.

The Smith chart is used both as impedance chart and admittance chart 1 Y 5 1/Z 2 . As admittance chart (normalized admittance y 5 Y/Yo 5 g 1 jb 2 , the g- and b-circles correspond to r- and x-circles, respectively. 11_Sadiku_Ch11.indd 577 25/09/17 5:24 PM ge ne ra t or 578 CHAPTER 11 TRANSMISSION LINES d loa FIGURE 11.14 (a) Smith chart illustrating scales around the periphery and movements around the chart. (b) Corresponding movements along the transmission line. Based on these important properties, the Smith chart may be used to determine, among other things, (a) G 5 0 G 0 lu G and s; (b) Zin or Yin; and (c) the locations of Vmax and Vmin provided that we are given Zo, ZL, l, and the length of the line. Some examples will clearly show how we can find all these and much more with the aid of the Smith chart, a compass, and a plain straightedge. A complete Smith chart is available in Appendix D. You may copy this. 11_Sadiku_Ch11.indd 578 25/09/17 5:24 PM 11.5 The Smith Chart 579 EXAMPLE 11.4 A lossless transmission line with Zo 5 50 V is 30 m long and operates at 2 MHz. The line is terminated with a load ZL 5 60 1 j40 V. If u 5 0.6c on the line, find (a) The reflection coefficient G (b) The standing wave ratio s (c) The input impedance Solution: This problem will be solved with and without using the Smith chart. Method 1 (without the Smith chart): 60 1 j40 2 50 10 1 j40 ZL 2 Zo (a) G 5 5 5 ZL 1 Zo 50 1 j40 1 50 110 1 j40 5 0.3523l56º i 1 1 0G0 1 1 0.3523 (b) s 5 5 5 2.088 1 2 0.3523 1 2 0G0 (c) Since u 5 v/b, or b 5 v/u, b, 5 2p 1 2 3 106 2 1 30 2 v, 2p 5 5 5 120 º 8 u 3 0.6 1 3 3 10 2 Note that b is the electrical length of the line. Zin 5 Zo c ZL 1 jZo tan b, Zo 1 jZL tan b, d 50 1 60 1 j40 1 j50 tan 120º 2 3 50 1 j 1 60 1 j40 2 tan 120º 4 50 1 6 1 j4 2 j5"3 2 5 5 24.01l3.22° ii 1 5 1 4"3 2 j6"3 2 5 5 23.97 1 j1.35 V Method 2 (using the Smith chart): (a) Calculate the normalized load impedance zL 5 60 1 j40 ZL 5 Zo 50 5 1.2 1 j0.8 Locate zL on the Smith chart of Figure 11.15 at point P, where the r 5 1.2 circle and the x 5 0.8 circle meet. To get G at zL, extend OP to meet the r 5 0 circle at Q and measure OP and OQ.

Since OQ corresponds to 0 G 0 5 1, then at P, 0G0 5 11_Sadiku_Ch11.indd 579 OP 3.2 cm 5 5 0.3516 OQ 9.1 cm 25/09/17 5:24 PM 580 CHAPTER 11 TRANSMISSION LINES FIGURE 11.15 Smith chart for Example 11.4. Note that OP 5 3.2 cm and OQ 5 9.1 cm were taken from the Smith chart used by the author; the Smith chart in Figure 11.15 is reduced, but the ratio of OP/OQ remains the same. Angle G is read directly on the chart as the angle between OS and OP; that is, u G 5 angle POS 5 56º Thus G 5 0.3516 l56º i (b) To obtain the standing wave ratio s, draw a circle with radius OP and center at O. This is the constant s or 0 G 0 circle. Locate point S where the s-circle meets the Gr-axis. [This is easily shown by setting Gi 5 0 in eq. (11.49a).] The value of r at this point is s; that is, s 5 r 1 for r $ 1 2 5 2.1 11_Sadiku_Ch11.indd 580 25/09/17 5:24 PM 11.5 The Smith Chart 581 (c) To obtain Zin, first express in terms of l or in degrees: 0.6 1 3 3 108 2 u 5 5 90 m f 2 3 106 30 l 720º , 5 30 m 5 l5 S 5 240º 90 3 3 l5 Since l corresponds to an angular movement of 720° on the chart, the length of the line corresponds to an angular movement of 240°. That means we move toward the generator (or away from the load, in the clockwise direction) 240° on the s-circle from point P to point G. At G, we obtain zin 5 0.47 1 j0.03 Hence Zin 5 Zozin 5 50 1 0.47 1 j0.03 2 5 23.5 1 j1.5 V Although the results obtained using the Smith chart are only approximate, for engineering purposes they are close enough to the exact ones obtained by Method 1. However, an inexpensive modern calculator can handle the complex algebra in less time and with much less effort than are needed to use the Smith chart. The value of the Smith chart is that it allows us to observe the variation of Zin with . PRACTICE EXERCISE 11.4 A 70 lossless line has s 5 1.6 and u G 5 300°. If the line is 0.6l long, obtain (a) G, ZL, Zin (b) The distance of the first minimum voltage from the load Answer: (a) 0.228 l300º , 80.5 2 j33.6 V, 47.6 2 j17.5 V, (b) /6. i i EXAMPLE 11.5 A load of 100 1 j150 V is connected to a 75 lossless line. Find: (a) G (b) s (c) The load admittance YL (d) Zin at 0.4l from the load (e) The locations of Vmax and Vmin with respect to the load if the line is 0.6l long (f) Zin at the generator. Solution: (a) We can use the Smith chart to solve this problem. The normalized load impedance is zL 5 11_Sadiku_Ch11.indd 581 100 1 j150 ZL 5 5 1.33 1 j2 Zo 75 25/09/17 5:24 PM 582 CHAPTER 11 TRANSMISSION LINES We locate this at point P on the Smith chart of Figure 11.16. At P, we obtain OP 6 cm 5 5 0.659 OQ 9.1 cm u G 5 angle POS 5 40º 0G0 5 Hence, G 5 0.659 l40º i Check: G5 100 1 j150 2 75 ZL 2 Zo 5 ZL 1 Zo 100 1 j150 1 75 5 0.6598 — / 39.94° — FIGURE 11.16 Smith chart for Example 11.5. 11_Sadiku_Ch11.indd 582 25/09/17 5:24 PM 11.5 The Smith Chart 583 (b) Draw the constant s-circle passing through P and obtain s 5 4.82 Check: s5 1 1 0G0 1 1 0.659 5 5 4.865 1 2 0.659 1 2 0G0 (c) To obtain YL, extend PO to POP and note point P where the constant s-circle meets POP. At P, obtain yL 5 0.228 2 j0.35 The load admittance is YL 5 YoyL 5 Check: YL 5 1 1 0.228 2 j0.35 2 5 3.04 2 j4.67 mS 75 1 1 5 5 3.07 2 j4.62 mS ZL 100 1 j150 (d) The 0.4l corresponds to an angular movement of 0.4 3 720 º 5 288º on the constant s-circle. From P, we move 288° toward the generator (clockwise) on the s-circle to reach point R. At R, zin 5 0.3 1 j0.63 Hence Zin 5 Zozin 5 75 1 0.3 1 j0.63 2 5 22.5 1 j47.25 V Check: b, 5 2p 1 0.4l 2 5 360° 1 0.4 2 5 144° l Zin 5 Zo c 5 ZL 1 jZo tan b, Zo 1 jZL tan b, d 75 1 100 1 j150 1 j75 tan 144° 2 3 75 1 j 1 100 1 j150 2 tan 144° 4 5 54.41l65.25 º ii 11_Sadiku_Ch11.indd 583 25/09/17 5:24 PM 584 CHAPTER 11 TRANSMISSION LINES or Zin 5 21.9 1 j47.6 V (e) The 0.6l corresponds to an angular movement of 0.6 3 720º 5 1 revolution 1 72º Thus, we start from P (load end), move clockwise along the s-circle 432°, or one revolution plus 72°, and reach the generator at point G. Note that to reach G from P, we have passed through point T (location of Vmin) once and point S (location of Vmax) twice. Thus, from the load, 1st Vmax is located at 40º l 5 0.055l 720º 2nd Vmax is located at 0.0555l 1 l 5 0.555l 2 and the only Vmin is located at 0.055l 1 l/4 5 0.3055l (f) At G (generator end), zin 5 1.8 2 j2.2 Zin 5 75 1 1.8 2 j2.2 2 5 135 2 j165 V This can be checked by using eq. (11.34), where b, 5 2p 1 0.6l 2 5 216°. l We can see how much time and effort are saved by using the Smith chart. PRACTICE EXERCISE 11.5 A lossless 60 V line is terminated by a load of 60 1 j60 V. (a) Find G and s. If Zin 5 120 2 j60 V, how far (in terms of wavelengths) is the load from the generator? Solve this without using the Smith chart.

(b) Use the Smith chart to solve the problem in part (a). Calculate Zmax and Zin, min. How far (in terms of l) is the first maximum voltage from the load? l Answer: (a) 0.4472 — / 63.43°, / 62°, — 2.618, 8 1 1 1 4n 2 , n 5 0, 1, 2, . . . , (b) 0.4457 — l 2.612, 1 1 1 4n 2 , 157.1 V, 22.92 V, 0.0861 l. 8 11_Sadiku_Ch11.indd 584 25/09/17 5:25 PM 11.6 Some Applications of Transmission Lines 585 11.6 SOME APPLICATIONS OF TRANSMISSION LINES Transmission lines are used to serve different purposes. Here we consider how transmission lines are used for load matching and impedance measurements. A. Quarter-Wave Transformer (Matching) When Zo 2 ZL, we say that the load is mismatched and a reflected wave exists on the line. However, for maximum power transfer, it is desired that the load be matched to the transmission line 1 Zo 5 ZL 2 so that there is no reflection 1 0 G 0 5 0 or s 5 1 2 . The matching is achieved by using shorted sections of transmission lines. We recall from eq. (11.34) that when , 5 l/4 or b, 5 1 2p/l 2 1 l/4 2 5 p/2, that is, Zin 5 Zo c ZL 1 jZo tan p/2 Zo 1 jZL tan p/2 d 5 Z2o (11.56) ZL Zo Zin 5 Zo ZL or zin 5 1 S yin 5 zL(11.57) zL Thus by adding a l/4 line on the Smith chart, we obtain the input admittance corresponding to a given load impedance. Also, a mismatched load ZL can be properly matched to a line (with characteristic impedance Zo) by inserting prior to the load a transmission line l/4 long (with characteristic impedance Z or) as shown in Figure 11.17.

The l/4 section of the transmission line is called a quarter-wave transformer because it is used for impedance matching like an ordinary transformer. From eq. (11.56), Zro is selected such that 1 Zin 5 Zo 2 Zro 5 "ZoZL (11.58) FIGURE 11.17 Load matching using a l/4 transformer. 11_Sadiku_Ch11.indd 585 25/09/17 5:25 PM 586 CHAPTER 11 TRANSMISSION LINES FIGURE 11.18 Voltage standing wave pattern of mismatched load: (a) without a l/4 transformer, (b) with a l/4 transformer.

where Zro, Zo and ZL are all real. If, for example, a 120 V load is to be matched to a 75 V line, the quarter-wave transformer must have a characteristic impedance of ! 1 75 2 1 120 2 . 95 V. This 95 V quarter-wave transformer will also match a 75 V load to a 120 V line. The voltage standing wave patterns without and with the l/4 transformer are shown in Figure 11.18(a) and (b), respectively.

From Figure 11.18, we observe that although a standing wave still exists between the transformer and the load, there is no standing wave to the left of the transformer due to the matching. However, the reflected wave (or standing wave) is eliminated only at the desired wavelength (or frequency f); there will be reflection at a slightly different wavelength. Thus, the main disadvantage of the quarter-wave transformer is that it is a narrow-band or frequency-sensitive device. B. Single-Stub Tuner (Matching) The major drawback of using a quarter-wave transformer as a line-matching device is eliminated by using a single-stub tuner. The tuner consists of an open or shorted section of transmission line of length d connected in parallel with the main line at some distance from the load, as in Figure 11.19. Notice that the stub has the same characteristic impedance as the main line, although stubs may be designed with different values of Zo. It is more difficult to use a series stub although it is theoretically feasible. An open-circuited stub radiates some energy at high frequencies. Consequently, shunt short-circuited parallel stubs are preferred. Since we intend that Zin 5 Zo, that is, zin 5 1 or yin 5 1 at point A on the line, we first draw the locus y 5 1 1 jb 1 r 5 1 circle 2 on the Smith chart as shown in Figure 11.20.

If a shunt stub of admittance ys 5 2jb is introduced at A, then yin 5 1 1 jb 1 ys 5 1 1 jb 2 jb 5 1 1 j0(11.59) FIGURE 11.19 Matching with a single-stub tuner. S 11_Sadiku_Ch11.indd 586 25/09/17 5:25 PM 11.6 Some Applications of Transmission Lines 587 L FIGURE 11.20 Using the Smith chart to determine and d of a shunt-shorted single-stub tuner. Psc as desired. Since b could be positive or negative, two possible values of , 1 ,l/2 2 can be found on the line. At A, ys 5 2jb, , 5 ,A and at B, ys 5 jb, , 5 ,B as in Figure 11.20. Because the stub is shorted 1 yrL 5 ` 2 , we determine the length d of the stub by finding the distance from Psc (at which zrL 5 0 1 j0) to the required stub admittance ys.

For the stub at A, we obtain d 5 dA as the distance from Psc to A, where A corresponds to ys 5 2jb located on the periphery of the chart as in Figure 11.20.

Similarly, we obtain d 5 dB as the distance from Psc to B 1 ys 5 jb 2 . Thus we obtain d 5 dA and d 5 dB, corresponding to A and B, respectively, as shown in Figure 11.20. Note that dA 1 dB 5 l/2 always. Since we have two possible shunted stubs, we normally choose to match the shorter stub or one at a position closer to the load. Instead of having a single stub shunted across the line, we may have two stubs. This arrangement, which is called double-stub matching, allows for the adjustment of the load impedance. C. Slotted Line (Impedance Measurement) At high frequencies, it is very difficult to measure current and voltage because measuring devices become significant in size and every circuit becomes a transmission line. The slotted line is a simple device used in determining the impedance of an unknown load at high frequencies up into the region of gigahertz. It consists of a section of an air (lossless) line with a slot in the outer conductor as shown in Figure 11.21. The line has a probe, along the E field (see Figure 11.4), which samples the E field and consequently measures the potential difference between the probe and its outer shield. The slotted line is primarily used in conjunction with the Smith chart to determine the standing wave ratio s (the ratio of maximum voltage to the minimum voltage) and the load impedance ZL. The value of s is read directly on the detection meter when the load is connected. To determine ZL, we first replace the load by a short circuit and note the locations of voltage minima (which are more accurately determined than the maxima because of the sharpness of the turning point) on the scale. Since impedances repeat every half-wavelength, any of the minima may be selected as the load reference point. We now determine the distance from the selected reference point to the load by replacing the short circuit by the load and noting the locations of voltage minima. The distance (distance of 11_Sadiku_Ch11.indd 587 25/09/17 5:25 PM 588 CHAPTER 11 TRANSMISSION LINES S To ge C FIGURE 11.21 (a) Typical slotted line; (b) Determining the location of the load ZL and Vmin on the line.

Vmin toward the load) expressed in terms of l is used to locate the position of the load of an s-circle on the chart as shown in Figure 11.22. The procedure for using the slotted line can be summarized as follows. 1. With the load connected, read s on the detection meter. With the value of s, draw the s-circle on the Smith chart. 2. With the load replaced by a short circuit, locate a reference position for ZL at a voltage minimum point. 3. With the load on the line, note the position of Vmin and determine . 4. On the Smith chart, move toward the load a distance from the location of Vmin. Find ZL at that point. load s-C FIGURE 11.22 Determining the load impedance from the Smith chart by using the data obtained from the slotted line. V 11_Sadiku_Ch11.indd 588 25/09/17 5:25 PM 11.6 Some Applications of Transmission Lines 589 EXAMPLE 11.6 With an unknown load connected to a slotted air line, s 5 2 is recorded by a standing wave indicator, and minima are found at 11 cm, 19 cm, . . . , on the scale. When the load is replaced by a short circuit, the minima are at 16 cm, 24 cm, ... If Zo 5 50 V, calculate l, f, and ZL. Solution: Consider the standing wave patterns as in Figure 11.23(a).

From this, we observe that l 5 19 2 11 5 8 cm or 5 16 cm 2 f5 u 3 3 108 5 5 1.875 GHz l 16 3 1022 Electrically speaking, the load can be located at 16 cm or 24 cm. If we assume that the load is at 24 cm, the load is at a distance from Vmin, where , 5 24 2 19 5 5 cm 5 W W 11_Sadiku_Ch11.indd 589 5 l 5 0.3125 l 16 FIGURE 11.23 Determining ZL by using the slotted line: (a) wave pattern, (b) Smith chart for Example 11.6. 25/09/17 5:25 PM 590 CHAPTER 11 TRANSMISSION LINES This corresponds to an angular movement of 0.3125 3 720° 5 225° on the s 5 2 circle. By starting at the location of Vmin and moving 225° toward the load (counterclockwise), we reach the location of zL as illustrated in Figure 11.23(b). Thus zL 5 1.4 1 j0.75 and ZL 5 ZozL 5 50 1 1.4 1 j0.75 2 5 70 1 j37.5 V PRACTICE EXERCISE 11.6 The following measurements were taken by means of the slotted line technique: with load, s 5 1.8, Vmax occurred at 23 cm, 33.5 cm, . . . ; with short, s 5 ` , Vmax occurred at 25 cm, 37.5 cm, ... If Zo 5 50 V, determine ZL. Answer: 32.5 2 j17.5 V. EXAMPLE 11.7 An antenna with an impedance of 40 1 j30 V is to be matched to a 100 V lossless line with a shorted stub. Determine (a) The required stub admittance (b) The distance between the stub and the antenna (c) The stub length (d) The standing wave ratio on each segment of the system Solution: (a) zL 5 40 1 j30 ZL 5 5 0.4 1 j0.3 Zo 100 Locate zL on the Smith chart as in Figure 11.24 and from this draw the s-circle so that yL can be located diametrically opposite zL. Thus yL 5 1.6 2 j1.2. Alternatively, we may find yL by using yL 5 Zo 100 5 5 1.6 2 j1.2 ZL 40 1 j30 Locate points A and B where the s-circle intersects the g 5 1 circle. At A, ys 5 2j1.04 and at B, ys 5 1j1.04. Thus the required stub admittance is Ys 5 Yo ys 5 6j1.04 1 5 6j10.4 mS 100 Both j10.4 mS and 2j10.4 mS are possible values. 11_Sadiku_Ch11.indd 590 25/09/17 5:25 PM 11.6 Some Applications of Transmission Lines 591 FIGURE 11.24 Smith chart for Example 11.7. (b) From Figure 11.24, we determine the distance between the load (antenna in this case) yL and the stub. At A, ,A 5 and at B: 1 62° 2 239° 2 l l 2 5 0.36l 2 720° ,B 5 11_Sadiku_Ch11.indd 591 1 62° 2 39° 2 l 5 0.032l 720° 25/09/17 5:25 PM 592 CHAPTER 11 TRANSMISSION LINES (c) Locate points A and B corresponding to stub admittance 2j1.04 and j1.04, respectively. Determine the stub length (distance from Psc to A and B): dA 5 88° l 5 0.1222l 720° dB 5 272° l 5 0.3778l 720° Notice that dA 1 dB 5 0.5l as expected.

(d) From Figure 11.24, s 5 2.7. This is the standing wave ratio on the line segment between the stub and the load (see Figure 11.18); s 5 1 to the left of the stub because the line is matched, and s 5 ` along the stub because the stub is shorted. PRACTICE EXERCISE 11.7 A 75 V lossless line is to be matched to a load of 100 2 j80 with a shorted stub. Calculate the stub length, its distance from the load, and the necessary stub admittance. Answer: ,A 5 0.093l, ,B 5 0.272l, dA 5 0.126l, dB 5 0.374l, 6j12.67 mS. † 11.7 TRANSIENTS ON TRANSMISSION LINES In our discussion so far, we have assumed that a transmission line operates at a single frequency. In computer networks and in certain other practical applications, pulsed signals may be sent through the line. From Fourier analysis, a pulse can be regarded as a superposition of waves of many frequencies.

Thus, sending a pulsed signal on the line may be regarded as the same as simultaneously sending waves of different frequencies. As in circuit analysis, when a pulse generator or battery connected to a transmission line is switched on, it takes some time for the current and voltage on the line to reach steady values. This transitional period is called the transient. The transient behavior just after closing the switch (or due to lightning strokes) is usually analyzed in the frequency domain by using Laplace transformation.

For the sake of convenience, we treat the problem in the time domain. Consider a lossless line of length and characteristic impedance Zo as shown in Figure 11.25(a). Suppose that the line is driven by a pulse generator of voltage Vg with internal impedance Zg at z 5 0 and terminated with a purely resistive load ZL. At the instant t 5 0 that the switch is closed, the starting current “sees” only Zg and Zo, so the initial situation can be described by the equivalent circuit of Figure 11.25(b). From the figure, the starting current at z 5 0, t 5 01 is given by Vg I 1 0, 01 2 5 Io 5 (11.60) Zg 1 Zo 11_Sadiku_Ch11.indd 592 25/09/17 5:25 PM 11.7 Transients on Transmission Lines 593 FIGURE 11.25 Transients on a transmission line: (a) a line driven by a pulse generator, (b) the equivalent circuit at z 5 0, t 5 01. and the initial voltage is V 1 0, 01 2 5 Vo 5 IoZo 5 Zo V (11.61) Zg 1 Zo g After the switch is closed, waves I1 5 Io and V1 5 Vo propagate toward the load at the speed u5 1 "LC (11.62) Since this speed is finite, it takes some time for the waves traveling in the positive direction to reach the load and interact with it. The presence of the load has no effect on the waves before the transit time given by , t1 5 (11.63) u After t1 seconds, the waves reach the load. The voltage (or current) at the load is the sum of the incident and reflected voltages (or currents). Thus and V 1 ,, t1 2 5 V1 1 V2 5 Vo 1 GLVo 5 1 1 1 GL 2 Vo(11.64) I 1 ,, t1 2 5 I1 1 I2 5 Io 2 GLIo 5 1 1 2 GL 2 Io(11.65) where GL is the load reflection coefficient given in eq. (11.36); that is, GL 5 ZL 2 Zo (11.66) ZL 1 Zo The reflected waves V2 5 GLVo and I2 5 2GLIo travel back toward the generator in addition to the waves Vo and Io already on the line. At time t 5 2t1, the reflected waves have reached the generator, so V 1 0, 2t1 2 5 V1 1 V2 5 GGGLVo 1 1 1 1 GL 2 Vo 11_Sadiku_Ch11.indd 593 15/11/17 5:17 PM 594 CHAPTER 11 TRANSMISSION LINES FIGURE 11.26 Bounce diagram for (a) a voltage wave and (b) a current wave. or and or V 1 0, 2t1 2 5 1 1 1 GL 1 GGGL 2 Vo(11.67) I 1 0, 2t1 2 5 I1 1 I2 5 2GG 1 2GLIo 2 1 1 1 2 GL 2 Io I 1 0, 2t1 2 5 1 1 2 GL 1 GLGG 2 Io(11.68) where GG is the generator reflection coefficient given by Zg 2 Zo GG 5 (11.69) Zg 1 Zo Again the reflected waves (from the generator end) V1 5 GGGLVo and I1 5 GGGLIo propagate toward the load, and the process continues until the energy of the pulse is actually absorbed by the resistors Zg and ZL. Instead of tracing the voltage and current waves back and forth, it is easier to keep track of the reflections using a bounce diagram, otherwise known as a lattice diagram. The bounce diagram consists of a zigzag line indicating the position of the voltage (or current) wave with respect to the generator end, as shown in Figure 11.26. On the bounce diagram, the voltage (or current) at any time may be determined by adding those values that appear on the diagram above that time. EXAMPLE 11.8 11_Sadiku_Ch11.indd 594 For the transmission line of Figure 11.27, calculate and sketch (a) The voltage at the load and generator ends for 0 , t , 6 ms (b) The current at the load and generator ends for 0 , t , 6 ms 25/09/17 5:25 PM 11.7 Transients on Transmission Lines 595 FIGURE 11.27 For Example 11.8. Solution: (a) We first calculate the voltage reflection coefficients at the generator and load ends: GG 5 GL 5 The transit time t1 5 Zg 2 Zo Zg 1 Zo 5 100 2 50 1 5 100 1 50 3 ZL 2 Zo 200 2 50 3 5 5 ZL 1 Zo 200 1 50 5 , 100 5 8 5 1 ms. u 10 The initial voltage at the generator end is Vo 5 Zo 50 1 12 2 5 4 V Vg 5 Zo 1 Zg 150 The 4 V is sent out to the load. The leading edge of the pulse arrives at the load at t 5 t1 5 1 ms. A portion of it, 4 1 3/5 2 5 2.4 V, is reflected back and reaches the generator at t 5 2t1 5 2 ms. At the generator, 2.4 1 1/3 2 5 0.8 is reflected and the process continues.

The whole process is best illustrated in the voltage bounce diagram of Figure 11.28. FIGURE 11.28 Voltage bounce diagram for Example 11.8. 11_Sadiku_Ch11.indd 595 25/09/17 5:25 PM 596 CHAPTER 11 TRANSMISSION LINES From the bounce diagram, we can sketch V 1 0, t 2 and V 1 ,, t 2 as functions of time as shown in Figure 11.29.

Notice from Figure 11.29 that as t S `, the voltages approach an asymptotic value of V` 5 ZL 200 1 12 2 5 8 V V 5 ZL 1 Zg g 300 This should be expected because the equivalent circuits at t 5 0 and t 5 ` are as shown in Figure 11.30. (b) The current reflection coefficients at the generator and load ends are 2GG 5 21/3 and 2GL 5 23/5, respectively. The initial current is Io 5 Vo 4 5 5 80 mA Zo 50 FIGURE 11.29 Voltage (not to scale) for Example 11.8: (a) at the generator end, (b) at the load end. 11_Sadiku_Ch11.indd 596 25/09/17 5:25 PM 11.7 Transients on Transmission Lines 597 FIGURE 11.30 Equivalent circuits for the line in Figure 11.27 for (a) t 5 0 and (b) t 5 ` .

Again, I 1 0, t 2 and I 1 ,, t 2 are easily obtained from the current bounce diagram shown in Figure 11.31. These currents are sketched in Figure 11.32. Note that I 1 ,, t 2 5 V 1 ,, t 2 /ZL. Hence, Figure 11.32(b) can be obtained either from the current bounce diagram of Figure 11.31 or by scaling Figure 11.29(b) by a factor of 1/ZL 5 1/200. Notice from Figures 11.30(b) and 11.32 that the currents approach an asymptotic value of I` 5 Vg Zg 1 ZL 5 12 5 40 mA 300 FIGURE 11.31 Current bounce diagram for Example 11.8. 11_Sadiku_Ch11.indd 597 25/09/17 5:25 PM 598 CHAPTER 11 TRANSMISSION LINES FIGURE 11.32 Current (not to scale) for Example 11.8: (a) at the generator end, (b) at the load end. PRACTICE EXERCISE 11.8 Repeat Example 11.8 if the transmission line is (a) Short-circuited (b) Open-circuited Answer: (a) See Figure 11.33, (b) See Figure 11.34. 11_Sadiku_Ch11.indd 598 25/09/17 5:25 PM 11.7 Transients on Transmission Lines 599 0V 4V FIGURE 11.33 For Practice Exercise 11.8(a). 11_Sadiku_Ch11.indd 599 25/09/17 5:25 PM 600 CHAPTER 11 TRANSMISSION LINES 12 V 8V 4V 0A 12 V 4V 4V FIGURE 11.34 For Practice Exercise 11.8(b).

11_Sadiku_Ch11.indd 600 25/09/17 5:25 PM 11.7 Transients on Transmission Lines 601 EXAMPLE 11.9 A 75 V transmission line of length 60 m is terminated by a 100 V load. If a rectangular pulse of width 5 ms and magnitude 4 V is sent out by the generator connected to the line, sketch I 1 0, t 2 and I 1 ,, t 2 for 0 , t , 15 ms. Take Zg 5 25 V and u 5 0.1c. Solution: In the previous example, the switching on of a battery created a step function, a pulse of infinite width. In this example, the pulse is of finite width of 5 ms. We first calculate the voltage reflection coefficients: GG 5 Zg 2 Zo 25 2 75 21 5 5 Zg 1 Zo 25 1 75 2 GL 5 ZL 2 Zo 100 2 75 1 5 5 ZL 1 Zo 100 1 75 7 The initial voltage and transit time are given by Vo 5 Zo 75 142 5 3 V Vg 5 Zo 1 Zg 100 t1 5 , 60 5 5 2 ms u 0.1 1 3 3 108 2 The time taken by Vo to go forth and back is 2t1 5 4 ms, which is less than the pulse duration of 5 ms. Hence, there will be overlapping. The current reflection coefficients are 2GL 5 2 The initial current Io 5 Vg Zg 1 Zo 5 1 7 and 2GG 5 1 2 4 5 40 mA. 100 Let i and r denote incident and reflected pulses, respectively. At the generator end: 0 , t , 5 ms, 4 , t , 9, 11_Sadiku_Ch11.indd 601 8 , t , 13, Ir 5 Io 5 40 mA 1 Ii 5 2 1 40 2 5 25.714 7 1 Ir 5 1 25.714 2 5 22.857 2 1 Ii 5 2 1 22.857 2 5 0.4082 7 1 Ir 5 1 0.4082 2 5 0.2041 2 25/09/17 5:25 PM 602 CHAPTER 11 TRANSMISSION LINES 12 , t , 17, 1 Ii 5 2 1 0.2041 2 5 20.0292 7 Ir 5 1 1 20.0292 2 5 20.0146 2 and so on. Hence, the plot of I 1 0, t 2 versus t is as shown in Figure 11.35(a). FIGURE 11.35 For Example 11.9 (not to scale). 11_Sadiku_Ch11.indd 602 25/09/17 5:25 PM 11.7 Transients on Transmission Lines 603 At the load end: 0 , t , 2 ms, V50 2 , t , 7, Vi 5 3 1 Vr 5 1 3 2 5 0.4296 7 1 Vi 5 2 1 0.4296 2 5 20.2143 2 6 , t , 11, 1 Vr 5 1 20.2143 2 5 20.0306 7 1 Vi 5 2 1 20.0306 2 5 0.0154 2 10 , t , 14, 1 Vr 5 1 0.0154 2 5 0.0022 7 and so on. From V 1 ,, t 2 , we can obtain I 1 ,, t 2 as I 1 ,, t 2 5 V 1 ,, t 2 V 1 ,, t 2 5 ZL 100 The plots of V 1 ,, t 2 and I 1 ,, t 2 are shown in Figure 11.35(b) and (c). PRACTICE EXERCISE 11.9 Repeat Example 11.9, replacing the rectangular pulse by the triangular pulse of Figure 11.36. Answer: 1 Io 2 max 5 100 mA. See Figure 11.37 for the current waveforms. FIGURE 11.36 Triangular pulse for Practice Exercise 11.9. 11_Sadiku_Ch11.indd 603 25/09/17 5:25 PM 604 CHAPTER 11 TRANSMISSION LINES FIGURE 11.37 Current waves for Practice Exercise 11.9. † 11.8 APPLICATION NOTE—MICROSTRIP LINES AND CHARACTERIZATION OF DATA CABLES † A. Microstrip Transmission Lines Microstrip lines belong to a group of lines known as parallel-plate transmission lines. They are widely used in present-day electronics. Apart from being the most commonly used form of transmission lines for microwave integrated circuits, microstrips are used for circuit components such as filters, couplers, resonators, and antennas. In comparison with coaxial lines, microstrip lines allow for greater flexibility and compactness of design. A microstrip line consists of a single ground plane and an open strip conductor separated by dielectric substrate as shown in Figure 11.38. It is constructed by the FIGURE 11.38 Microstrip line. 11_Sadiku_Ch11.indd 604 25/09/17 5:25 PM 11.8 Application Note—Microstrip Lines and Characterization of Data Cables 605 FIGURE 11.39 Pattern of the EM field of a microstrip line. (From D. Roddy, Microwave Technology, 1986, by permission of Prentice-Hall.) - hotographic processes used for integrated circuits. Analytical derivation of the charp acteristic properties of the line is cumbersome. We will consider only some basic, valid empirical formulas necessary for calculating the phase velocity, impedance, and losses of the line. Owing to the open structure of the microstrip line, the EM field is not confined to the dielectric, but is partly in the surrounding air as in Figure 11.39. Provided the frequency is not too high, the microstrip line will propagate a wave that, for all practical purposes, is a TEM wave. Because of the fringing, the effective relative permittivity eff is less than the relative permittivity r of the substrate. If w is the line width and h is the substrate thickness, an approximate value of eff is given by eeff 5 1 er 1 1 2 2 1 1 er 2 1 2 2"1 1 12h/w (11.70) The characteristic impedance is given by the following approximate formulas: 60 Zo 5 µ "eeff 1 "eeff lna 8h w 1 b w 4h 120p 3 w/h 1 1.393 1 0.667 ln 1 w/h 1 1.444 2 4 w/h # 1 (11.71) w/h $ 1 The characteristic impedance of a wide strip is often low, while that of a narrow strip is high. 11_Sadiku_Ch11.indd 605 25/09/17 5:25 PM 606 CHAPTER 11 TRANSMISSION LINES For design purposes, if r and Zo are known, the ratio w/h necessary to achieve Zo is given by 8eA , e 22 2 w 5 f e B 2 1 2 ln 1 2B 2 1 2 p h er 2 1 0.61 1 c ln 1 B 2 1 2 1 0.39 2 d f, er 2er 2A where A5 B5 w/h # 2 (11.72) w/h . 2 er 2 1 Zo er 1 1 0.11 1 a0.23 1 b er 60 Å 2 er 1 1 60p2 Zo"er (11.73a) (11.73b) From the knowledge of eff and Zo, the phase constant and the phase velocity of a wave propagating on the microstrip are given, respectively, by v"eeff (11.74a) c c (11.74b) u5 "eeff b5 where c is the speed of light in a vacuum. The attenuation due to conduction (or ohmic) loss is (in dB/m) ac . 8.686 Rs (11.75) wZo 1 is the skin resistance of the conductor. The attenuation due to dielectric s cd loss is (in dB/m) where Rs 5 ad .

27.3 1 eeff 2 1 2 er tan u (11.76) 1 er 2 1 2 "eeff l where l 5 u/f is the line wavelength and tan u 5 s/ve is the loss tangent of the substrate. The total attenuation constant is the sum of the ohmic attenuation constant ac and the dielectric attenuation constant ad; that is, a 5 ac 1 ad(11.77) Sometimes ad is negligible in comparison with ac. Although they offer an advantage of flexibility and compactness, microstrip lines are not useful for long transmission because attenuation is excessive. 11_Sadiku_Ch11.indd 606 25/09/17 5:25 PM 11.8 Application Note—Microstrip Lines and Characterization of Data Cables 607 EXAMPLE 11.10 A certain microstrip line has fused quartz 1 er 5 3.8 2 as a substrate.

If the ratio of line width to substrate thickness is w/h 5 4.5, determine (a) The effective relative permittivity of the substrate (b) The characteristic impedance of the line (c) The wavelength of the line at 10 GHz Solution: (a) For w/h 5 4.5, we have a wide strip. From eq. (11.70), eeff 5 (b) From eq.

(11.71), Zo 5 4.8 2.8 12 21/2 1 5 3.131 c1 1 d 2 2 4.5 120p "3.131 3 4.5 1 1.393 1 0.667 ln 1 4.5 1 1.444 2 4 5 30.08 V (c) l 5 u c 3 3 108 5 5 f 1010 "3.131 f "eeff 5 1.69 3 1022 m 5 16.9 mm PRACTICE EXERCISE 11.10 Repeat Example 11.10 for w/h 5 0.8. Answer: (a) 2.75, b) 84.03 V, (c) 18.09 mm. EXAMPLE 11.11 At 10 GHz, a microstrip line has the following parameters: h 5 1 mm w 5 0.8 mm er 5 6.6 tan u 5 1024 sc 5 5.8 3 107 S/m Calculate the attenuation due to conduction loss and dielectric loss. 11_Sadiku_Ch11.indd 607 25/09/17 5:25 PM 608 CHAPTER 11 TRANSMISSION LINES Solution: The ratio w/h 5 0.8. Hence from eqs.

(11.70) and (11.71) eeff 5 7.6 5.6 12 21/2 1 5 4.5 a1 1 b 2 2 0.8 Zo 5 60 "4.5 ln a 0.8 8 1 b 0.8 4 5 65.69 V The skin resistance of the conductor is Rs 5 pfmo 1 p 3 10 3 109 3 4p 3 1027 5 5 s cd Å s c Å 5.8 3 107 5 2.609 3 1022 V/m2 Using eq. (11.75), we obtain the conduction attenuation constant as ac 5 8.686 3 2.609 3 1022 0.8 3 1023 3 65.69 5 4.31 dB/m To find the dielectric attenuation constant, we need l: l5 u c 3 3 108 5 5 f f "eeff 10 3 109 "4.5 5 1.414 3 1022 m Applying eq. (11.76), we have ad 5 27.3 3 3.5 3 6.6 3 1024 5.6 3 "4.5 3 1.414 3 1022 5 0.3754 dB/m PRACTICE EXERCISE 11.11 Calculate the attenuation due to ohmic losses at 20 GHz for a microstrip line constructed of copper conductor having a width of 2.5 mm on an alumina substrate.

Take the characteristic impedance of the line as 50 V. Answer: 2.564 dB/m. 11_Sadiku_Ch11.indd 608 25/09/17 5:25 PM 11.8 Application Note—Microstrip Lines and Characterization of Data Cables 609 B. Characterization of Data Cables Data communication has become a vital part of our daily life, the educational system, and business enterprises. Cables (copper or optical fiber) play an important role in data communication because they constitute the vehicle that transmits electrical signals from one point to another. Before such cables are installed, they must meet certain requirements specified in terms of parameters including insertion loss (or attenuation), return loss (RL), near-end crosstalk (NEXT), far-end crosstalk (FEXT), attenuation-to-crosstalk ratio (ACR), power sum NEXT (PSNEXT), propagation delay, propagation delay skew, equal level farend crosstalk (ELFEXT), and power sum ELFEXT (PSELFEXT). In this section, we focus on the most popular measures: attenuation, RL, NEXT, and ELFEXT. Attenuation Attenuation (also known as insertion loss) is one of the greatest concerns of any cabling infrastructure. It is the reduction of signal strength during transmission. It is the opposite of amplification. Although it is normal to expect attenuation, a signal that attenuates too much becomes unintelligible, which is why most networks require repeaters at regular intervals. The factors that contribute to a cable’s attenuation include conductor size, material, insulation, frequency (bandwidth), speed, and distance. Attenuation describes the phenomenon of reduction of power intensity according to the law dP 5 22aP S P 5 Poe22az(11.78) dz where it is assumed that signal propagates along z, a is the attenuation coefficient, and Po is the power at z 0. Thus, attenuation describes how energy is lost or dissipated. Energy loss occurs as a transformation from one type of energy to another. Attenuation increases with both frequency and length.

Attenuation is usually expressed in decibels. For a cable of length L, attenuation (or loss) through the cable is A 5 8.686La dB (11.79) Since it is a loss, it is usually expressed as a negative value. Thus, –12 dB is a weaker signal than 10 dB. Return Loss Return loss (RL) is a measure of the reflected energy caused by impedance mismatches in a cabling system. It is a measure of the dissimilarity between impedances in metallic transmission lines and loads. It may also be regarded as the ratio, at the junction of a transmission line and a terminating impedance or other discontinuity, of the amplitude of the reflected wave to the amplitude of the incident wave.

Return loss is important in applications that use simultaneous bidirectional transmission.

Possible causes of excessive return loss include fluctuation in characteristic impedance, cable kinks, excessive bends, and cable jacket. 11_Sadiku_Ch11.indd 609 25/09/17 5:25 PM 610 CHAPTER 11 TRANSMISSION LINES Return loss is defined as the ratio of the incident power to the reflected power: RL 5 10 log10 P1 5 220 log10 0 G 0 dB(11.80) P2 since P 2 5 0 G 0 2P 1 and the reflection coefficient is given by G5 ZL 2 Zo (11.81) ZL 1 Zo where Zo is the characteristic impedance of the cable and ZL is the load impedance. Thus, return loss is a number that indicates the amount of signal that is reflected back into the cable from the terminating equipment. It is generally specified in decibels, and larger values are better because they indicate less reflection. Ideally, there would be no reflection, and return loss would have a value of infinity.

Generally, values of 35 to 40 dB or higher are considered acceptable. A value of 40 dB indicates that only 1% of the signal is reflected. NEXT Crosstalk is a major impairment in any two-wire transmission system. Within a cable, there are usually several active pairs. Because these pairs are in close physical proximity over long distances, coupling takes place, and the pairs “crosstalk” in each other. Thus, the idea of crosstalk refers to interference that enters a communication channel through some coupling path. There are two types of crosstalk in multipair cables: near-end crosstalk (NEXT) and far-end crosstalk (FEXT). When current flows in a wire, an electromagnetic field is created which can interfere with signals on adjacent wires. As frequency increases, this effect becomes stronger. Each pair is twisted because this allows opposing fields in the wire pair to cancel each other. The tighter the twist, the more effective the cancellation and the higher the data rate supported by the cable. If wires are not tightly twisted, the result is near-end crosstalk (NEXT). If you have ever been talking on the telephone and could hear another conversation faintly in the background, you have experienced crosstalk. In fact, the term derives from telephony applications where ‘talk’ came ‘across.’ In local-area networks, NEXT occurs when a strong signal on one pair of wires is picked up by an adjacent pair of wires. NEXT is the portion of the transmitted signal that is electromagnetically coupled back into the received signal, as illustrated in Figure 11.40. In many cases, excessive crosstalk is due to poorly twisted terminations at connection points. Since NEXT is a measure of difference in signal strength between a disturbing pair and a disturbed pair, a larger number (less crosstalk) is more desirable than a smaller number (more crosstalk). Because NEXT varies significantly with frequency, it is important to measure it across a range of frequencies, typically 1–250 MHz. Twisted-pair coupling becomes less effective for higher frequencies. ELFEXT Far-end crosstalk (FEXT) is similar to NEXT, except that the signal is sent from the local end as shown in Figure 11.41, and crosstalk is measured at the far end. As a result of attenuation, signals that induce FEXT can be much weaker, especially for longer cable lengths. 11_Sadiku_Ch11.indd 610 25/09/17 5:25 PM 11.8 Application Note—Microstrip Lines and Characterization of Data Cables 611 Transmitter Transmitted signal Remote transceiver Receiver Transmitter Crosstalk signal Remote transceiver Receiver FIGURE 11.40 Near-end crosstalk (NEXT) in a paired cable. Transmitter Transmitted signal Remote transceiver Receiver Transmitter Crosstalk signal Remote transceiver Receiver FIGURE 11.41 Far-end crosstalk (FEXT) in a paired cable. For that reason, FEXT results are not meaningful without an indication of the corresponding attenuation on the link. Thus, FEXT is measured but rarely reported. FEXT results are used to derive equal-level far-end crosstalk (ELFEXT). Noise occurring at the far end can be difficult to measure. It is common in practice to eliminate the attenuation effects and look at the pure noise taking place. ELFEXT is used when one is looking at the noise minus the effects of attenuation. ELFEXT is a measure of the unwanted signal coupling from a transmitter at the near end into a neighboring pair measured at the far end relative to the received signal level measured on the same pair. Unlike attenuation, return loss, NEXT, and FEXT, ELFEXT is a calculated rather than a measured quantity. It is derived by subtracting the attenuation of the disturbing pair from the FEXT this pair induces in an adjacent pair.

That is, if the disturbing pair is i and the disturbed pair is j, 11_Sadiku_Ch11.indd 611 25/09/17 5:25 PM 612 CHAPTER 11 TRANSMISSION LINES ELFEXTij 5 FEXTij 2 attenuationj(11.82) This normalizes the results for length. Since both FEXT and attenuation are measured in decibels, ELFEXT is also measured in decibels. High ELFEXT is indicative of excessive attenuation, higher than expected FEXT, or both. 11.9 APPLICATION NOTE—METAMATERIALS In 1967, Russian physicist Viktor Veselago studied theoretically the problem of timeharmonic monochromatic plane wave propagation in a material whose permittivity and permeability he assumed to be simultaneously negative at the frequency of interest. He showed that such a material could possess interesting electromagnetic features such as anomalous refraction. At the turn of the century, American physicists Richard Shelby, David Smith, Sheldon Schultz, and their group constructed such a composite medium by embedding split rings in a host medium. They experimentally showed in the microwave region the presence of anomalous refraction in the medium.

These complex materials, called metamaterials, have gained considerable attention. metamaterial with negative refractive index Ref nal ra c ref us o enti onv om alo in c An ion tio n ract ia med FIGURE 11.42 Showing anomalous refraction. 11_Sadiku_Ch11.indd 612 25/09/17 5:25 PM 11.10 Application Note—Microwave Imaging 613 Metamaterials are synonymously called left-handed materials, negative refractive index materials, and double-negative materials. A conventional material inherits its electromagnetic properties from the constituent atoms. But metamaterials derive their properties from the constituent units embedded in a host medium. They can be engineered as we desire. The phenomena of radiation, scattering, guidance in metamaterials, and complex structures made by pairing them with layers of conventional materials are being actively investigated. Researchers envision exciting possibilities for the development of improved computer chips, DVDs with vastly increased memory, cheaper and better-performing wireless communication systems, and improved medical imaging equipment. Several important ideas and developments that should lead to fabrication of components such as phase conjugators, unconventional guided-wave structures, compact thin cavities, highimpedance surfaces, and perfect lens are expected to emerge from these studies. The concepts underlying metamaterials also led to the development of artificial left-handed transmission lines. Metamaterials are artificial materials engineered to have specific properties that are not found in nature. Unlike naturally occurring materials, the behavior of a metamaterial is determined not only by the properties of its components, but also by the size, geometry, orientation, and arrangement of those components. Metamaterials have gained attention owing to their unique ability to bend waves rather than reflect them, rendering an object surrounded by a metamaterial invisible. Section 11.10 describes some applications of these unusual materials to microwave i- maging. 11.10 APPLICATION NOTE—MICROWAVE IMAGING Section 10.2 introduced mmWave technology, which takes advantage of the properties of electromagnetic waves in the 30–300 GHz frequency band, or mm-waves, which correspond to wavelengths in the range of 10–1 mm. Many optically opaque objects are rendered transparent when they are imaged with signals only a few millimeters in wavelength. This makes mm-wave imaging attractive for a wide variety of commercial, defense, and scientific applications.

Examples include nondestructive testing (NDT) for structural integrity, material characterization, security scanning, and medical screening. Microwaves and millimeter waves have been used extensively to image dielectric bodies.

They can penetrate into many optically opaque media such as living tissues, wood, ceramics, plastics, clothing, concrete, soil, fog, and foliage. In the past, many people believed that microwaves could be used just for target detection and tracking. But they have also been used for decades in remote sensing, that is, for the imaging of weather patterns or the surfaces of remote planets, for underground surveillance, and so on. More recently, microwave and millimeter-wave systems have been deployed for a variety of short-range applications, most notably in concealed-weapon detection (Figure 11.43) and throughthe-wall imaging. Additional information about cloaking applications for metamaterials is presented in S ection 12.10. 11_Sadiku_Ch11.indd 613 25/09/17 5:25 PM 614 CHAPTER 11 TRANSMISSION LINES FIGURE 11.43 A concealed weapon revealed. The main measures of performance of an imaging system are considered to be the spatial resolution in lateral and range directions and the image dynamic range offered. With the availability of more channels, combined with the powerful digital signal processing (DSP) capabilities of modern computers, the performance of mm-wave imaging systems is advancing rapidly. The most commonly known imaging systems are based on X-ray technology. Their applications are in, for example, computed tomography (CT) for medical

diagnostics, NDT applications, and luggage inspection at security checkpoints. These systems work in a transmission setup. Furthermore, backscatter X-ray systems, which work in a reflection setup, have been investigated over the past few years, especially for the screening of airline passengers for concealed objects. On the one hand, X-ray images have an inherent high lateral resolution owing to the extremely short wavelength (0.01–10 nm). But on the other hand, the energy of the photons is high enough to ionize organic and inorganic matter. Therefore, health aspects are critical with respect to the imaging of humans, especially people who must be imaged frequently, such as airport personnel. In contrast, electromagnetic mm-waves offer a contactless inspection of materials with nonionizing radiation and a high spatial resolution. Since spatial resolution and penetration depth are conflicting parameters regarding the wavelength, the E-band (60–90 GHz with wavelengths of 5–3.3 mm) is a good compromise for NDT applications to detect flaws, material inhomogeneities, and inclusions in dielectrics. A lateral resolution of about 2 mm is sufficient for many applications (e.g., personnel screening at airport security checkpoints). The application of microwaves in biomedical imaging and diagnostics, however, remains a field with many uncharted territories. So far, the contributions of microwave technology to medical imaging have been limited except for radio-frequency component design for magnetic resonance imaging (MRI) systems. From the imaging of isolated organs to the imaging of the brain, the bones, or the breast, the microwave community is striving to make an entrance in the highly competitive world of medical imaging. 11_Sadiku_Ch11.indd 614 25/09/17 5:25 PM 11.10 Application Note—Microwave Imaging 615 MATLAB 11.1 % This script computes the voltage and current waveforms as functions % of length along % a transmission line terminated with a complex load clear syms w Z0 ZL ZG VG gamma z Zin % symbolic variables for frequency, % characteristic impedance, % load impedance, source impedance, source voltage, propagation % constant, distance % Enter the frequency (in rad/s) wn = input('Enter the angular frequency > '); % Enter the propagation constant gamma (in a+j*b format) gamman = input('Enter the propagation constant > '); % Enter the length (m) L = input('Enter the length > '); % Enter the characteristic impedance Z0n = input('Enter the characteristic impedance > '); % Enter the load impedance (in a+j*b format) disp('Enter the complex load impedance '); ZLn = input('(in a+j*b format) > '); % Enter the source impedance (in a+j*b format) disp('Enter the complex source impedance '); ZGn = input('(in a+j*b format) > '); % Enter the source voltage (in a*exp(j*b) format) disp('Enter the source voltage '); VGn = input(' (in a*exp(j*b) format) > '); % The expression for the input impedance as a function of % length along the line % This expression at this point is purely symbolic and % contains no numerical data, it will be used in line 37 Zin = Z0*(ZL - Z0 * tanh(gamma *z)) / (Z0 - ... ZL * tanh(gamma * z)); % Output the key parameters % Reflection coefficient at load GammaL = (ZLn - Z0n) / (ZLn + Z0n); disp('The reflection coefficient at the load is'); disp(sprintf(' %0.2f+j%0.2f',real(GammaL), imag(GammaL))) 11_Sadiku_Ch11.indd 615 15/11/17 7:51 PM 616 CHAPTER 11 TRANSMISSION LINES % SWR SWR=(1+abs(GammaL))/(1-abs(GammaL)); disp(sprintf('The SWR at the load is %0.2f', SWR)) % Input impedance seen at source ZinG=subs(Zin,{Z0,ZL,gamma,z},{Z0n,ZLn, gamman,L}); disp('The input impedance seen by the generator ') disp(sprintf(' is %0.2f+j%0.2f', real(ZinG), imag(ZinG))) % now determine the forward-traveling voltage coefficient % V0plus at the load (z = 0) from the equation % V(z) = (V0+) e^(-j B z) (1 + GammaL exp (2 j B z)) % by setting z = -L % First Determine the voltage at the source V0G = VGn * ZinG / (ZGn + ZinG); V0plus = V0G * exp(-gamman*L)/(1 + GammaL *exp(-2*gamman*L)); % Generate vector of voltage over length z=-L:L/1000:0; % vector of length Vz = V0plus*exp(gamman*z).*(1 + GammaL*exp(2*gamman*z)); % voltage vector Iz = V0plus/Z0n*exp(gamman*z).*(1 - GammaL*exp(2*gamman*z)); % voltage vector %(notice the .* for multiplying two vectors % together element-by-element) % Plot figure subplot(2,1,1) % generate a subplot within one figure window % the two plots will be one on top of the other % (2,`,1) means there are two rows, one column, and this plot % goes into the top subplot plot(z, abs(Vz)) % plot only voltage magnitude axis([-L 0 0 abs(2*V0plus)]) title('Voltage on transmission line as a function of length') xlabel('distance from load (m)') ylabel('magnitude of voltage (V)') subplot(2,1,2) plot(z, abs(Iz)) % plot only current magnitude axis([-L 0 0 abs(2*V0plus/Z0n)]) title('Current on transmission line as a function of length') xlabel('distance from load (m)') ylabel('magnitude of current (A)') 11_Sadiku_Ch11.indd 616 15/11/17 7:51 PM Summary 617 SUMMARY 1. A transmission line is commonly described by its distributed parameters R (in V/m), L (in H/m), G (in S/m), and C (in F/m). Formulas for calculating R, L, G, and C are provided in Table 11.1 for coaxial, two-wire, and planar lines. 2. The distributed parameters are used in an equivalent circuit model to represent a differential length of the line. The transmission line equations are obtained by applying Kirchhoff 's laws and allowing the length of the line to approach zero. The voltage and current waves on the line are 2az az V 1 z, t 2 5 V1 cos 1 vt 2 bz 2 1 V2 oe o e cos 1 vt 1 bz 2 I 1 z, t 2 5 V1 V2 o 2az o az e cos 1 vt 2 bz 2 2 e cos 1 vt 1 bz 2 Zo Zo showing that there are two waves traveling in opposite directions on the line. 3. The characteristic impedance Zo (analogous to the intrinsic impedance h of plane waves in a medium) of a line is given by Zo 5 R 1 jvL Å G 1 jvC and the propagation constant g (per meter) is given by g 5 a 1 jb 5 " 1 R 1 jvL 2 1 G 1 jvC 2 The wavelength and wave velocity are l5 2p , b u5 v 5 fl b 4. The general case is that of the lossy transmission line 1 G 2 0 2 R 2 considered earlier. For a lossless line, R 5 0 5 G; for a distortionless line, R/L 5 G/C. It is desirable that power lines be lossless and telephone lines be distortionless. 5. The voltage reflection coefficient at the load end is defined as GL 5 V2 ZL 2 Zo o 1 5 Vo ZL 1 Zo and the standing wave ratio is s5 where ZL is the load impedance. 11_Sadiku_Ch11.indd 617 1 1 0 GL 0 1 2 0 GL 0 15/11/17 5:17 PM 618 CHAPTER 11 TRANSMISSION LINES 6. At any point on the line, the ratio of the phasor voltage to phasor current is the impedance at that point looking toward the load and would be the input impedance to the line if the line were that long. For a lossy line, Z1z2 5 Vs 1 z 2 ZL 1 Zo tanh g, 5 Zin 5 Zo c d Zo 1 ZL tanh g, Is 1 z 2 here is the distance from load to the point. For a lossless line (a 5 0), tanh w g, 5 j tan b,; for a shorted line, ZL 5 0; for an open-circuited line, ZL 5 `; and for a matched line, ZL 5 Zo. 7. The Smith chart is a graphical means of obtaining line characteristics such as G, s, and Zin. It is constructed within a circle of unit radius and based on the formula for GL given in eq. (11.36). For each r and x, there are two explicit circles (the resistance and reactance circles) and one implicit circle (the constant s-circle).

The Smith chart is conveniently used in determining the location of a stub tuner and its length.

It is also used with the slotted line to determine the value of the unknown load impedance. 8. When a dc voltage is suddenly applied at the sending end of a line, a pulse moves forth and back on the line. The transient behavior is conveniently analyzed by using bounce diagrams. 9. Microstrip transmission lines are useful in microwave integrated circuits. Useful formulas for constructing microstrip lines and determining losses on the lines have been presented. 10. Some parameters that are commonly used in characterizing data communication cables are presented. These parameters include attenuation, return loss, NEXT, and ELFEXT. REVIEW QUESTIONS 11.1 Which of the following statements are not true of the line parameters R, L, G, and C? (a) R and L are series elements. (b) G and C are shunt elements. 1 (c) G 5 . R (d) LC 5 me and RG 5 se. (e) Both R and G depend on the conductivity of the conductors forming the line. (f) Only R depends explicitly on frequency. (g) The parameters are not lumped but distributed. 11.2 For a lossy transmission line, the characteristic impedance does not depend on (a) The operating frequency of the line (b) The length of the line (c) The load terminating the line (d) The conductivity of the conductors (e) The conductivity of the dielectric separating the conductors 11_Sadiku_Ch11.indd 618 15/11/17 9:22 PM Review Questions 619 11.3 Which of the following conditions will not guarantee a distortionless transmission line? (a) R 5 0 5 G (b) RC 5 GL (c) Very low frequency range 1 R W vL, G W vC 2 (d) Very high frequency range 1 R V vL, G V vC 2 11.4 Which of these is not true of a lossless line? (a) Zin 5 2jZo for a shorted line with , 5 l/8. (b) Zin 5 j` for a shorted line with , 5 l/4. (c) Zin 5 jZo for an open line with , 5 l/2. (d) Zin 5 Zo for a matched line. (e) At a half-wavelength from a load, Zin 5 ZL and repeats for every half-wavelength thereafter. 11.5 A lossless transmission line of length 50 cm with L 5 10 mH/m, C 5 40 pF/m is operated at 30 MHz. Its electrical length is (a) 20l (b) 0.2l (c) 108° (d) 40p (e) None of the above 11.6 Match the following normalized impedances with points A, B, C, D, and E on the Smith chart of Figure 11.44. (i) 0 1 j0 (ii) 1 1 j0 (iii) 0 2 j1 (iv) 0 1 j1 (v) ` 1 j` (vii) Z in c d Zo max (vi) c Zin d Zo min (viii) Matched load 1 G 5 0 2 11.7 A 500 m lossless transmission line is terminated by a load that is located at P on the Smith chart of Figure 11.45. If l 5 150 m, how many voltage maxima exist on the line? (a) 7 (c) 5 (e) None (b) 6 (d) 3 11.8 Write true (T) or false (F) for each of the following statements. (a) All r- and x-circles pass through point 1 Gr, Gi 2 5 1 1, 0 2 . (b) Any impedance repeats itself every l/4 on the Smith chart. (c) An s 5 2 circle is the same as 0 G 0 5 0.5 circle on the Smith chart. 11_Sadiku_Ch11.indd 619 15/11/17 5:17 PM 620 CHAPTER 11 TRANSMISSION LINES (d) The basic principle of any matching scheme is to eliminate the reflected wave between the source and the matching device. (e) The slotted line is used to determine ZL only. (f) At any point on a transmission line, the current reflection coefficient is the reciprocal of the voltage reflection coefficient at that point. 11.9 In an air line, adjacent maxima are found at 12.5 cm and 37.5 cm. The operating frequency is (a) 1.5 GHz (c) 300 MHz FIGURE 11.44 Smith chart for Review Question 11.6. (b) 600 MHz (d) 1.2 GHz FIGURE 11.45 Smith chart for Review Question 11.7. 11.10 Two identical pulses each of magnitude 12 V and width 2 ms are incident at t 5 0 on a lossless transmission line of length 400 m terminated with a load. If the two pulses are separated 3 ms and u 5 2 3 108 m/s, when does the contribution to VL 1 ,, t 2 by the second pulse start overlapping that of the first? (a) t 5 0.5 ms (b) t 5 2 ms (c) t 5 5 ms (d) t 5 5.5 ms (e) t 5 6 ms Answers: 11.1c,d,e, 11.2b,c, 11.3c, 11.4a,c, 11.5c, 11.6 (i) D,e (ii) A, (iii) E, (iv) C, (v) B, (vi) D, (vii) B, (viii) A, 11.7a, 11.8 (a) T, (b) F, (c) F, (d) T, (e) F, (f) F, 11.9b, 11.10e. 11_Sadiku_Ch11.indd 620 15/11/17 9:15 PM Problems 621 PROBLEMS Section 11.2—Transmission Line Parameters 11.1 An air-filled planar line with w 5 30 cm, d 5 1.2 cm, t 5 3 mm has conducting plates with sc 5 7 3 107 S/m. Calculate R, L, C, and G at 500 MHz. 11.2 A coaxial cable has an inner conductor of radius a 0.8 mm and an outer conductor of radius b 2.6 mm. The conductors have sc 5.28 107 S/m, mc mo, and ec eo; they are separated by a dielectric material having s 105 S/m, m mo, e 3.5 eo. At 80 MHz, calculate the line parameters L, C, G, and R. 11.3 A coaxial cable has inner radius a and outer radius b. If the inner and outer conductors are separated by a material with conductivity σ, show that the conductance per unit length is G5 2ps b ln a 11.4 A planar transmission line is made of copper strips of width 30 mm and are separated by a dielectric of thickness 2 mm, s 5 10 23 S/m, e 5 4eo, m 5 mo. The conductivity of copper is 5.8 107 S/m. Assuming that the line operates at 200 MHz, (a) find R, L, G, and C, (b) determine g and Zo. 11.5 The copper leads of a diode are 16 mm in length and have a radius of 0.3 mm. They are separated by a distance of 2 mm as shown in Figure 11.46. Find the capacitance between the leads and the ac resistance at 10 MHz. Section 11.3—Transmission Line Equations *11.6 A TV twin-lead is made of two parallel copper wires with a 1.2 mm. The wires are separated by 1.5 cm of a dielectric material with «r = 4. Calculate L, C, and Zo. 11.7 A small section Dz of a transmission line may be represented by the equivalent circuit in Figure 11.47. Determine the voltage–current relationship for the section. FIGURE 11.46 The diode of Problem 11.5. 11_Sadiku_Ch11.indd 621 15/11/17 5:17 PM 622 CHAPTER 11 TRANSMISSION LINES I(z.t) I(z+Δz.t) LΔz + V(z.t) + CΔz V(z+Δz.t) – – Δz FIGURE 11.47 For Problem 11.7. 11.8 (a) Show that at high frequencies (R V vL, G V vL), g.a R C G L 1 b 1 jv"LC 2Å L 2ÅC (b) Obtain a similar formula for Zo. 11.9 A transmission line operates at 12 MHz and has the following parameters: R 0.2 V/m, G 4 mS/m, L 40 mH/m, C 25 mH/m (a) Compute g using eq. (11.11). (b) Compute g using the result of Problem 11.8(a). 11.10 The current along a transmission line is given by I(z,t) 5 I1(vt 2 bz) 1 I2(vt 1 bz) (a) Determine which of the two components represents a wave traveling from source to load. (b) Find the corresponding voltage V(z, t).

11.11 At 60 MHz, the following characteristics of a lossy line are measured: Zo 5 50 V, a 5 0.04 dB/m, b 5 2.5 rad/m Calculate R, L, C, and G of the line. 11.12 A 78 lossless planar line was designed but did not meet a requirement. What fraction of the widths of the strip should be added or removed to get the characteristic impedance of 75 ? 11.13 A telephone line operating at 1 kHz has R 5 6.8 V/mi, L 5 3.4 mH/mi, C 5 8.4 nF/mi, and G 5 0.42 mS/mi. Find (a) Z and g, (b) phase velocity, (c) wavelength. 11.14 A TV antenna lead-in wire 10 cm long has a characteristic impedance of 250 Ω and is open-circuited at its end. If the line operates at 400 MHz, determine its input impedance. 11_Sadiku_Ch11.indd 622 15/11/17 7:51 PM Problems 623 11.15 A coaxial cable has its conductors made of copper 1 sc 5 5.8 3 107 S/m 2 and its dielectric made of polyethylene 1 er 5 2.25, mr 5 1 2 . If the radius of the outer conductor is 3 mm, determine the radius of the inner conductor so that Zo 5 75 . 11.16 For a lossless two-wire transmission line, show that 1 (a) The phase velocity u 5 c 5 !LC 120 d (b) The characteristic impedance Zo 5 cosh21 2a !er Is part (a) true of other lossless lines? 11.17 A twisted line, which may be approximated by a two-wire line, is very useful in the telephone industry. Consider a line comprising two copper wires of diameter 0.12 cm that have a 0.32 cm center-to-center spacing. If the wires are separated by a dielectric material with e 5 3.5eo, find L, C, and Zo. 11.18 A distortionless cable is 4 m long and has a characteristic impedance of 60 Ω. An attenuation of 0.24 dB is observed at the receiving end. Also, a signal applied to the cable is delayed by 80 ms before it is measured at the receiving end. Find R, G, L, and C for the cable. 11.19 A distortionless line operating at 120 MHz has R 5 20 V/m, L 5 0.3 mH/m, and C 5 63 pF/m. (a) Determine g, u, and Zo. (b) How far will a voltage wave travel before it is reduced to 20% of its initial magnitude? (c) How far will it travel to suffer a 45° phase shift? 11.20 On a distortionless line, the voltage wave is given by V 1 ,r 2 5 120e0.0025,r cos 1 108t 1 2,r 2 1 60e20.0025,r cos 1 108t 2 2,r 2 where is the distance from the load. If ZL 5 300 V, find (a) a, b, and u, (b) Zo and I 1 ,r 2 . 11.21 The voltage on a line is given by V 1 , 2 5 80e10 2 3, cos 1 2p 3 104t 1 0.01, 2 1 60e 210 2 3, cos 1 2p 3 104t 1 0.01, 2 V where , is the distance from the load. Calculate and u. 11.22 A distortionless transmission line satisfies RC = LG. If the line has R 5 10 mV/m, C = 82 pF/m, and L 5 0.6 mH/m, calculate its characteristic impedance and propagation constant. Assume that the line operates at 80 MHz. 11.23 A coaxial line 5.6 m long has distributed parameters R 5 6.5 V/m, L 5 3.4 mH/m, G 5 8.4 mS/m, and C 5 21.5 pF/m. If the line operates at 2 MHz, calculate the characteristic impedance and the end-to-end propagation time delay. 11.24 A lossy transmission line of length 2.1 m has characteristic impedance of 80 1 j60 . When the line is short-circuited, the input impedance is 30 – j12 . (a) Determine a and b. (b) Find the input impedance when the short circuit is replaced by ZL 5 40 1 j30 . 11.25 A lossy transmission line with characteristic impedance of 75 1 j60 is connected to a 200 load. If attenuation is 1.4 Np/m and phase constant is 2.6 rad/m, find the input impedance for 5 0.5 m. 11_Sadiku_Ch11.indd 623 15/11/17 5:18 PM 624 CHAPTER 11 TRANSMISSION LINES Section 11.4—Input Impedance, Standing Wave Ratio, and Power 11.26 (a) Show that a transmission coefficient may be defined as tL 5 VL 2ZL 5 1 1 GL 5 V1 Z o L 1 Zo (b) Find tL when the line is terminated by (i) a load whose value is nZo, (ii) an open circuit, (iii) a short circuit, (iv) ZL 5 Zo (matched line). 11.27 (a) Show that ZL 5 11G Zo 12G (b) A 50 V line has a reflection coefficient of 0.6/45°. Find the load impedance. 11.28 A 120 V lossless line is terminated at a load impedance 200 2 j240 V. Find GL and s. 11.29 A lossy transmission line has R 3.5 V/m, L 2 mH/m, C 120 pF/m, and G < 0. At 400 MHz, determine a, b, Zo, and u. 11.30 Find the input impedance of a short-circuited coaxial transmission line of Figure 11.48 if Zo 5 65 1 j38 V, g 5 0.7 1 j2.5/m, , 5 0.8 m. 11.31 Calculate the reflection coefficient due to ZL 5 (1 1 2j)Z0. 11.32 Refer to the lossless transmission line shown in Figure 11.49. (a) Find G and s. (b) Determine Zin at the generator. 11.33 A 60 V lossless line is connected to a source with Vg 5 10l0º Vrms and Zg 5 50 2 j40 V and terminated with a load of j40 V. If the line is 100 m long and b 5 0.25 rad/m, calculate Zin and V at (a) The sending end (c) 4 m from the load (b) The receiving end (d) 3 m from the source FIGURE 11.48 For Problem 11.30. λ/6 Zg Zo = 50 Ω Vg 120 Ω FIGURE 11.49 For Problem 11.32. 11_Sadiku_Ch11.indd 624 15/11/17 5:18 PM Problems 625 FIGURE 11.50 For Problem 11.34: (a) network, (b) lossy line. X Y Z 75 Ω 50 Ω λ/4 λ/2 150 Ω FIGURE 11.51 For Problem 11.38. *11.34 Consider the two-port network shown in Figure 11.50(a).

The relation between the input and output variables can be written in matrix form as c V1 A d 5 c I1 C B V2 dc d D 2I2 For the lossy line in Figure 11.50(b), show that the ABCD matrix is cosh g, Zo sinh g, £ 1 § sinh g, cosh g, Zo Section 11.5—The Smith Chart 11.35 Normalize the following impedances with respect to 50 V and locate them on the Smith chart: (a) Za 5 80 V, (b) Zb 5 60 1 j40 V, (c) Zc 5 30 2 j120 V. 11.36 A quarter-wave lossless 100 V line is terminated by a load ZL 5 210 V. If the voltage at the receiving end is 80 V, what is the voltage at the sending end? 11.37 Determine the impedance at a point l/4 distant from a load of impedance (1 + j2)Zo. 11.38 11_Sadiku_Ch11.indd 625 Two lines are cascaded as shown in Figure 11.51. Determine: (a) The input impedance (b) The standing wave ratio for sections XY and YZ (c) The reflection coefficient at Z 15/11/17 5:18 PM 626 CHAPTER 11 TRANSMISSION LINES 11.39 A 50 V lossless line operates at 600 MHz and is terminated by a load of ZL. If the line is 0.1 m long and Zin 5 100 2 j120, find ZL and s. Assume u = 0.8 c. 11.40 A lossless transmission line, with characteristic impedance of 50 V and electrical length of , 5 0.27l, is terminated by a load impedance 40 – j25 V. Determine GL, s, and Zin.

11.41 A lossless 100 V transmission line is terminated in an unknown impedance ZL. The standing wave ratio is 2.4, and the nearest voltage minimum is 0.2 l from the load.

Find ZL and G. 11.42 The distance from the load to the first minimum voltage in a 50 V line is 0.12l, and the standing wave ratio s = 4. (a) Find the load impedance ZL. (b) Is the load inductive or capacitive? (c) How far from the load is the first maximum voltage? 11.43 A lossless 50 V line is terminated by a load ZL 75 j60 V. Using a Smith chart, determine (a) the reflection coefficient G, (b) the standing wave ratio s, (c) the input impedance at 0.2l from the load, (d) the location of the first minimum voltage from the load, (e) the shortest distance from the load at which the input impedance is purely resistive. 11.44 A transmission line is terminated by a load with admittance YL 5 1 0.6 1 j0.8 2 /Zo. Find the normalized input impedance at l/6 from the load. 11.45 Using the Smith chart, determine the admittance of Z = 100 1 j60 V with respect to Zo 5 50 V. 11.46 A 50 V transmission line operates at 160 MHz and is terminated by a load of 50 1 j30 V. If its wave speed is c/2 and the input impedance is to be made real, calculate the minimum possible length of the line and the corresponding input impedance.

11.47 A 50 V transmission line, /4 in length, is connected to a /2 section of 100 V line terminated by a 60 V resistor. Calculate the input impedance to the 50 V line. 11.48 (a) Calculate the reflection coefficient corresponding to ZL 5 (0.5 2 j)Zo. (b) Determine the load impedance corresponding to the reflection coefficient 0.4 l25°. 11.49 An 80 V transmission line operating at 12 MHz is terminated by a load ZL. At 22 m from the load, the input impedance is 100 2 j120 V. If u 5 0.8c, (a) Calculate GL, Zin, max, and Zin, min. (b) Find ZL, s, and the input impedance at 28 m from the load.

(c) How many Zin, max and Zin, min are there between the load and the 100 2 j120 V input impedance? 11.50 An antenna, connected to a 150 V lossless line, produces a standing wave ratio of 2.6. If measurements indicate that voltage maxima are 120 cm apart and that the last maximum is 40 cm from the antenna, calculate (a) The operating frequency (b) The antenna impedance (c) The reflection coefficient (assume that u 5 c). 11_Sadiku_Ch11.indd 626 15/11/17 5:18 PM Problems 627 FIGURE 11.52 Double section transformer of Problem 11.53. FIGURE 11.53 For Problems 11.54 and 11.55. in Line 1 λ /4 200 Ω FIGURE 11.54 For Problem 11.56. Zo λ /4 Line 3 Zo Line 2 λ /4 Zo Short circuit FIGURE 11.55 For Problem 11.57. 11_Sadiku_Ch11.indd 627 15/11/17 5:18 PM 628 CHAPTER 11 TRANSMISSION LINES 11.51 An 80 V lossless line has ZL j60 V and Zin j40 V. (a) Determine the shortest length of the line. (b) Calculate s and GL. 11.52 A 50 V air-filled line is terminated in a mismatched load of 40 1 j25 V. Find the shortest distance from the load at which the voltage has the smallest magnitude. 11.53 Two l/4 transformers in tandem are to connect a 50 V line to a 75 V load as in Figure 11.52. (a) Determine the characteristic impedance Zo1 if Zo2 5 30 V and there is no reflected wave to the left of A. (b) If the best results are obtained when c Zo 2 Zo1 Zo2 2 5 c d 5 d Zo1 Zo2 ZL determine Zo1 and Zo2 for this case. 11.54 Two identical antennas, each with input impedance 74 V, are fed with three identical 50 V quarter-wave lossless transmission lines as shown in Figure 11.53. Calculate the input impedance at the source end. 11.55 If the lines in Figure 11.53 are connected to a voltage source of 120 V with an internal impedance of 80 V, calculate the average power delivered to either antenna. 11.56 Consider the three lossless lines in Figure 11.54. If Zo 5 50 V, calculate: (a) Zin looking into line 1 (b) Zin looking into line 2 (c) Zin looking into line 3 11.57 A section of lossless transmission line is shunted across the main line as in Figure 11.55. If ,1 5 l/4, ,2 5 l/8, and ,3 5 7l/8, find yin1, yin2, and yin3 given that Zo 5 100 V, ZL 5 200 1 j150 V. Repeat the calculations as if the shorted section were open. Section 11.6—Some Applications of Transmission Lines 11.58 A load ZL 5 75 1 j100 V is to be matched to a 50 V line. A shorted shunt-stub tuner is preferred. Find the length of the stub in terms of l. 11.59 A stub of length 0.12l is used to match a 60 V lossless line to a load. If the stub is located at 0.3l from the load, calculate (a) The load impedance ZL (b) The length of an alternative stub and its location with respect to the load (c) The standing wave ratio between the stub and the load 11.60 A 50 V lossless transmission line that is 20 m long is terminated into a 120 j220 V load. To perfectly match, what should be the length and location of a short-circuited stub line? Assume an operating frequency of 10 MHz. 11.61 On a lossless line, measurements indicate s 5 4.2 with the first maximum voltage at l/4 from the load. Determine how far from the load a short-circuited stub should be located and calculate its length. 11_Sadiku_Ch11.indd 628 15/11/17 5:18 PM Problems 629 11.62 A 60 V lossless line terminated by load ZL has a voltage wave as shown in Figure 11.56. Find s, G, and ZL. FIGURE 11.56 For Problem 11.62. 2Zo 27 V t=0 Zo , γ 0.5 Zo z=, z=0 FIGURE 11.57 For Problem 11.65. Rs = 4Zo Is(t) Zo, u RL = 2Zo Is 10 mA t 0 z FIGURE 11.58 For Problem 11.66. 11.63 A 50 V air-filled slotted line is applied in measuring a load impedance. Adjacent minima are found at 14 cm and 22.5 cm from the load when the unknown load is connected, and Vmax 5 0.95 V and Vmin 5 0.45 V. When the load is replaced by a short circuit, the minima are 3.2 cm to the load. Determine s, f, G, and ZL. 11_Sadiku_Ch11.indd 629 15/11/17 5:18 PM 630 CHAPTER 11 TRANSMISSION LINES Section 11.7—Transients on Transmission Lines 11.64 A 50 V coaxial cable is connected to an 80 V resistive load and a dc source with zero internal resistance.

Calculate the voltage reflection coefficients at the source and at the load. 11.65 The switch in Figure 11.57 is closed at t 5 0. Sketch the voltage and current at the right side of the switch for 0 , t , 6,/u. Take Zo 5 50 V and ,/u 5 2 ms. Assume a lossless transmission line.

11.66 A step current of 10 mA is turned on at t = 0, at z = 0 of a transmission line as shown in Figure 11.58. Determine the load voltage and current as functions of time. Let Zo 5 50 V, m 5 2 3 108 m/s. You may Thévenin equivalent at z = 0. Carry out your analysis for 0 , t , 6,/u. + + – – FIGURE 11.59 For Problem 11.68. 11.67 A 50-cm-long cable, of characteristic impedance 75 V and wave velocity 2 3 108 m/s, is used to connect a source of internal impedance 32 V to an amplifier with an input impedance of 2 M V. If the source voltage changes from 0 to 8 V at t = 0, obtain the voltage seen by the amplifier at t = 20 ns. *11.68 Refer to Figure 11.59, where Zo 5 50 V, Zg 5 40 V, Vg 5 12V, Zt 5 100V, l = 40 cm Assuming wave speed u 5 2.5 3 108 m/s, find V(0, t) and V(l, t). 11.69 Using a time-domain reflectometer, the voltage waveform shown in Figure 11.60 was observed at the sending end of a lossless transmission line in response to a step voltage. If Zg = 60 V and Zo = 50 V, calculate the generator voltage and the length of the line. Assume u 5 c. 11.70 A 12 V battery with an internal resistance of 10 V is connected to a 20 m length of 50 V coaxial cable with phase velocity of 2 108 m/s. If the receiving end is short-circuited, sketch the sending voltage V(0, t) and the receiving-end voltage V(,, t).

11.71 Suppose ZL 5 ZG and a dc voltage is turned on at t = 0 (i.e., a unit step function) of amplitude Vo. The voltage is launched on a lossless line with characteristic impedance Zo. Find the voltage across the line after a very long time.

Section 11.8—Application Note: Microstrip and Characterization of Data Cables 11.72 A microstrip line is 1 cm thick and 1.5 cm wide. The conducting strip is made of brass 1 sc 5 1.1 3 107 S/m 2 , while the substrate is a dielectric material with er 5 2.2 and tan u 5 0.02. If the line operates at 2.5 GHz, find (a) Zo and eff, (b) ac and ad, (c) the distance down the line before the wave drops by 20 dB. 11_Sadiku_Ch11.indd 630 15/11/17 5:18 PM Problems 631 11.73 A 50 V microstrip line has a phase shift of 45° at 8 GHz. If the substrate thickness is h 5 8 mm with er 5 4.6, find (a) the width of the conducting strip, (b) the length of the microstrip line. 11.74 An alumina substrate 1 e 5 9.6eo 2 of thickness 2 mm is used for the construction of a microstrip circuit. If the circuit designer can choose a line width between 0.4 mm and 8.0 mm, what is the range of characteristic impedance of the line? V (0, t) εr B 8V t w 5V FIGURE 11.61 For Problem 11.75.

0 t 4 µs FIGURE 11.60 For Problem 11.69.

11.75 A strip transmission line is shown in Figure 11.61. An approximate expression for the characteristic impedance is Zo 5 377 ln e 1 1 4b 8b 8b 2 1 6.27 1 a c b df pwr pwr Å pwr 2p"er 5b t where w 5 w 1 lna b. Determine Zo for w = 0.5 cm, t = 0.1 cm, b= 1.2 cm, r = 2.

3.2 t 11.76 Design a 75 V microstrip line on a 1.2 mm thick duroid 1 er 5 2.3 2 substrate. Find the width of the conducting strip and the phase velocity. 11.77 Find the return loss due to a 150 V cable terminated by a 100 V load. 11.78 The effective relative permittivity «e is given by eq. (11.70). Use MATLAB to plot «e for 0.1 , w/h , 10. Assume «r = 2.2 (Teflon). 11_Sadiku_Ch11.indd 631 15/11/17 9:16 PM RF MEMS MEMS stands for micro electro mechanical systems. These devices and systems range in size from a few micrometers to a few millimeters. The field of MEMS (or nanotechnology) encompasses all aspects of science and technology and is involved with things on a smaller scale. MEMS technology gives scientists and engineers the tools to build things that have been impossible or prohibitively expensive with other t echnologies. MEMS technology has emerged with a major application area in telecommunications, particularly in optical switching and wireless communication. Wireless communication is expanding at an incredible pace for applications ranging from mobile phones to satellite communications. Radiofrequency MEMS technologies are helping fuel this expansion. The integration of MEMS into traditional RF circuits has resulted in systems with superior performance levels and lower manufacturing costs. RF MEMS are providing critical reductions in power consumption and signal loss to extend battery life and reduce weight. RF MEMS devices have a broad range of potential applications in military and commercial wireless communication and in navigation and sensor systems. Though RF MEMS devices are small, they can be very complex, commonly encompassing multiple interdependent engineering disciplines. RF MEMS application areas are in phased arrays and reconfigurable apertures for defense and communication systems and in switching networks for satellite communications. As a breakthrough technology, allowing unparalleled synergy between apparently unrelated fields of endeavor such as biology and microelectronics, many new MEMS - applications will emerge, expanding beyond that which is currently identified or known. Traditionally, the training of MEMS engineers and scientists has entailed a graduate education at one of a few research universities, with the student working under the d irection of an experienced faculty member to design, fabricate, and test a MEMS device. A graduate education in MEMS technology is very costly and comparatively time-consuming. Consequently, the current output from our universities of technical persons trained in MEMS technology does not meet the requirement for personnel to support the projected growth of MEMS industry. If your university offers classes in MEMS, take as many as possible. Better still, if your university has a MEMS laboratory, consider doing your senior design or your graduate thesis in that area. That should prepare you well enough for the job market. (Note: For an additional discussion of RF MEMS, see Section 6.8.) 12_Sadiku_Ch12.indd 632 25/09/17 5:27 PM CHAPTER 12 WAVEGUIDES Reading makes a full man; conference makes a ready man; and writing makes an accurate man. —ANONYMOUS 12.1 INTRODUCTION As mentioned in the preceding chapter, a transmission line can be used to guide EM energy from one point (generator) to another (load). A waveguide is another means of achieving the same goal. However, a waveguide differs from a transmission line in some respects, although we may regard the latter as a special case of the former. In the first place, a transmission line can support only a transverse electromagnetic (TEM) wave, whereas a waveguide can support many possible field configurations. Second, at microwave frequencies (roughly 3–300 GHz), transmission lines become inefficient as a result of skin effect and dielectric losses; waveguides are used at that range of frequencies to obtain larger bandwidth and lower signal attenuation. Moreover, a transmission line may operate from dc 1 f 5 0 Hz 2 to a very high frequency; a waveguide can operate only above a certain frequency called the cutoff frequency and therefore acts as a high-pass filter. Thus, waveguides cannot transmit dc, and they become excessively large at frequencies below microwave frequencies.

Although a waveguide may assume any arbitrary but uniform cross section, common waveguides are either rectangular or circular. Typical waveguides1 are shown in Figure 12.1. Analysis of circular waveguides is involved and requires familiarity with Bessel functions, which are beyond our scope.2 We will consider only hollow rectangular waveguides. By assuming lossless waveguides 1 sc . `, s < 0 2 , we shall apply Maxwell's equations with the appropriate boundary conditions to obtain different modes of wave propagation and the corresponding E and H fields. When we close both ends of a waveguide, a cavity is formed. We will also consider optical fiber guide, which is basic to optical communications. 1 For other types of waveguides, see J. A.

Seeger, Microwave Theory, Components and Devices. E nglewood Cliffs, NJ: Prentice-Hall, 1986, pp. 128–133. 2 Analysis of circular waveguides can be found in advanced EM or EM-related texts (e.g., S. Y. Liao, Microwave Devices and Circuits, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1990, pp. 119–141). 633 12_Sadiku_Ch12.indd 633 25/09/17 5:27 PM 634 CHAPTER 12 WAVEGUIDES FIGURE 12.1 Typical waveguides. 12.2 RECTANGULAR WAVEGUIDES Consider the rectangular waveguide shown in Figure 12.2, where a and b are the inner dimensions of the waveguide. We shall assume that the waveguide is filled with a sourcefree 1 rv 5 0, J 5 0 2 lossless dielectric material 1 s . 0 2 and that its walls are perfectly conducting 1 sc .

` 2 . From eqs. (10.17) and (10.19), we recall that for a lossless medium, Maxwell's equations in phasor form become =2Es 1 k2Es 5 0(12.1) =2Hs 1 k2Hs 5 0(12.2) FIGURE 12.2 A rectangular waveguide with perfectly conducting walls, filled with a lossless material. a 12_Sadiku_Ch12.indd 634 25/09/17 5:27 PM 12.2 Rectangular Waveguides 635 where k 5 v !me(12.3) and the time factor e jt is assumed. If we let Es 5 1 Exs, Eys, Ezs 2 and Hs 5 1 Hxs, Hys, Hzs 2 each of eqs. (12.1) and (12.2) comprises three scalar Helmholtz equations. In other words, to obtain the E and H fields, we have to solve six scalar equations. For the z-component, for example, eq. (12.1) becomes '2Ezs '2Ezs '2Ezs 1 k2Ezs 5 0(12.4) 2 1 2 1 'x 'y 'z2 which is a partial differential equation.

From Example 6.5, we know that eq. (12.4) can be solved by separation of variables (product solution). So we let Ezs 1 x, y, z 2 5 X 1 x 2 Y 1 y 2 Z 1 z 2 (12.5) where X(x), Y(y), and Z(z) are functions of x, y, and z, respectively. Substituting eq. (12.5) into eq. (12.4) and dividing by XYZ gives Xs Ys Zs 1 1 5 2k2(12.6) X Y Z Since the variables are independent, each term in eq. (12.6) must be constant, so the equation can be written as 2k2x 2 k2y 1 g2 5 2k2(12.7) where 2k2x, 2k2y, and 2 are separation constants. Thus, eq. (12.6) is separated as Xs 1 k2xX 5 0(12.8a) Ys 1 k2yY 5 0(12.8b) Zs 2 g2Z 5 0(12.8c) The choice of 2 is due to the realization that the guided waves propagate along the guide axis z in the positive or negative direction, and the propagation may result in Ezs and Hzs that approach zero as z S 6`. By following the same argument as in Example 6.5, we obtain the solution to eq. (12.8) as 12_Sadiku_Ch12.indd 635 X 1 x 2 5 c1 cos kxx 1 c2 sin kxx(12.9a) Y 1 y 2 5 c3 cos kyy 1 c4 sin kyy(12.9b) Z 1 z 2 5 c5egz 1 c6e2gz(12.9c) 25/09/17 5:27 PM 636 CHAPTER 12 WAVEGUIDES Substituting eq. (12.9) into eq. (12.5) gives Ezs 1 x, y, z 2 5 1 c1 cos kxx 1 c2 sin kxx 2 1 c3 cos kyy 1 c4 sin kyy 2 1 c5egz 1 c6e2gz 2 (12.10) As usual, if we assume that the wave propagates along the waveguide in the 1z-direction, the multiplicative constant c5 5 0 because the wave has to be finite at infinity [i.e., Ezs 1 x, y, z 5 ` 2 5 0]. Hence eq. (12.10) is reduced to Ezs 1 x, y, z 2 5 1 A1 cos kxx 1 A2 sin kxx 2 1 A3 cos kyy 1 A4 sin kyy 2 e2gz(12.11) where A1 5 c1c6, A2 5 c2c6, A3 c3c6, and A4 c4c6. By taking similar steps, we get the solution of the z-component of eq. (12.2) as Hzs 1 x, y, z 2 5 1 B1 cos kxx 1 B2 sin kxx 2 1 B3 cos kyy 1 B4 sin kyy 2 e2gz(12.12) Instead of solving for other field components Exs, Eys, Hxs, and Hys in eqs. (12.1) and (12.2) in the same manner, it is more convenient to use Maxwell's equations to determine them from Ezs and Hzs. From = 3 Es 5 2jvmHs and = 3 Hs 5 jveEs we obtain 'Eys 'Ezs 2 5 2jvmHxs(12.13a) 'y 'z 'Hys 'Hzs 2 5 jveExs (12.13b) 'y 'z 'Ezs 'Exs 2 5 2jvmHys(12.13c) 'z 'x 'Hzs 'Hxs 2 5 jveEys (12.13d) 'z 'x 'Eys 'Exs 2 5 2jvmHzs(12.13e) 'x 'y 'Hys 'Hxs 2 5 jveEzs (12.13f) 'x 'y We will now express Exs, Eys, Hxs, and Hys in terms of Ezs and Hzs. For Exs, for example, we combine eqs. (12.13b) and (12.13c) and obtain 12_Sadiku_Ch12.indd 636 25/09/17 5:27 PM 12.2 Rectangular Waveguides 637 jveExs 5 'Hzs '2Ezs '2Exs 1 b(12.14) a 2 2 'y jvm 'z 'x'z From eqs. (12.11) and (12.12), it is clear that all field components vary with z according to e2gz, that is, Ezs , e2gz, Exs , e2gz Hence 'Ezs 5 2gEzs, 'z '2Exs 5 g2Exs 'z2 and eq. (12.14) becomes jveExs 5 or 2 'Hzs 'Ezs 1 1 ag2Exs 1 g b 'y jvm 'x 'Hzs g 'Ezs 1 1 g2 1 v 2me 2 Exs 5 1 jvm jvm 'x 'y Thus, if we let h2 5 g2 1 v 2me 5 g2 1 k2, Exs 5 2 jvm 'Hzs g 'Ezs 2 2 2 h 'x h 'y Similar manipulations of eqs. (12.13) yield expressions for Eys, Hxs, and Hys in terms of Ezs and Hzs. Thus, Exs 5 2 jvm 'Hzs g 'Ezs 2 'x 2 h h2 'y (12.15a) Eys 5 2 jvm 'Hzs g 'Ezs 2 'y 1 h h2 'x (12.15b) Hxs 5 jve 'Ezs g 'Hzs 2 2 2 h 'y h 'x Hys 5 2 jve 'Ezs g 'Hzs 2 2 2 h 'x h 'y (12.15c) (12.15d) where 12_Sadiku_Ch12.indd 637 h2 5 g2 1 k2 5 k2x 1 k2y(12.16) 25/09/17 5:27 PM 638 CHAPTER 12 WAVEGUIDES FIGURE 12.3 Components of EM fields in a rectangular waveguide: (a) TE mode Ez 5 0, (b) TM mode, Hz 5 0. Thus we can use eqs. (12.15) in conjunction with eqs. (12.11) and (12.12) to obtain Exs, Eys, Hxs, and Hys. From eqs. (12.11), (12.12), and (12.15), we notice that the field patterns or configurations come in different types. Each of these distinct field patterns is called a mode. Four different mode categories can exist, namely: 1. Ezs 5 0 5 Hzs (TEM mode): In the transverse electromagnetic mode, both the E and H fields are transverse to the direction of wave propagation. From eq.

(12.15), all field components vanish for Ezs 5 0 5 Hzs. Consequently, we conclude that a hollow rectangular waveguide cannot support TEM mode.

2. Ezs 5 0, Hzs 2 0 (TE modes): For this case, the remaining components (Exs and Eys) of the electric field are transverse to the direction of propagation az.

Under this condition, fields are said to be in transverse electric (TE) modes. See Figure 12.3(a). 3. Ezs 2 0, Hzs 5 0 (TM modes): In this case, the H field is transverse to the direction of wave propagation. Thus we have transverse magnetic (TM) modes. See Figure 12.3(b). 4. Ezs 2 0, Hzs 2 0 (HE modes): In this case neither the E nor the H field is transverse to the direction of wave propagation. Sometimes these modes are referred to as hybrid modes. We should note the relationship between k in eq. (12.3) and b of eq. (10.43a). The phase constant b in eq.

(10.43a) was derived for TEM mode. For the TEM mode, h 5 0, so from eq. (12.16), g2 5 2k2 S g 5 a 1 jb 5 jk; that is, b 5 k. For other modes, b 2 k. In the subsequent sections, we shall examine the TM and TE modes of propagation separately.

12.3 TRANSVERSE MAGNETIC MODES For the TM case, the magnetic field has its components transverse (or normal) to the direction of wave propagation. This implies that we set Hz 5 0 and determine Ex, Ey, Ez, Hx, and Hy by using eqs. (12.11) and (12.15) and the boundary conditions. We shall solve for Ez and later determine other field components from Ez. At the walls (perfect conductors) of 12_Sadiku_Ch12.indd 638 25/09/17 5:27 PM 12.3 Transverse Magnetic Modes 639 the w aveguide in Figure 12.2, the tangential components of the E field must be continuous; that is, Ezs 5 0 at Ezs 5 0 at Ezs 5 0 at y 5 0 (bottom wall) (12.17a) y 5 b (top wall) (12.17b) x 5 0 (right wall) (12.17c) x 5 a (left wall) (12.17d) Ezs 5 0 at Equations (12.17a) and (12.17c) require that A1 5 0 5 A3 in eq. (12.11), so eq. (12.11) becomes Ezs 5 Eo sin kxx sin kyy e2gz(12.18) where Eo 5 A2A4.

Also eqs. (12.17d) and (12.17b) when applied to eq. (12.18) require, respectively, that sin kxa 5 0, sin kyb 5 0(12.19) This implies that kxa 5 mp, m 5 1, 2, 3, . . .(12.20a) kyb 5 np, n 5 1, 2, 3, . . .(12.20b) or kx 5 mp , a ky 5 np (12.21) b The negative integers are not chosen for m and n in eq. (12.20a) for the reason given in Example 6.5. Substituting eq. (12.21) into eq. (12.18) gives Ezs 5 Eo sin a npy 2gz mpx b sin a b e (12.22) a b We obtain other field components from eqs. (12.22) and (12.15), bearing in mind that Hzs 5 0. Thus 12_Sadiku_Ch12.indd 639 npy 2gz g mp mpx b Eo cos a b sin a b e (12.23a) 2 a a a b h npy 2gz g np mpx Eys 5 2 2 a b Eo sin a b cos a b e (12.23b) a b b h jve np npy 2gz mpx Hxs 5 2 a b Eo sin a b cos a b e (12.23c) a b b h jve mp npy 2gz mpx Hys 5 2 2 a b Eo cos a b sin a b e (12.23d) a a b h Exs 5 2 25/09/17 5:27 PM 640 CHAPTER 12 WAVEGUIDES where h2 5 k2x 1 k2y 5 c mp 2 np 2 d (12.24) d 1 c a b which is obtained from eqs. (12.16) and (12.21). Notice from eqs. (12.22) and (12.23) that each set of integers m and n gives a different field pattern or mode, referred to as TMmn mode, in the waveguide. Integer m equals the number of half-cycle variations in the x-direction, and integer n is the number of half-cycle variations in the y-direction. We also notice from eqs. (12.22) and (12.23) that if 1 m, n 2 is 1 0, 0 2 , 1 0, n 2 , or 1 m, 0 2 , all field components vanish. Thus neither m nor n can be zero. Consequently, TM11 is the lowest-order mode of all the TMmn modes. By substituting eq. (12.21) into eq. (12.16), we obtain the propagation constant g5 mp 2 np 2 d 1 c d 2 k2(12.25) Å a b c where k 5 v !me as in eq. (12.3). We recall that, in general, g 5 a 1 jb. In the case of eq. (12.25), we have three possibilities depending on k (or), m, and n: CASE 1 (cutoff) If mp 2 np 2 d 1 c d a b g 5 0 or a 5 0 5 b k2 5 v 2me 5 c The value of that causes this is called the cutoff angular frequency c; that is, vc 5 1 mp 2 np 2 d 1 c c d (12.26) b !me Å a No propagation takes place at this frequency. CASE 2 (evanescent) If k2 5 v 2me , c g 5 a, mp 2 np 2 d 1 c d a b b50 In this case, we have no wave propagation at all.

These nonpropagating modes are said to be evanescent. 12_Sadiku_Ch12.indd 640 25/09/17 5:27 PM 12.3 Transverse Magnetic Modes 641 CASE 3 (propagation) If mp 2 np 2 d d 1 c a b a50 k2 5 v 2me . c g 5 jb, that is, from eq. (12.25) the phase constant b becomes b5 Å k2 2 c mp 2 np 2 d 2 c d (12.27) a b This is the only case in which propagation takes place, because all field components will have the factor e2gz 5 e2jbz. Thus for each mode, characterized by a set of integers m and n, there is a corresponding cutoff frequency fc. The cutoff frequency is the operating frequency below which attenuation occurs and above which propagation takes place. The waveguide therefore operates as a high-pass filter. The cutoff frequency is obtained from eq. (12.26) as fc 5 or fc 5 where ur 5 vc 1 mp 2 np 2 5 d 1 c d c 2p b 2p"me Å a 1 ur m 2 n 2 a b 1 a b (12.28) 2 Å a b 5 phase velocity of uniform plane wave in the lossless dielectric "me edium 1 s 5 0, m, e 2 filling the waveguide. The cutoff wavelength lc is given by m lc 5 ur fc or 12_Sadiku_Ch12.indd 641 lc 5 2 2 m n 2 a b 1a b Å a b (12.29) 25/09/17 5:27 PM 642 CHAPTER 12 WAVEGUIDES Note from eqs. (12.28) and (12.29) that TM11 has the lowest cutoff frequency (or the longest cutoff wavelength) of all the TM modes. The phase constant b in eq. (12.27) can be written in terms of fc as or b 5 v !me b 5 br fc 2 12 c d Å f fc 2 1 2 c d (12.30) Å f where br 5 v/ur 5 v !me 5 phase constant of uniform plane wave in the dielectric medium. It should be noted that g for evanescent mode can be expressed in terms of fc, namely, g 5 a 5 br fc 2 a b 2 1(12.30) Å f The phase velocity up and the wavelength in the guide are, respectively, given by up 5 up v 2p ,l 5 5 (12.31) br b f The intrinsic wave impedance of the mode is obtained from eq. (12.23) as 1 g 5 jb 2 hTM 5 5 or Ey Ex 52 Hy Hx fc 2 m b 12 c d 5 ve Å e Å f hTM 5 hr fc 2 1 2 c d (12.32) Å f where hr 5 !m/e is the intrinsic impedance of a uniform plane wave in the medium. Note the difference between u, b, and h, and u, b, and h. The primed quantities are wave characteristics of the dielectric medium unbounded by the waveguide, as discussed in Chapter 10 (i.e., for TEM mode). For example, u would be the velocity of the wave if the waveguide were removed and the entire space were filled with the dielectric. The unprimed quantities are the wave characteristics of the medium bounded by the waveguide. 12_Sadiku_Ch12.indd 642 25/09/17 5:27 PM 12.4 Transverse Electric Modes 643 End view Side view E H FIGURE 12.4 Field configuration for TM21 mode. As mentioned before, the integers m and n indicate the number of half-cycle variations in the x–y cross section of the guide. Thus for a fixed time, the field configuration of Figure 12.4 results for TM21 mode, for example. 12.4 TRANSVERSE ELECTRIC MODES In the TE modes, the electric field is transverse (or normal) to the direction of wave propagation. We set Ez 5 0 and determine other field components Ex, Ey, Hx, Hy, and Hz from eqs. (12.12) and (12.15) and the boundary conditions just as we did for the TM modes. The boundary conditions are obtained from the requirement that the tangential components of the electric field be continuous at the walls (perfect conductors) of the waveguide; that is, Exs 5 0 Exs 5 0 Eys 5 0 Eys 5 0 at at at at y 5 0(12.33a) y 5 b(12.33b) x 5 0(12.33c) x 5 a(12.33d) From eqs. (12.15) and (12.33), the boundary conditions can be written as 'Hzs 50 'y at y 5 0(12.34a) 'Hzs 50 'y at y 5 b(12.34b) at x 5 0(12.34c) at x 5 a(12.34d) 12_Sadiku_Ch12.indd 643 'Hzs 50 'x 'Hzs 50 'x 25/09/17 5:27 PM 644 CHAPTER 12 WAVEGUIDES Imposing these boundary conditions on eq. (12.12) yields Hzs 5 Ho cos a npy 2gz mpx b e (12.35) b cos a a b where Ho 5 B1B3. Other field components are easily obtained from eqs. (12.35) and (12.15) as jvm np npy 2gz mpx a b Ho cos a b sin a b e (12.36a) a b b h2 Exs 5 Eys 5 2 Hxs 5 Hys 5 npy 2gz jvm mp mpx b e (12.36b) b Ho sin a b cos a 2 a a a b h npy 2gz g mp mpx b Ho sin a b cos a b e (12.36c) 2 a a a b h npy 2gz g np mpx b sin a b Ho cos a b e (12.36d) 2 a a b b h where m 5 0, 1, 2, 3, . . . ; and n 5 0, 1, 2, 3, . . . ; h and g remain as defined for the TM modes. Again, m and n denote the number of half-cycle variations in the x–y cross section of the guide. For TE32 mode, for example, the field configuration is in Figure 12.5. The cutoff frequency fc, the cutoff wavelength lc, the phase constant b, the phase velocity up, and the wavelength l for TE modes are the same as for TM modes [see eqs. (12.28) to (12.31)]. For TE modes, 1 m, n 2 may be 1 0, 1 2 or 1 1, 0 2 but not 1 0, 0 2 .

Both m and n cannot be zero at the same time because this will force the field components in eq. (12.36) to vanish. This implies that the lowest mode can be TE10 or TE01 depending on the values of a and b, the dimensions of the guide. It is standard practice to have a . b so that 1/a2 , 1/b2 in ur ur eq. (12.28). Thus TE10 is the lowest mode because fcTE 5 , fcTE 5 . This mode is 2a 2b 10 End view 01 Top view E H FIGURE 12.5 Field configuration for TE32 mode.

12_Sadiku_Ch12.indd 644 25/09/17 5:27 PM 12.4 Transverse Electric Modes 645 called the dominant mode of the waveguide and is of practical importance. The cutoff frequency for the TE10 mode is obtained from eq. (12.28) as 1 m 5 1, n 5 0 2 fc10 5 ur (12.37) 2a and the cutoff wavelength for TE10 mode is obtained from eq. (12.29) as lc10 5 2a(12.38) Note that from eq. (12.28) the cutoff frequency for TM11 is ur 3 a2 1 b2 4 1/2 2ab which is greater than the cutoff frequency for TE10. Hence, TM11 cannot be regarded as the dominant mode. The dominant mode is the mode with the lowest cutoff frequency (or longest cutoff wavelength). Also note that any EM wave with frequency f , fc10 1 or l . lc10 2 will not be propagated in the guide. The intrinsic impedance for the TE mode is not the same as for TM modes. From eq. (12.36), it is evident that 1 g 5 jb 2 hTE 5 5 Ey vm Ex 52 5 Hy Hx b m Åe or 1 fc 2 12 c d Å f hr hTE 5 fc Å 12 c d f 2 (12.39) Note from eqs. (12.32) and (12.39) that hTE and hTM are purely resistive and vary with - frequency, as shown in Figure 12.6. Also note that hTE hTM 5 hr2(12.40) Important equations for TM and TE modes are listed in Table 12.1 for convenience and quick reference. 12_Sadiku_Ch12.indd 645 25/09/17 5:27 PM 646 CHAPTER 12 WAVEGUIDES FIGURE 12.6 Variation of wave impedance with frequency for TE and TM modes. η TE η TM TABLE 12.1 Important Equations for TM and TE Modes TM Modes Exs 5 2 TE Modes npy 2gz jb mp mpx a b Eo cos a b sin a b e a b h2 a npy 2gz jb np mpx a b Eo sin a b cos a be a b a b h2 b npy 2gz mpx Ezs 5 Eo sin a b sin a be a b npy 2gz jve np mpx Hxs 5 2 a b Eo sin a b cos a be b a b h Eys 5 2 Hys 5 2 npy 2gz jve mp mpx b Eo cos a b sin a be 2 a a a b h Hzs 5 0 h 5 hr fc 2 12a b Å f Exs 5 npy 2gz jvm mp mpx a b Ho cos a b sin a be a b a b h2 Eys 5 2 npy 2gz jvm mp mpx a b Ho sin a b cos a be a a b h2 Ezs 5 0 Hxs 5 npy 2gz jb mp mpx a b Ho sin a b cos a be a b h2 a npy 2gz jb np mpx b Ho cos a b sin a be 2 a b a b h npy 2gz mpx Hzs 5 Ho cos a b cos a be a b hr h5 fc 2 12a b Å f Hys 5 ur m 2 n 2 a b 1a b 2 Å a b ur lc 5 fc fc 2 b 5 br 1 2 a b Å f v up 5 5 f l b fc 5 where h2 5 a 12_Sadiku_Ch12.indd 646 m mp 2 np 2 1 v , br 5 , hr 5 b 1 a b , ur 5 a b ur Å e "me 25/09/17 5:27 PM 12.4 Transverse Electric Modes 647 From eqs. (12.22), (12.23), (12.35), and (12.36), we obtain the field patterns for the TM and TE modes. For the dominant TE10 mode, m 5 1 and n 5 0, so eq. (12.35) becomes Hzs 5 Ho cosa In the time domain, px 2jbz b e (12.41) a Hz 5 Re 1 Hzse jvt 2 or Hz 5 Ho cosa Similarly, from eq. (12.36), px b cos 1 vt 2 bz 2 (12.42) a vma px Ho sina b sin 1 vt 2 bz 2 (12.43a) p a Ey 5 Hx 5 2 Ez 5 Ex 5 Hy 5 0 (12.43c) ba px H sina b sin 1 vt 2 bz 2 (12.43b) p o a FIGURE 12.7 Variation of the field components with x for TE10 mode. 12_Sadiku_Ch12.indd 647 25/09/17 5:27 PM 648 CHAPTER 12 WAVEGUIDES FIGURE 12.8 Field lines for TE10 mode, corresponding to components (a), (b), and (c) in Figure 12.7. End view Side view Top view E H The variation of the E and H fields with x in an xy-plane, say plane cos 1 vt 2 bz 2 5 1 for Hz, and plane sin 1 vt 2 bz 2 5 1 for Ey and Hx, is shown in Figure 12.7 for the TE10 mode.

The corresponding field lines are shown in Figure 12.8. EXAMPLE 12.1 A rectangular waveguide with dimensions a 5 2.5 cm, b 5 1 cm is to operate below 15.1 GHz. How many TE and TM modes can the waveguide transmit if the guide is filled with a medium characterized by s 5 0, e 5 4 eo, mr 5 1? Calculate the cutoff frequencies of the modes. Solution: The cutoff frequency is given by fcmn 5 12_Sadiku_Ch12.indd 648 ur m2 n2 1 2 Å a2 b2 25/09/17 5:27 PM 12.4 Transverse Electric Modes 649 where a 5 2.5b or a/b 5 2.5, and ur 5 Hence, 1 "me 5 c "mr er 5 c 2 c a2 m2 1 2 n2 4a Å b 3 3 108 5 "m2 1 6.25n2 4 1 2.5 3 1022 2 fcmn 5 or fcmn 5 3"m2 1 6.25n2 GHz(12.1.1) We are looking for fcmn , 15.1 GHz. A systematic way of doing this is to fix m or n and increase the other until fcmn is greater than 15.1 GHz. From eq. (12.1.1), it is evident that fixing m and increasing n will quickly give us an fcmn that is greater than 15.1 GHz. For TE01 mode 1 m 5 0, n 5 1 2 , fc01 5 3 1 2.5 2 5 7.5 GHz TE02 mode 1 m 5 0, n 5 2 2 , fc02 5 3 1 5 2 5 15 GHz TE03 mode, fc03 5 3 1 7.5 2 5 22.5 GHz Thus for fcmn , 15.1 GHz, the maximum n 5 2. We now fix n and increase m until fcmn is greater than 15.1 GHz. For TE10 mode 1 m 5 1, n 5 0 2 , fc10 5 3 GHz TE20 mode, fc20 5 6 GHz TE30 mode, fc30 5 9 GHz TE40 mode, fc40 5 12 GHz TE50 mode, fc50 5 15 GHz 1 the same as for TE02 2 TE60 mode, fc60 5 18 GHz That is, for fcmn , 15.1 GHz, the maximum m 5 5. Now that we know the maximum m and n, we try other possible combinations in between these maximum values. For TE11, TM11 (degenerate modes), fc11 5 3"7.25 5 8.078 GHz TE21, TM21, fc21 5 3"10.25 5 9.6 GHz TE31, TM31, fc31 5 3"15.25 5 11.72 GHz TE41, TM41, fc41 5 3"22.25 5 14.14 GHz TE12, TM12, fc12 5 3"26 5 15.3 GHz 12_Sadiku_Ch12.indd 649 25/09/17 5:27 PM 650 CHAPTER 12 WAVEGUIDES FIGURE 12.9 Cutoff frequencies of rectangular waveguide with a 5 2.5b; for Example 12.1. Those modes, whose cutoff frequencies are less than or equal to 15.1 GHz, will be transmitted—that is, 11 TE modes and 4 TM modes (all the foregoing modes except TE12, TM12, TE60, and TE03). The cutoff frequencies for the 15 modes are illustrated in Figure 12.9. PRACTICE EXERCISE 12.1 Consider the waveguide of Example 12.1. Calculate the phase constant, phase velocity, and wave impedance for TE10 and TM11 modes at the operating frequency of 15 GHz. Answer: For TE10, b 5 615.6 rad/m, u 5 1.531 3 108 m/s, hTE 5 192.4 V; for TM11, b 5 529.4 rad/m, u 5 1.78 3 108 m/s, hTM 5 158.8 V. EXAMPLE 12.2 Write the general instantaneous field expressions for the TM and TE modes. Deduce those for TE01 and TM12 modes. Solution: The instantaneous field expressions are obtained from the phasor forms by using E 5 Re 1 Ese jvt 2 and H 5 Re 1 Hse jvt 2 Applying these to eqs. (12.22) and (12.23) while replacing g with jb gives the following field components for the TM modes: Ex 5 Ey 5 npy b mp mpx d Eo cos a b sin a b sin 1 vt 2 bz 2 2c a a b h npy b np mpx c d Eo sin a b cos a b sin 1 vt 2 bz 2 a b h2 b Ez 5 Eo sin a 12_Sadiku_Ch12.indd 650 npy mpx b sin a b cos 1 vt 2 bz 2 a b 25/09/17 5:28 PM 12.4 Transverse Electric Modes 651 npy ve np mpx d Eo sin a b cos a b sin 1 vt 2 bz 2 2 c a b b h npy ve mp mpx Hy 5 2 c d Eo cos a b sin a b sin 1 vt 2 bz 2 a a b h Hz 5 0 Hx 5 2 Similarly, for the TE modes, eqs. (12.35) and (12.36) become npy vm np mpx d Ho cos a b sin a b sin 1 vt 2 bz 2 2 c a b b h npy vm mp mpx Ey 5 2 c d Ho sin a b cos a b sin 1 vt 2 bz 2 a a b h Ez 5 0 Ex 5 2 npy mp mpx d Ho sin a b cos a b sin 1 vt 2 bz 2 a a b npy np mpx c d Ho cos a b sin a b sin 1 vt 2 bz 2 a b b npy mpx Hz 5 Ho cos a b cos a b cos 1 vt 2 bz 2 a b b h2 b Hy 5 2 2 h Hx 5 2 c For the TE01 mode, we set m 5 0, n 5 1 to obtain p 2 h2 5 c d b Ex 5 2 py vmb Ho sin a b sin 1 vt 2 bz 2 p b Ey 5 0 5 Ez 5 Hx Hy 5 2 py bb H sin a b sin 1 vt 2 bz 2 p o b Hz 5 Ho cos a py b b cos 1 vt 2 bz 2 For the TM12 mode, we set m 5 1, n 5 2 to obtain 2py p px a b Eo cos a b sin a b sin 1 vt 2 bz 2 a a b 2py 2p px a b Eo sin a b cos a b sin 1 vt 2 bz 2 a b b h2 2py px Ez 5 Eo sin a b sin a b cos 1 vt 2 bz 2 a b b h2 Ey 5 2 h Ex 5 12_Sadiku_Ch12.indd 651 25/09/17 5:28 PM 652 CHAPTER 12 WAVEGUIDES 2py ve 2p px b Eo sin a b cos a b sin 1 vt 2 bz 2 2 a a b b h 2py ve p px Hy 5 2 a b Eo cos a b sin a b sin 1 vt 2 bz 2 a a b h Hz 5 0 Hx 5 2 where p 2 2p 2 h2 5 c d 1 c d a b PRACTICE EXERCISE 12.2 At 15 GHz, an air-filled 5 cm 2 cm waveguide has Ezs 5 20 sin 40px sin 50py e2jbz V/m (a) What mode is being propagated? (b) Find b. (c) Determine Ey /Ex. Answer: (a) TM21, (b) 241.3 rad/m, (c) 1.25 tan 40px cot 50py.

EXAMPLE 12.3 In a rectangular waveguide for which a 5 1.5 cm, b 5 0.8 cm, s 5 0, m 5 mo, and e 5 4eo, Hx 5 2 sin a 3py px b cos a b sin 1 p 3 1011t 2 bz 2 A/m a b Determine (a) The mode of operation (b) The cutoff frequency (c) The phase constant b (d) The propagation constant g (e) The intrinsic wave impedance h Solution: (a) It is evident from the given expression for Hx and the field expressions in Example 12.2 that m 5 1, n 5 3; that is, the guide is operating at TM13 or TE13. Suppose we choose TM13 mode (the possibility of having TE13 mode is left as an exercise in Practice Exercise 12.3). 12_Sadiku_Ch12.indd 652 25/09/17 5:28 PM 12.4 Transverse Electric Modes 653 fcmn 5 (b) ur 5 ur m2 n2 1 2 Å a2 b2 1 "me 5 c "mrer 5 c 2 Hence fc13 5 (c) c 1 9 22 2 1 3 4 3 4 Å 1.5 3 10 0.8 3 1022 4 2 3 3 108 1 "0.444 1 14.06 2 3 102 5 28.57 GHz 4 b 5 v"me fc 2 v"er fc 2 12 c d 5 12 c d Å f f v 5 2pf 5 p 3 1011 or f 5 b5 (d) 1011 5 50 GHz 2 p 3 1011 1 2 2 28.57 2 12 c d 5 1718.81 rad/m 8 Å 50 3 3 10 g 5 jb 5 j1718.81 /m fc 2 377 28.57 2 12 c d 5 12 c d Å f 50 "er Å 5 154.7 V hTM13 5 hr (e) PRACTICE EXERCISE 12.3 Repeat Example 12.3 if TE13 mode is assumed. Determine other field components for this mode. Answer: fc 5 28.57 GHz, b 5 1718.81 rad/m, g 5 jb, hTE13 5 229.69 V 12_Sadiku_Ch12.indd 653 Ex 5 2584.1 cos a Ey 5 2459.4 sin a Hy 5 11.25 cos a 3py px b sin a b sin 1 vt 2 bz 2 V/m a b 3py px b cos a b sin 1 vt 2 bz 2 V/m, a b Ez 5 0 3py px b sin a b sin 1 vt 2 bz 2 A/m a b Hz 5 27.96 cos a 3py px b cos a b cos 1 vt 2 bz 2 A/m a b 25/09/17 5:28 PM 654 CHAPTER 12 WAVEGUIDES 12.5 WAVE PROPAGATION IN THE GUIDE Examination of eq. (12.23) or (12.36) shows that the field components all involve the terms sine or cosine of 1 mp/a 2 x or 1 np/b 2 y times e2gz. Since sin u 5 cos u 5 1 ju 1 e 2 e2ju 2 (12.44a) 2j 1 ju 1 e 1 e2ju 2 (12.44b) 2 a wave within the waveguide can be resolved into a combination of plane waves reflected from the waveguide walls. For the TE10 mode, for example, jvma px sin a b e2jbz p a vma jpx/a 1e 52 2 e2jpx/a 2 e2jbz (12.45) 2p vma 2jb1z1px/ba2 3e 5 2 e2jb1z2px/ba2 4 2p Eys 5 2 where Ho 1. The first term of eq. (12.45) represents a wave traveling in the positive z-direction at an angle u 5 tan21 a p b(12.46) ba with the z-axis. The second term of eq. (12.45) represents a wave traveling in the positive z-direction at an angle 2u. The field may be depicted as a sum of two plane TEM waves propagating along zigzag paths between the guide walls at x 5 0 and x 5 a as illustrated in Figure 12.10(a). The decomposition of the TE10 mode into two plane waves can be extended to any TE and TM mode. When n and m are both different from zero, four plane waves result from the decomposition. The wave component in the z-direction has a different wavelength from that of the plane waves. This wavelength along the axis of the guide is called the waveguide wavelength and is given by lr l5 fc Å 12 c d f 2 (12.47) where lr 5 ur/f . As a consequence of the zigzag paths, we have three types of velocity: the medium velocity u, the phase velocity up, and the group velocity ug. Figure 12.10(b) illustrates the relationship between the three different velocities. The medium velocity ur 5 1/ !me is as 12_Sadiku_Ch12.indd 654 25/09/17 5:28 PM 12.5 Wave Propagation in the Guide 655 FIGURE 12.10 (a) Decomposition of the TE10 mode into two plane waves.

(b) Relationship between u, up, and ug. /2 Wave path explained in the preceding sections. The phase velocity up is the velocity at which loci of constant phase are propagated down the guide and is given by eq. (12.31); that is, up 5 v (12.48a) b or up 5 ur 5 cos u ur fc Å 12 c d f 2 (12.48b) This shows that up $ ur, since cos u # 1. If ur 5 c, then up is greater than the speed of light in vacuum. Does this violate Einstein's relativity theory that messages cannot travel faster than the speed of light? Not really, because information (or energy) in a waveguide generally does not travel at the phase velocity. Information travels at the group velocity, which must be less than the speed of light. The group velocity ug is the velocity with which the resultant repeated reflected waves are traveling down the guide and is given by ug 5 1 (12.49a) 'b/'v or 12_Sadiku_Ch12.indd 655 ug 5 ur cos u 5 ur fc 2 1 2 c d (12.49b) Å f 25/09/17 5:28 PM 656 CHAPTER 12 WAVEGUIDES Although the concept of group velocity is fairly complex and is beyond the scope of this chapter, a group velocity is essentially the velocity of propagation of the wave-packet envelope of a group of frequencies. It is the energy propagation velocity in the guide and is always less than or equal to u. From eqs. (12.48) and (12.49), it is evident that upug 5 ur2(12.50) This relation is similar to eq. (12.40). Hence the variation of up and ug with frequency is similar to that in Figure 12.6 for hTE and hTM. EXAMPLE 12.4 A standard air-filled rectangular waveguide with dimensions a 5 8.636 cm, b 5 4.318 cm is fed by a 4 GHz carrier from a coaxial cable. Determine whether a TE10 mode will be propagated. If so, calculate the phase velocity and the group velocity. Solution: For the TE10 mode, fc 5 ur/2a. Since the waveguide is air filled, ur 5 c 5 3 3 108. Hence, fc 5 3 3 108 5 1.737 GHz 2 3 8.636 3 1022 As f 5 4 GHz . fc, the TE10 mode will propagate. ur up 5 5 "1 2 1 fc /f 2 2 5 3.33 3 108 m/s ug 5 3 3 108 "1 2 1 1.737/4 2 2 ur2 9 3 1016 5 5 2.702 3 108 m/s up 3.33 3 108 PRACTICE EXERCISE 12.4 Repeat Example 12.4 for the TM11 mode. Answer: 12.5 3 108 m/s, 7.2 3 107 m/s. 12.6 POWER TRANSMISSION AND ATTENUATION To determine power flow in the waveguide, we first find the average Poynting vector [given earlier as eq. (10.78)], 12_Sadiku_Ch12.indd 656 ave 5 1 Re 1 Es 3 H*s 2 (12.51) 2 25/09/17 5:28 PM 12.6 Power Transmission and Attenuation 657 In this case, the Poynting vector is along the z-direction so that ave 5 1 Re 1 ExsH*ys 2 EysH*xs 2 az 2 0 Exs 0 2 1 0 Eys 0 2 5 2h (12.52) az where h 5 hTE for TE modes or h 5 hTM for TM modes. The total average power transmitted across the cross section of the waveguide is Pave 5 3 ave # dS a 53 0 Exs 0 2 1 0 Eys 0 2 b 3 x50 y50 2h (12.53) dy dx Of practical importance is the attenuation in a lossy waveguide.

In our analysis thus far, we have assumed lossless waveguides 1 s 5 0, sc . ` 2 for which a 5 0, g 5 jb. When the dielectric medium is lossy 1 s 2 0 2 and the guide walls are not perfectly conducting 1 sc 2 ` 2 , there is a continuous loss of power as a wave propagates along the guide. According to eq. (10.79), the power flow in the guide is of the form Pave 5 Poe22az(12.54) In general, a 5 ac 1 ad(12.55) where ac and ad are attenuation constants due to ohmic or conduction losses 1 sc 2 ` 2 and dielectric losses 1 s 2 0 2 , respectively. To determine ad, recall that we started with eq. (12.1) assuming a lossless dielectric medium 1 s 5 0 2 . For a lossy dielectric, we need to incorporate the fact that s 2 0. All our equations still hold except that g 5 jb needs to be modified.

This is achieved by replacing « in eq. (12.25) by the complex permittivity of eq. (10.40). Thus, we obtain where g 5 ad 1 jbd 5 mp 2 np 2 b 1 a b 2 v 2mec(12.56) Å a b a ec 5 er 2 jes 5 e 2 j s (12.57) v Substituting eq. (12.57) into eq. (12.56) and squaring both sides of the equation, we obtain 12_Sadiku_Ch12.indd 657 g2 5 a2d 2 b2d 1 2jadbd 5 a mp 2 np 2 b 1 a b 2 v 2me 1 jvms a b 25/09/17 5:28 PM 658 CHAPTER 12 WAVEGUIDES Equating real and imaginary parts, we have a2d 2 b2d 5 a mp 2 np 2 b 1 a b 2 v 2me(12.58a) a b 2adbd 5 vms or ad 5 vms (12.58b) 2bd Assuming that a2d V b2d, a2d 2 b2d . 2b2d, so eq. (12.58a) gives bd 5 Å v 2me 2 a 5 v"me Å mp 2 np 2 b 2a b a b fc 2 (12.59) 12a b f which is the same as b in eq. (12.30). Substituting eq. (12.59) into eq. (12.58b) gives ad 5 shr fc 2 Å 12a b f 2 (12.60) where hr 5 !m/e.

The determination of ac for TMmn and TEmn modes is time-consuming and tedious. We shall illustrate the procedure by finding ac for the TE10 mode. For this mode, only Ey, Hx, and Hz exist. Substituting eq. (12.43a) into eq. (12.53) yields Pave 5 3 a x50 y50 2 3 b 2 3 0 Eys 0 2 2h H2ob vma Pave 5 4p2h dx dy 5 a v 2m2a2H2o b px dy sin2 dx 3 3 2 a 2p h 0 0 (12.61) The total power loss per unit length in the walls is PL 5 PL 0 y50 1 PL 0 y5b 1 PL 0 x50 1 PL 0 x5a 5 2 1 PL 0 y50 1 PL 0 x50 2 (12.62) since the same amount is dissipated in the walls y 5 0 and y 5 b or x 5 0 and x 5 a. For the wall y 5 0, 12_Sadiku_Ch12.indd 658 1 Re c hc 3 1 0 Hxs 0 2 1 0 Hzs 0 2 2 dx d ` 2 y50 a 2 2 a b a 2 2 px 1 px 5 Rs c 3 dx 1 3 H2o cos2 dx d (12.63) 2 Ho sin a a 2 p 0 0 b 2a 2 RsaH2o 5 a1 1 2 b a 4 b p PL 0 y50 5 25/09/17 5:28 PM 12.6 Power Transmission and Attenuation 659 where Rs is the real part of the intrinsic impedance hc of the conducting wall. From eq. (10.56), we write pfm 1 Rs 5 5 (12.64) s cd Å s c where d is the skin depth. The skin resistance of the wall Rs may be regarded as the resistance of 1 m by d by 1 m of the conducting material. For the wall x 5 0, PL 0 x50 5 1 1 Re c hc 3 1 0 Hzs 0 2 2 dyd 0 x50 5 Rs 3 H2o dy 2 2 0 b RsbH2o 5 2 (12.65) Substituting eqs. (12.63) and (12.65) into eq. (12.62) gives PL 5 RsH2o c b 1 b 2a 2 a a1 1 2 b d (12.66) 2 p For energy to be conserved, the rate of decrease in Pave must equal the time-average power loss PL per unit length; that is, dPave PL 5 2 5 2aPave dz or PL a5 (12.67) 2Pave Finally, substituting eqs. (12.61) and (12.66) into eq. (12.67), we have ac 5 RsH2o c b 1 b 2a 2 a a1 1 2 b d 2p2h 2 p (12.68a) 2 2 3 2 v m a Ho b It is convenient to express ac in terms of f and fc. After some manipulations, we obtain for the TE10 mode ac 5 2Rs bhr fc 2 12 c d Å f a0.5 1 b fc 2 c d b(12.68b) a f By following the same procedure, the attenuation constant for the TEmn modes 1 n 2 0 2 can be obtained as ac 0 TE 12_Sadiku_Ch12.indd 659 b b 2 a m 1 n2 b fc 2 a a b fc 2 5 ≥ 1 1 2 a1 b c d a1 c d b ¥ (12.69) a f b2 2 f f 2 2 bhr 1 2 c c d m 1 n a2 Å f 2Rs 25/09/17 5:28 PM 660 CHAPTER 12 WAVEGUIDES and for the TMmn modes as ac 0 TM 5 2Rs bhr fc 2 12 c d Å f c (b/a)3 m2 1 n2 d (12.70) (b/a)2 m2 1 n2 The total attenuation constant a is obtained by substituting eqs. (12.60) and (12.69) or (12.70) into eq. (12.55). † 12.7 WAVEGUIDE CURRENT AND MODE EXCITATION For either TM or TE modes, the surface current density K on the walls of the waveguide may be found by using K 5 an 3 H(12.71) where an is the unit outward normal to the wall and H is the field intensity evaluated on the wall. The current flow on the guide walls for TE10 mode propagation can be found by using eq. (12.71) with eqs. (12.42) and (12.43).

The result is sketched in Figure 12.11. The surface charge density rS on the walls is given by rS 5 an # D 5 an # eE(12.72) where E is the electric field intensity evaluated on the guide wall. A waveguide is usually fed or excited by a coaxial line or another waveguide.

Most often, a probe (central conductor of a coaxial line) is used to establish the field intensities of the desired mode and achieve a maximum power transfer. The probe is located so as to produce E and H fields that are roughly parallel to the lines of E and H fields of the desired mode. To excite the TE10 mode, for example, we know from eq. (12.43a) that Ey has maximum value at x 5 a/2. Hence, the probe is located at x 5 a/2 to excite the TE10 mode as x y z FIGURE 12.11   Surface current on guide walls for TE10 mode. 12_Sadiku_Ch12.indd 660 25/09/17 5:28 PM 12.7 Waveguide Current and Mode Excitation   661 (a) (b) FIGURE 12.12   Excitation of modes in a rectangular waveguide: (a) TE10 mode, (b) TM11 mode. shown in Figure 12.12(a), where the field lines are similar to those of Figure 12.8. Similarly, the TM11 mode is launched by placing the probe along the z-direction as in Figure 12.12(b). EXAMPLE 12.5 An air-filled rectangular waveguide of dimensions a 5 4 cm, b 5 2 cm transports energy in the dominant mode at a rate of 2 mW. If the frequency of operation is 10 GHz, determine the peak value of the electric field in the waveguide. Solution: The dominant mode for a .
b is TE10 mode. The field expressions corresponding to this mode 1 m 5 1, n 5 0 2 are in eq. (12.36) or (12.43), namely, Exs 5 0, fc 5 Eys 5 2jEo sin a ur 3 3 108 5 3.75 GHz 5 2a 2 1 4 3 1022 2 hr h 5 hTE 5 fc / 12_Sadiku_Ch12.indd 661 px 2jbz be , a 12 c d f 2 5 where Eo 5 377 3.75 2 12 c d / 10 vma Ho p 5 406.7 V 25/09/17 5:28 PM 662   CHAPTER 12   WAVEGUIDES From eq.
(12.53), the average power transmitted is Pave 5 3 b 3 a y50 x50 5 0 Eys 0 2 2h E2o ab 4h dx dy 5 a E2o b 2 px 3 dy 3 sin a a b dz 2h 0 0 Hence, 4hPave 4 1 406.7 2 3 2 3 1023 5 5 4067 ab 8 3 1024 Eo 5 63.77 V/m E2o 5 PRACTICE EXERCISE   12.5 In Example 12.5, calculate the peak value Ho of the magnetic field in the guide if a 5 2 cm, b 5 4 cm, while other things remain the same. Answer: 63.34 mA/m. EXAMPLE 12.6 A copper-plated waveguide 1 sc 5 5.8 3 107 S/m 2 operating at 4.8 GHz is supposed to deliver a minimum power of 1.2 kW to an antenna. If the guide is filled with polystyrene 1 s 5 10217 S/m, e 5 2.55eo 2 and its dimensions are a 5 4.2 cm, b 5 2.6 cm, calculate the power dissipated in a length 60 cm of the guide in the TE10 mode. Solution: Let Pd 5 power loss or dissipated Pa 5 power delivered to the antenna Po 5 input power to the guide so that Po 5 Pd 1 Pa From eq. (12.54), Pa 5 Poe22az Hence, or Pa 5 1 Pd 1 Pa 2 e22az Pd 5 Pa 1 e2az 2 1 2 12_Sadiku_Ch12.indd 662 25/09/17 5:28 PM 12.7 Waveguide Current and Mode Excitation   663 Now we need to determine from a 5 ad 1 ac From eq. (12.60), shr ad 5 2 fc 2 12 c d Å f Since the loss tangent s 5 ve 10217 1029 3 2.55 36p V 1 1 lossless dielectric medium 2 2p 3 4.8 3 109 3 5 1.47 3 10217 then m 377 5 5 236.1 Åe "er c 1 5 5 1.879 3 108 m/s ur 5 "me "er ur 1.879 3 108 5 fc 5 5 2.234 GHz 2a 2 3 4.2 3 1022 hr . 10217 3 236.1 ad 5 2 Å 12 c 2.234 2 d 4.8 ad 5 1.334 3 10215 Np/m For the TE10 mode, eq. (12.68b) gives 2Rs ac 5 bhr fc 2 12 c d Å f a0.5 1 b fc 2 c d b a f where Rs 5 pfm 1 p 3 4.8 3 109 3 4p 3 1027 5 5 s cd Å s c Å 5.8 3 107 5 1.808 3 1022 V 12_Sadiku_Ch12.indd 663 25/09/17 5:28 PM 664   CHAPTER 12   WAVEGUIDES Hence 2 3 1.808 3 1022 a0.5 1 ac 5 2.6 3 1022 3 236.1 5 4.218 3 10 23 Å 2.6 2.234 2 c d b 4.2 4.8 12 c 2.234 2 d 4.8 Np/m Note that ad V ac, showing that the loss due to the finite conductivity of the guide walls is more important than the loss due to the dielectric medium. Thus a 5 ad 1 ac . ac 5 4.218 3 1023 Np/m and the power dissipated in the guide if it is Pd 5 Pa 1 e2az 2 1 2 5 1.2 3 103 1 e234.218310 23 5 6.089 W 30.6 2 12 PRACTICE EXERCISE   12.6 A brass waveguide 1 sc 5 1.1 3 107 S/m 2 of dimensions a 5 4.2 cm, b 5 1.5 cm is filled with Teflon 1 er 5 2.6, s 5 10215 S/m 2 . The operating frequency is 9 GHz. For the TE10 mode: (a)   Calculate ad and ac. (b)   Find the loss in decibels in the guide if it is 40 cm long. Answer: (a) 1.205 3 10213 Np/m, 2 3 1022 Np/m, (b) 0.06945 dB. EXAMPLE 12.7 Sketch the field lines for the TM11 mode. Derive the instantaneous expressions for the surface current density of this mode. Solution: From Table 12.1, we obtain the fields for TM11 mode 1 m 5 1, n 5 1 2 as py p px a b Eo cos a b sin a b sin 1 vt 2 bz 2 a a b py p px a b Eo sin a b cos a b sin 1 vt 2 bz 2 a b b py px Ez 5 Eo sin a b sin a b cos 1 vt 2 bz 2 a b py ve p px Hx 5 2 2 a b Eo sin a b cos a b sin 1 vt 2 bz 2 a b b h b h2 b Ey 5 2 h Ex 5 12_Sadiku_Ch12.indd 664 25/09/17 5:28 PM 12.7 Waveguide Current and Mode Excitation   665 py ve p px a b Eo cos a b sin a b sin 1 vt 2 bz 2 a b h2 a Hz 5 0 Hy 5 For the electric field lines, dy dx 5 Ey Ex For the magnetic field lines, dy dx 5 Hy Hx 5 py a px tan a b cot a b a b b py b px 5 2 cot a b tan a b a a b Notice that 1 Ey /Ex 2 1 Hy /Hx 2 5 21, showing that electric and magnetic field lines are mutually orthogonal. This should also be observed in Figure 12.13, where the field lines are sketched. The surface current density on the walls of the waveguide is given by K 5 an 3 H 5 an 3 1 Hx, Hy, 0 2 At x 5 0, an 5 ax, K 5 Hy 1 0, y, z, t 2 az; that is, K5 py ve p b sin 1 vt 2 bz 2 az 2 a b Eo sin a a b h At x 5 a, an 5 2ax, K 5 2Hy 1 a, y, z, t 2 az, or K5 py ve p a b Eo sin a b sin 1 vt 2 bz 2 az b h2 a At y 5 0, an 5 ay, K 5 2Hx 1 x, 0, z, t 2 az, or K5 ve p px b sin 1 vt 2 bz 2 az 2 a b Eo sin a a b h At y 5 b, an 5 2ay, K 5 Hx 1 x, b, z, t 2 az, or K5 ve p px a b Eo sin a b sin 1 vt 2 bz 2 az a h2 b FIGURE 12.13   Field lines for TM11 mode; for Example 12.7. E H 12_Sadiku_Ch12.indd 665 25/09/17 5:28 PM 666   CHAPTER 12   WAVEGUIDES End view Side view E H Top view FIGURE 12.14   Field lines for TE11 mode; for Practice Exercise 12.7. PRACTICE EXERCISE   12.7 Sketch the field lines for the TE11 mode. Answer: See Figure 12.14. The strength of the field at any point is indicated by the density of the lines; the field is strongest (or weakest) where the lines are closest together (or farthest apart). 12.8 WAVEGUIDE RESONATORS Resonators are primarily used for energy storage. At high frequencies (100 MHz) the RLC circuit elements are inefficient when used as resonators because the dimensions of the circuits are comparable to the operating wavelength, and consequently, there is unwanted radiation. Therefore, at high frequencies the RLC resonant circuits are replaced by electromagnetic cavity resonators.
Such resonator cavities are used in klystron tubes, bandpass filters, and wave meters.
The microwave oven essentially consists of a power supply, a waveguide feed, and an oven cavity. Consider the rectangular cavity (or closed conducting box) shown in Figure 12.15. We notice that the cavity is simply a rectangular waveguide shorted at both ends. We therefore expect to have standing wave and also TM and TE modes of wave propagation. Depending on how the cavity is excited, the wave can propagate in the x-, y-, or z-direction. We will choose the 1z-direction as the “direction of wave propagation.” In fact, there is no wave propagation. Rather, there are standing waves. We recall from Section 10.9 that a standing wave is a combination of two waves traveling in opposite directions.
12_Sadiku_Ch12.indd 666 25/09/17 5:28 PM 12.8 Waveguide Resonators   667 FIGURE 12.15   Rectangular cavity. A.   TM Mode to z For propagation to z in TM mode, Hz 5 0 and we let Ezs 1 x, y, z 2 5 X 1 x 2 Y 1 y 2 Z 1 z 2 (12.73) be the product solution of eq. (12.1). We follow the same procedure taken in Section 12.2 and obtain X 1 x 2 5 c1 cos kxx 1 c2 sin kxx(12.74a) Y 1 y 2 5 c3 cos kyy 1 c4 sin kyy(12.74b) Z 1 z 2 5 c5 cos kzz 1 c6 sin kzz(12.74c) where k2 5 k2x 1 k2y 1 k2z 5 v 2me(12.75) The boundary conditions are: Ez 5 0 at x 5 0, a(12.76a) Ez 5 0 at y 5 0, b(12.76b) Ey 5 0, Ex 5 0 at z 5 0, c(12.76c) As shown in Section 12.3, the conditions in eqs. (12.76a,b) are satisfied when c1 5 0 5 c3 and kx 5 mp , a ky 5 np (12.77) b where m 5 1, 2, 3, . . .
, n 5 1, 2, 3, ... To invoke the conditions in eq. (12.76c), we notice that eq. (12.14) 1 with Hzs 5 0 2 yields 12_Sadiku_Ch12.indd 667 jveExs 5 '2Ezs '2Exs 1 a 2 2 b(12.78) jvm 'z 'z 'x 20/11/17 7:41 PM 668   CHAPTER 12   WAVEGUIDES Similarly, combining eqs. (12.13a) and (12.13d) 1 with Hzs 5 0 2 results in jveEys 5 '2Eys '2Ezs 1 2 a b(12.79) 2jvm 'y 'z 'z2 From eqs. (12.78) and (12.79), it is evident that eq. (12.76c) is satisfied if 'Ezs 5 0 at 'z z 5 0, c(12.80) This implies that c6 5 0 and sin kzc 5 0 5 sin pp. Hence, kz 5 pp (12.81) c where p 5 0, 1, 2, 3, ... Substituting eqs. (12.77) and (12.81) into eq. (12.74) yields Ezs 5 Eo sin a npy ppz mpx b sin a b cos a b(12.82) a c b where Eo 5 c2c4c5. Other field components are obtained from eqs.
(12.82) and (12.13). The phase constant b is obtained from eqs. (12.75), (12.77), and (12.81) as b2 5 k2 5 c pp 2 mp 2 np 2 d 1 c d (12.83) d 1 c a c b Since b2 5 v 2me, from eq. (12.83), we obtain the resonant frequency fr 2pfr 5 v r 5 or fr 5 b "me 5 bur p 2 ur m 2 n 2 c d 1 c d 1 c d (12.84) c 2 Å a b The corresponding resonant wavelength is lr 5 ur 5 fr 2 p 2 m n 2 c d 1 c d 1 c d c Å a b 2 (12.85) From eq. (12.84), we notice that the lowest-order TM mode is TM110. 12_Sadiku_Ch12.indd 668 25/09/17 5:28 PM 12.8 Waveguide Resonators   669 B.   TE Mode to z For propagation to z in TE mode, Ez 5 0 and Hzs 5 1 b1 cos kxx 1 b2 sin kxx 2 1 b3 cos kyy 1 b4 sin kyy 2 1 b5 cos kzz 1 b6 sin kzz 2 (12.86) The boundary conditions in eq. (12.76c) combined with eq. (12.13) yields Hzs 5 0 at z 5 0, c(12.87a) 'Hzs 5 0 at 'x x 5 0, a(12.87b) 'Hzs 5 0 at 'y y 5 0, b(12.87c) Imposing the conditions in eq. (12.87) on eq. (12.86) in the same manner as for TM mode to z leads to Hzs 5 Ho cos a npy ppz mpx b cos a b(12.88) b sin a a c b where m 5 0, 1, 2, 3, . . . , n 5 0, 1, 2, 3, . . . , and p 5 1, 2, 3, ... Other field components can be obtained from eqs. (12.13) and (12.88). The resonant frequency is the same as that of eq. (12.84) except that m or n (but not both at the same time) can be zero for TE modes. It is impossible for m and n to be zero at the same time because the field components will be zero if m and n are zero.
The mode that has the lowest resonant frequency for a given cavity size (a, b, c) is the dominant mode. If a . b , c, it implies that 1/a , 1/b . 1/c; hence, the dominant mode is TE101. Note that for a . b , c, the resonant frequency of TM110 mode is higher than that for TE101 mode; hence, TE101 is dominant. When different modes have the same resonant frequency, we say that the modes are degenerate; one mode will dominate others depending on how the cavity is e xcited. A practical resonant cavity has walls with finite conductivity sc and is, therefore, capable of losing stored energy.
The quality factor Q is a means of determining the loss. The quality factor is also a measure of the bandwidth of the cavity resonator. It may be defined as Q 5 2p # 12_Sadiku_Ch12.indd 669 time average energy stored energy loss per cycle of oscillation 5 2p # W W 5 v (12.89) PLT PL 25/09/17 5:28 PM 670   CHAPTER 12   WAVEGUIDES where T 5 1/f 5 the period of oscillation, PL is the time-average power loss in the cavity, and W is the total time-average energy stored in electric and magnetic fields in the cavity. Usually Q is very high for a cavity resonator compared with Q for an RLC resonant circuit. By following a procedure similar to that used in deriving ac in Section 12.6, it can be shown that the quality factor for the dominant TE101 is given by3 QTE101 5 where d 5 EXAMPLE 12.8 1 "pf101mosc 1 a2 1 c2 2 abc d 3 2b 1 a 1 c3 2 1 ac 1 a2 1 c2 2 4 3 (12.90) is the skin depth of the cavity walls. An air-filled resonant cavity with dimensions a 5 5 cm, b 5 4 cm, and c 5 10 cm is made of copper 1 sc 5 5.8 3 107 S/m 2 . Find (a) The five lowest-order modes (b) The quality factor for TE101 mode Solution: (a) The resonant frequency is given by fr 5 p 2 ur m 2 n 2 c d 1 c d 1 c d c 2 Å a b where ur 5 1 "me 5c Hence fr 5 2 2 2 p 3 3 108 m n 1 1 c d c d c d 2 Å 5 3 1022 4 3 1022 10 3 1022 5 15"0.04m2 1 0.0625n2 1 0.01p2 GHz Since c . a . b or 1/c , 1/a , 1/b, the lowest-order mode is TE101. Notice that TM101 and TE100 do not exist because m 5 1, 2, 3, . . . , n 5 1, 2, 3, . . . , and p 5 0, 1, 2, 3, . . . for the TM modes, and m 5 0, 1, 2, . . . , n 5 0, 1, 2, . . . , and p 5 1, 2, 3, . . . for the TE modes. The resonant frequency for the TE101 mode is fr101 5 15"0.04 1 0 1 0.01 5 3.354 GHz 3 For the proof, see S. V. Marshall and G. G. Skitek, Electromagnetic Concepts and Applications, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1990, pp. 440–442. 12_Sadiku_Ch12.indd 670 25/09/17 5:28 PM 12.8 Waveguide Resonators   671 The next higher mode is TE011 (TM011 does not exist), with fr011 5 15"0 1 0.0625 1 0.01 5 4.04 GHz The next mode is TE102 (TM102 does not exist), with fr102 5 15"0.04 1 0 1 0.04 5 4.243 GHz The next mode is TM110 (TE110 does not exist), with fr110 5 15"0.04 1 0.0625 1 0 5 4.8 GHz The next two modes are TE111 and TM111 (degenerate modes), with fr111 5 15"0.04 1 0.0625 1 0.01 5 5.031 GHz The next mode is TM103 with fr103 5 15"0.04 1 0 1 0.09 5 5.408 GHz Thus the five lowest order modes in ascending order are TE101 TE011 TE102 TM110 TE111 or TM111 (3.35 GHz) (4.04 GHz) (4.243 GHz) (4.8 GHz) (5.031 GHz) (b) The quality factor for TE101 is given by 1 a2 1 c2 2 abc d 3 2b 1 a3 1 c3 2 1 ac 1 a2 1 c2 2 4 1 25 1 100 2 200 3 1022 5 d 3 8 1 125 1 1000 2 1 50 1 25 1 100 2 4 QTE101 5 5 "pf101 mosc 1 5 61d 61 "p 1 3.35 3 109 2 4p 3 1027 1 5.8 3 107 2 61 5 14,358 5 PRACTICE EXERCISE   12.8 If the resonant cavity of Example 12.8 is filled with a lossless material 1 mr 5 1, er 5 3 2 , find the resonant frequency fr and the quality factor for TE101 mode. Answer: 1.936 GHz, 1.093 3 104. 12_Sadiku_Ch12.indd 671 25/09/17 5:28 PM 672   CHAPTER 12   WAVEGUIDES † 12.9   APPLICATION NOTE—OPTICAL FIBER In the mid-1970s, it was recognized that the existing copper technology would be unsuitable for future communication networks. In view of this, the telecommunication industry invested heavily in research into optical fibers. Optical fiber provides an attractive alternative to wire transmission lines such as twisted pair and coaxial cable (or coax). Optical fiber4 has the following advantages over copper: • Bandwidth: It provides a very high capacity for carrying information. It has sufficient bandwidth that bit-serial transmission can be used, thereby considerably reducing the size, cost, and complexity of the hardware. • Attenuation: It provides low attenuation and is therefore capable of transmitting over a long distance without the need of repeaters. • Noise susceptibility: It neither radiates nor is affected by electromagnetic interference. The immunity from EMI is due to the absence of metal parts, which means that there can be no conduction currents. • Security: It is more secure from malicious interception because it is not easy to tap a fiber-optic cable without interrupting communication. • Cost: The cost of optical fibers has fallen considerably since the turn of the century and will continue to fall. The cost of related components such as optical transmitters and receivers also is falling. These impressive advantages over electrical media have made fiber optics a popular transmission medium in recent times. Although optical fiber is more expensive and is used mainly for point-to-point links, there has been a rapid changeover from coax and twisted pair to optical fibers for telecommunication systems, instrumentation, cable TV networks, industrial automation, and data transmission systems.
An optical fiber is a dielectric waveguide operating at optical frequency. Optical frequencies are on the order of 100 THz. As shown in Figure 12.16, an optical fiber consists of three concentric cylindrical sections: the core, the cladding, and the jacket. The core consists of one or more thin strands made of glass or plastic. The cladding is the glass or plastic coating surrounding the core, which may be step index or graded index.
In the step-index core, the refractive index is uniform but undergoes an abrupt change at the core–cladding interface, while the graded-index core has a refractive index that varies with the radial distance from the center of the fiber. The jacket surrounds one or a bundle of cladded fibers. The jacket is made of plastic or other materials to protect against moisture, crushing, and other forms of damage. A ray of light entering the core will be internally reflected when incident in the denser medium and the angle of incidence is greater than a critical value. Thus a light ray is 4 There are several excellent books that can provide further exposition on optical fiber. See, for example, S. L. W. Meardon, The Elements of Fiber Optics, Englewood Cliffs, NJ: Regents/Prentice Hall, 1993.
12_Sadiku_Ch12.indd 672 25/09/17 5:28 PM 12.9 Application Note—Optical Fiber   673 Jacket Core Cladding Light at less than critical angle is absorbed in jacket Angle of incidence Angle of reflection FIGURE 12.16   Optical fiber. r eflected back into the original medium and the process is repeated as light passes down the core. This form of propagation is multimode, referring to the variety of angles that will r eflect, as shown in Figure 12.17. It causes the signal to spread out in time and limits the rate at which data can be accurately received. When the radius of the core is reduced a single-mode propagation occurs. This eliminates distortion. A fiber-optic system is similar to a conventional transmission system. As shown in Figure 12.18, a fiber-optic system consists of a transmitter, a transmission medium, and a receiver. The transmitter accepts and converts to optical signals electrical signals input in analog or digital form. The transmitter sends the optical signal by modulating the output of a light source (usually an LED or a laser) by varying its intensity. The optical signal is transmitted over the optical fiber to a receiver. At the receiver, the optical signal is converted back into an electrical signal by a photodiode. The performance of a fiber-optic link depends on the numerical aperture (NA), attenuation, and dispersion characteristics of the fiber.
As signals propagate through the fiber, they become distorted owing to attenuation and dispersion. Numerical Aperture The most important parameter of an optical fiber is its numerical aperture (NA). The value of NA is dictated by the refractive indices of the core and cladding. By definition, the refractive index n of a medium is defined as n5 12_Sadiku_Ch12.indd 673 speed of light in a vacuum speed of light in the medium 1 c !moeo 5 5 (12.91) um 1 !mmem 25/09/17 5:28 PM 674   CHAPTER 12   WAVEGUIDES Absorptive jacket Core Cladding (a) (b) (c) FIGURE 12.17.   Optical fiber transmission modes: (a) multimode, (b) multimode graded index, (c) single mode. (From W. Stallings, Local and Metropolitan Area Networks, 4th ed. New York: Macmillan, 1993, p. 85.) 12_Sadiku_Ch12.indd 674 25/09/17 5:28 PM 12.9 Application Note—Optical Fiber   675 Electrical data input Optical fiber cable Electrical-tooptical converter Transmission medium Light source Optical-toelectrical converter Light detector Electrical data output FIGURE 12.18   A typical fiber-optic system.
Since mm 5 mo in most practical cases, n5 em 5 "er(12.92) Å eo indicating that the refractive index is essentially the square root of the dielectric constant. Keep in mind that «r can be complex, as discussed in Chapter 10. For common materials, n 5 1 for air, n 5 1.33 for water, and n 5 1.5 for glass. As a light ray propagates from medium 1 to medium 2, Snell’s law must be satisfied. n1 sin u 1 5 n2 sin u 2(12.93) where 1 is the incident angle in medium 1 and 2 is the transmission angle in medium 2. The total reflection occurs when 2 5 90, resulting in u 1 5 u c 5 sin21 n2 (12.94) n1 where c is the critical angle for total internal reflection. Note that eq. (12.94) is valid only if n1 . n2, since the value of sin c must be less than or equal to 1. Another way of looking at the light-guiding capability of a fiber is to measure the acceptance angle a, which is the maximum angle over which light rays entering the fiber will be trapped in its core. We know that the maximum angle occurs when c is the critical angle, thereby satisfying the condition for total internal reflection. Thus, for a step-index fiber, NA 5 sin u a 5 n1 sin u c 5 "n21 2 n22(12.95) where n1 is the refractive index of the core and n2 is the refractive index of the cladding, as shown in Figure 12.19. Since most fiber cores are made of silica, n1 5 1.48. Typical values of NA range between 0.19 and 0.25. The larger the value of NA, the more optical power the fiber can capture from a source. Because such optical fibers may support the numerous modes, they are called a multimode step-index fibers. The mode volume V is given by 12_Sadiku_Ch12.indd 675 V5 pd "n21 2 n22(12.96) l 25/09/17 5:28 PM 676   CHAPTER 12   WAVEGUIDES Cladding n2 n0 1 n1 a a A Core c c FIGURE 12.19 Numerical aperture and acceptance angle. where d is the fiber core diameter and l is the wavelength of the optical source. From eq. (12.96), the number N of modes propagating in a step-index fiber can be estimated as N5 V2 (12.97) 2 Attenuation As discussed in Chapter 10, attentuation is the reduction in the power of the optical signal. Power attenuation (or fiber loss) in an optical fiber is governed by dP 5 2aP(12.98) dz where a is the attenuation and P is the optical power. In eq. (12.98), it is assumed that a wave propagates along z. By solving eq. (12.98), the power P 1 0 2 at the input of the fiber and the power P 1 , 2 of the light after are related as P 1 , 2 5 P 1 0 2 e2a,(12.99) It is customary to express attenuation a in decibels per kilometer and length of the fiber in kilometers.
In this case, eq. (12.99) becomes a, 5 10 log10 P102 (12.100) P1,2 Thus, the power of the light reduces by a decibels per kilometer as it propagates through the fiber. Equation (12.100) may be written as 12_Sadiku_Ch12.indd 676 P 1 , 2 5 P 1 0 2 # 102a,/10(12.101) 25/09/17 5:28 PM 12.9 Application Note—Optical Fiber   677 For , 5 100 km, P1,2 102100 , e 22 10 P102 for coaxial cable (12.102) for optical fiber indicating that much more power is lost in the coaxial cable than in optical fiber. Dispersion The spreading of pulses of light as they propagate down a fiber is called dispersion.
As the pulses representing 0s spread, they overlap epochs that represent 1s. If dispersion is beyond a certain limit, it may confuse the receiver. The dispersive effects in single-mode fibers are much smaller than in multimode fibers. EXAMPLE 12.9 A step-index fiber has a core diameter of 80 mm, a core refractive index of 1.62, and a numerical aperture of 0.21. Calculate (a) the acceptance angle, (b) the refractive index that the fiber can propagate at a wavelength of 0.8 mm, (c) the number of modes that the fiber can propagate at a wavelength of 0.8 mm. Solution: (a) Since sin u a 5 NA 5 0.21, then u a 5 sin21 0.21 5 12.12 (b) From NA 5 "n21 2 n22, we obtain (c) n2 5 "n21 2 NA2 5 "1.622 2 0.212 5 1.606 V5 5 Hence pd pd NA "n21 2 n22 5 l l p 1 80 3 1026 2 3 0.21 5 65.973 0.8 3 1026 N5 V2 5 2176 modes 2 PRACTICE EXERCISE   12.9 A silica fiber has a refractive index of 1.48. It is surrounded by a cladding material with a refractive index of 1.465. Find (a) the critical angle above which total internal reflection occurs, (b) the numerical aperture of the fiber.
Answer: (a) 81.83, (b) 0.21. 12_Sadiku_Ch12.indd 677 25/09/17 5:28 PM 678   CHAPTER 12   WAVEGUIDES EXAMPLE 12.10 Light pulses propagate through a fiber cable with an attenuation of 0.25 dB/km. Determine the distance through which the power of pulses is reduced by 40%. Solution: If the power is reduced by 40%, it means that Hence P1,2 5 1 2 0.4 5 0.6 P102 P102 10 log10 a P1,2 10 1 5 log10 0.25 0.6 ,5 5 8.874 km PRACTICE EXERCISE   12.10 A 10 km fiber with an attenuation of 0.2 dB/km serves as an optical link between two cities. How much of input power is received? Answer: 63.1%. 12.10   APPLICATION NOTE—CLOAKING AND INVISIBILITY The practice of using metamaterials to hide an object is called metamaterial cloaking. Metamaterials are ideal for cloaking because they are designed to have a negative refractive index. All materials have an index of refraction, a number that describes that amount of light, or electromagnetic wave, that is reflected as the wave passes through the material. All materials that are found in nature have a positive refraction index, allowing the reflected light to hit an observer’s eye, making the object visible. However, the negative refraction index of metamaterials can bend the wave around an object instead of reflecting the light, thus making the object invisible. Many attempts at cloaking an object have been made and have been successful to some degree, leaving only small reflections of the cloaked object.
Recently, however, researchers at Duke University discovered a method of cloaking an object completely, making it perfectly invisible. The research at Duke began in 2006, but the cloaking models suffered from the common problem of reflected light.
In 2011, David Smith and graduate student Nathan Landy modified the models by altering the arrangement of the metamaterial to a diamond-like configuration and shifting the metamaterial so that the reflections were canceled by its mirror image at each intersection. With this adjustment, illustrated schematically in Figure 12.20, perfect invisibility was achieved. 12_Sadiku_Ch12.indd 678 25/09/17 5:28 PM 12.10 Application Note—Cloaking and Invisibility   679 FIGURE 12.20   Showing bending of light. This perfect invisibility, however, comes at the price of a few caveats. An invisibility cloak has been created only on the centimeter scale. Also, the object surrounded by this metamaterial cloak is invisible only to microwaves. In other words, the researchers have been able to bend microwaves but have not yet achieved the bending of light waves, which are more difficult to bend because they have a higher frequency. Finally, the invisibility is unidirectional: that is, the object cloaked is invisible from only one specific angle. Nonetheless, this research at Duke University marks a breakthrough in metamaterial cloaking. These researchers, who were the first to bend waves without any reflection, now plan to further develop the cloak to make it omnidirectional, hiding the object from every angle. While using metamaterials to render objects invisible to the human eye may be decades away, invisibility to microwaves has many practical applications in telecommunications and defense. Potential applications include radar and sensor detection, battlefield communication, and infrastructure monitoring. MATLAB 12.1 % % % % % % This script computes the cutoff frequencies of the first 10 waveguide modes, allowing the user to enter the dimensions (assuming a > b) and relative material properties. The script first finds the lowest 100 modes by cutoff frequency for both TE and TM, creating a list of 200 total modes, from which the lowest 10 of all (TE and TM) are found clear % Enter the frequency (in rad/s) a = input(‘Enter the waveguide width > ‘); % Enter the propagation constant gamma (in a+j*b format) b = input(‘Enter the waveguide height > ‘); % Enter the relative permittivity er = input(‘Enter the relative permittivity > ‘); % Enter the propagation constant gamma (in a+j*b format) ur = input(‘Enter the relative permeability > ‘); % Determine the first 100 TM modes 12_Sadiku_Ch12.indd 679 15/11/17 7:56 PM 680 CHAPTER 12   WAVEGUIDES index=1; % start a count for m=1:10, for n=1:10, modes(index,1)=1; % store a 1 in row , and % column 1 for TM modes modes(index,2)=m; % store m in row , and column 2 modes(index,3)=n; % store n in row , and column 3 modes(index,4)=3e8/sqrt(er*ur)*sqrt((m*pi/a)^2+(n*pi/b)^2); % store cutoff in row , and column 4 index=index+1; % increment counter end end % Determine the first 100 TE modes for m=0:9, for n=0:9, if m | n % check if either m or n is nonzero and % compute mode modes(index,1)=2; modes(index,2)=m; modes(index,3)=n; modes(index,4)=3e8/sqrt(er*ur)*sqrt((m*pi/ a)^2+(n*pi/b)^2); index=index+1; else % do nothing, because m = n = 0 end end end % Sort these 100 modes by lowest cutoff % this command sorts the matrix by grouping the fourth % column (the frequencies) in ascending order) modes=sortrows(modes,4); % Print out the lowest 10 modes of the lowest 100 modes mode_string=’ME’; % ‘M’ is the first character, ‘E’ is the second disp(sprintf(‘’)); % format extra line for k = 1:10 disp(sprintf(‘Mode: T%c%d%d, ‘,... mode_string(modes(k,1)),modes(k,2), modes(k,3))) disp(sprintf(‘Cutoff frequency = %0.3f GHz’,... modes(k,4)/(2*pi*1e9))) end SUMMARY 12_Sadiku_Ch12.indd 680   1. Waveguides are structures used in guiding EM waves at high frequencies. Assuming a lossless rectangular waveguide 1 sc . `, s . 0 2 , we apply Maxwell’s equations in analyzing EM wave propagation through the guide. The resulting partial differential 15/11/17 7:56 PM Summary   681 equation is solved by using the method of separation of variables.
On applying the boundary conditions on the walls of the guide, the basic formulas for the guide are obtained for different modes of operation.   2. Two modes of propagation (or field patterns) are the TMmn and TEmn, where m and n are positive integers. For TM modes, m 5 1, 2, 3, .   .   . , and n 5 1, 2, 3, .   .   . and for TE modes, m 5 0, 1, 2, .   .   . , and n 5 0, 1, 2, .   .   . , n 5 m 2 0.   3. Each mode of propagation has an associated propagation constant and cutoff frequency. The propagation constant g 5 a 1 jb does not depend only on the constitutive para meters 1 e, m, s 2 of the medium as in the case of plane waves in an unbounded space; it depends on the cross-sectional dimensions 1 a, b 2 of the guide. The cutoff frequency is the frequency at which g changes from being purely real (attenuation) to purely imaginary (propagation). The dominant mode of operation is the lowest mode possible. It is the mode with the lowest cutoff frequency. If a . b, the dominant mode is TE10.   4. The basic equations for calculating the cutoff frequency fc, the phase constant b, and the phase velocity up are summarized in Table 12.1. Formulas for calculating the attenuation constants due to lossy dielectric medium and imperfectly conducting walls are also provided.   5. The group velocity (or velocity of energy flow) ug is related to the phase velocity up of the wave propagation by upug 5 ur2   where ur 5 1/"me is the medium velocity (i.e., the velocity of the wave in the dielectric medium unbounded by the guide).
Although up is greater than u, ug does not exceed u.   6. The mode of operation for a given waveguide is dictated by the method of excitation.   7.   A waveguide resonant cavity is used for energy storage at high frequencies. It is nothing but a waveguide shorted at both ends.
Hence its analysis is similar to that of a waveguide. The resonant frequency for both the TE and the TM modes to z is given by fr 5 p ur m 2 n 2 c d 1 c d 1 c d c 2Å a b For TM modes, m 5 1, 2, 3, .   .   . , n 5 1, 2, 3, .   .   . , and p 5 0, 1, 2, 3, .   .   . , and for TE modes, m 5 0, 1, 2, 3, .   .   . , n 5 0, 1, 2, 3, .   .   . , and p 5 1, 2, 3, .   .   . , m 5 n 2 0. If a . b , c, the dominant mode (one with the lowest resonant frequency) is TE101.   8. The quality factor, a measure of the energy loss in the cavity, is given by Q5v W PL   9. An optical fiber is a dielectric waveguiding structure operating at optical frequencies; it consists of a core region and a cladding region. 10. Advantages of optical fiber over copper wire include large bandwidth, low attenuation, immunity to electromagnetic interence, and low cost.
12_Sadiku_Ch12.indd 681 25/09/17 5:28 PM 682   CHAPTER 12   WAVEGUIDES REVIEW QUESTIONS   12.1 At microwave frequencies, we prefer waveguides to transmission lines for transporting EM energy because of all the following except that (a)   losses in transmission lines are prohibitively large. (b)   waveguides have larger bandwidths and lower signal attenuation.
(c)   transmission lines are larger than waveguides. (d)   transmission lines support only TEM mode.   12.2 An evanescent mode occurs when (a)   a wave is attenuated rather than propagated. (b)   the propagation constant is purely imaginary. (c)   m 5 0 5 n so that all field components vanish. (d)   the wave frequency is the same as the cutoff frequency.
12.3 The dominant mode for rectangular waveguides is (a) TE11 (c) TE101 (b) TM11 (d) TE10   12.4 The TM10 mode can exist in a rectangular waveguide. (a) True (b) False   12.5 For TE30 mode, which of the following field components exist? (a)   Ex (b)   Ey (c)   Ez (d)   Hx (e)   Hy   12.6 If in a rectangular waveguide for which a 5 2b, the cutoff frequency for TE02 mode is 12 GHz, the cutoff frequency for TM11 mode is (a)   3 GHz (b)   3!5 GHz (c)   12 GHz (d)   6!5 GHz (e)   None of the above   12.7 If a tunnel is 4 m by 7 m in cross section, a car in the tunnel will not receive an AM radio signal 1 e.g., f 5 10 MHz 2 . (a) True (b) False   12.8 When the electric field is at its maximum value, the magnetic energy of a cavity is (a)   at its maximum value (b) at !2 of its maximum value 1 (c) at of its maximum value !2 (d)   at 1/2 of its maximum value (e) zero 12_Sadiku_Ch12.indd 682 25/09/17 5:28 PM Problems   683   12.9 Which of these modes does not exist in a rectangular resonant cavity? (a) TE110 (b) TE011 (c) TM110 (d) TM111 12.10 How many degenerate dominant modes exist in a rectangular resonant cavity for which a 5 b 5 c? (a) 0 (d) 5 (b) 2 (e)   (c) 3 Answers: 12.1c, 12.2a,d, 12.3d, 12.4b, 12.5b,d, 12.6b, 12.7a, 12.8e, 12.9a, 12.10c. PROBLEMS Sections 12.3 and 12.4—TM and TE Modes   12.1 An air-filled rectangular waveguide has a cross section of 6 cm 3 4 cm.
(a)   Calculate the cutoff frequency of the dominant mode. (b) Determine how many modes are passed at three times cutoff frequency of dominant mode.   12.2 A square waveguide (a by a) can propagate only TE10 and not TE11 or higher modes. In order to achieve this, what must be the size a?   12.3 A rectangular waveguide (2.28 cm 1.01 cm) is filled with polyethylene 1 er 5 2.25 2 . Calculate the cutoff frequencies for the following modes: TE01, TE10, TE11, TE02, TE22, TM11, TM12, TM21.
Assume that polyethylene is lossless.   12.4 An air-filled waveguide has a cross section of 2.4 cm 1.2 cm. A microwave signal of 12 GHz propagates down the guide. (a) Calculate the cutoff frequencies of TE10, TE01, TE20, and TE02 modes. (b) Which modes will propagate?   12.5 Design a rectangular waveguide with an aspect ratio of 3 to 1 for use in the K-band (18–26.5 GHz). Assume that the guide is air filled.   12.6 A tunnel is modeled as an air-filled metallic rectangular waveguide with dimensions a 5 8 m and b 5 16 m. Determine whether the tunnel will pass (a) a 1.5 MHz AM broadcast signal, (b) a 120 MHz FM broadcast signal.   12.7 In an air-filled rectangular waveguide, the cutoff frequency of a TE10 mode is 5 GHz, whereas that of a TE01 mode is 12 GHz. Calculate (a)   The dimensions of the guide (b)   The cutoff frequencies of the next three higher TE modes (c) The cutoff frequency for TE11 mode if the guide is filled with a lossless material having er 5 2.25 and mr 5 1   12.8 An air-filled rectangular waveguide operates at 40 GHz. If the cutoff frequency of the TE12 mode is 25 GHz, calculate the wavelength, phase constant, phase velocity, and intrinsic impedance of this mode. 12_Sadiku_Ch12.indd 683 25/09/17 5:28 PM 684   CHAPTER 12   WAVEGUIDES 12.9 An air-filled rectangular waveguide of dimension 5 cm 3 3 cm operates on the TE10 mode at a frequency of 12.5 GHz. Find the phase constant, phase velocity, and the wave impedance. 12.10 An air-filled waveguide has a = 2b = 4 cm and operates at the TE10 mode. Determine fc, b, and l at 24 MHz. 12.11 An air-filled hollow rectangular waveguide is 150 m long and is capped at the end with a metal plate. If a short pulse of frequency 7.2 GHz is introduced into the input end of the guide, how long will it take the pulse to return to the input end? Assume that the cutoff frequency of the guide is 6.5 GHz. 12.12 A section of an air-filled rectangular waveguide (a 5 2.4 cm, b 5 1.2 cm) operates in the TE10 mode. The operating frequency is 25% higher than the cutoff frequency. Determine fc, f, and h. 12.13 A K-band waveguide (1.067 cm 3 0.533 cm) is filled by a dielectric material with er 5 6.8. If it operates in the dominant TE10 mode at 6 GHz, determine the following: (a)   The cutoff frequency (b)   The phase velocity (c)   The waveguide wavelength 12.14 Show that the attenuation due to a waveguide operating below cutoff is a 5 2p !me fc f 2 12a b Å fc 12.15 An air-filled rectangular waveguide has cross-sectional dimensions a 5 6 cm and b 5 3 cm. Given that Ez 5 5 sin a 3py 2px b sin a b cos 1 1012t 2 bz 2 V/m a b calculate the intrinsic impedance of this mode and the average power flow in the guide. 12.16 In an air-filled rectangular waveguide, a TE mode operating at 6 GHz has Ey 5 5 sin a py 2px b cos a b sin 1 vt 2 12z 2 V/m a b Determine (a) the mode of operation, (b) the cutoff frequency, (c) the intrinsic impedance, (d) Hx. 12.17 In an air-filled rectangular waveguide with a 5 2.286 cm and b 5 1.016 cm, the y-component of the TE mode is given by 12_Sadiku_Ch12.indd 684 Ey 5 sin a 3py 2px b cos a b sin 1 10p 3 1010t 2 bz 2 V/m a b Find (a) the operating mode, (b) the propagation constant g, (c) the intrinsic impedance h. 25/09/17 5:28 PM Problems   685 FIGURE 12.21   For Problem 12.18. 12.18 A rectangular waveguide with cross sections shown in Figure 12.21 has dielectric discontinuity. Calculate the standing wave ratio if the guide operates at 8 GHz in the dominant mode.
12.19 Analysis of a circular waveguide requires solution of the scalar Helmholtz equation in cylindrical coordinates, namely, =2Ezs 1 k2Ezs 5 0 or 'Ezs '2Ezs 1 ' 1 '2Ezs 1 1 k2Ezs 5 0 ar b1 2 2 r 'r 'r r 'f 'z2 By assuming the product solution Ezs 1 r, f, z 2 5 R 1 r 2 F 1 f 2 Z 1 z 2 show that the separated equations are Zs 2 k2z Z 5 0 Fs 1 k2f F 5 0 where r2Rs 1 rRr 1 1 k2r r2 2 k2f 2 R 5 0 k2r 5 k2 1 k2z 12.20 For an air-filled waveguide, use MATLAB to plot up and ug for 10 GHz , f , 100 GHz. Assume that fc 5 8 GHz. Section 12.5—Wave Propagation in the Guide 12.21 Determine the values of , p, g, and TE10 for a 7.2 cm 3 3.4 cm rectangular waveguide operating at 6.2 GHz (a) if the waveguide is air filled, (b) if the waveguide is filled with a material having « 5 2.25 «o, 5 o, 5 0. 12_Sadiku_Ch12.indd 685 25/09/17 5:28 PM 686   CHAPTER 12   WAVEGUIDES 12.22 Consider a WR284 waveguide (a 5 7.214 cm, b 5 3.404 cm). If it is filled with polyethylene («r 5 2.5) and operates at 4 GHz, determine up and ug. 12.23 In a certain medium, the phase velocity is up 5 c c lo l d 2 where c 5 3 3 108 m/s. Obtain the expression for the group velocity. 12.24 The group velocity of a dielectric-filled rectangular waveguide operating at 12 GHz is c/4. When the frequency becomes 15 GHz, the group velocity is c/3 for the same mode. Determine fc and «r. 12.25 A square waveguide operates at 4.5 GHz in the dominant mode. If the group velocity is determined to be 1.8 108 m/s, calculate the largest dimension of the waveguide. Assume that the waveguide is filled with oil (« 2.2«o). 12.26 A rectangular waveguide is filled with polyethylene 1 e 5 2.25eo 2 and operates at 24 GHz. If the cutoff frequency of a certain TE mode is 16 GHz, find the group velocity and intrinsic impedance of the mode.
Section 12.6—Power Transmission and Attenuation 12.27 The average power density is given by ave 5 Show that for a rectangular waveguide operating in the TE10 mode, ave 5 12.28 For TE01 mode, Exs 5 1 Re 3 Es 3 H*s 4 2 Find ave and Pave. jvmp bh2 vmba2 2 2 px Ho sin a baz a 2p2 Ho sin 1 py/b 2 e2gz, Eys 5 0 12.29 A 1 cm 3 2 cm waveguide is made of copper 1 sc 5 5.8 3 107 S/m 2 and filled with a dielectric material for which e 5 2.6eo, m 5 mo, sd 5 1024 S/m. If the guide operates at 12 GHz, evaluate ac and ad for (a) TE10 and (b) TM11. 12.30 A 4 cm square waveguide is filled with a dielectric with complex permittivity ec 5 16eo 1 1 2 j1024 2 and is excited with the TM21 mode. If the waveguide operates at 10% above the cutoff frequency, calculate attenuation ad. How far can the wave travel down the guide before its magnitude is reduced by 20%? 12.31 If the walls of the square waveguide in Problem 12.30 are made of brass 1 sc 5 1.5 3 107 S/m 2 , find ac and the distance over which the wave is attenuated by 30%.
12_Sadiku_Ch12.indd 686 25/09/17 5:28 PM Problems   687 12.32 An air-filled waveguide with dimensions a = 6 cm and b = 3 cm is excited at the level of |E| = 2.2 kV/m. If the dominant mode propagates at 4 GHz, determine the power transmitted. 12.33 A rectangular waveguide with a 5 2b 5 4.8 cm is filled with Teflon (er 5 2.11, loss tangent of 3 3 1024). Assume that the walls of the waveguide are coated with gold 1 sc 5 4.1 3 107 S/m 2 and that a TE10 wave at 4 GHz propagates down the waveguide. Find (a) ad, (b) ac. 12.34 Use MATLAB to plot the attenuation for the TE10 mode a of waveguide with copper walls as a function of frequency. Do this for frequencies above cutoff. Keep in mind that Rs varies with frequency. Take a 2b 1 cm, fc 10 GHz, and assume that the waveguide is filled with a dielectric having er 5 2.25.
12.35 An air-filled X-band rectangular waveguide has dimensions a 5 2.286 cm and b 5 1.016 cm. If the waveguide has copper walls (e 5 eo,m 5 mo,s 5 5.8 3 107 S/m), find the attenuation in dB/m due to the wall loss when the dominant mode is propagating at 8.4 GHz. 12.36 A rectangular, air-filled waveguide has dimensions a 5 3.8 cm and b 5 1.6 cm, and walls are made of copper. For the dominant mode at f 5 10 GHz, calculate (a) the group velocity (b) the attenuation dB/m 12.37 A rectangular waveguide has transverse dimensions a 5 2.5 cm and b 5 1.5 cm and operates at 7.5 GHz in the dominant mode.
If the waveguide is filled with a lossy dielectric material with er 5 2.26, mr 5 1, s 5 10 24 S/m and the walls are made of brass (so 5 1.1 3 107 S/m), calculate b, ad, ac, up, ug, and lc.
12.38 A rectangular brass 1 sc 5 1.37 3 107 S/m 2 waveguide with dimensions a 5 2.25 cm and b 5 1.5 cm operates in the dominant mode at frequency 5 GHz. If the waveguide is filled with Teflon 1 mr 5 1, er 5 2.11, s . 0 2 , determine (a) the cutoff frequency for the dominant mode, (b) the attenuation constant due to the loss in the guide walls. 12.39 For a square waveguide, show that attenuation ac is minimum for the TE10 mode when f 5 2.962fc. Section 12.8—Waveguide Resonators 12.40 Show that for propagation from the TE mode to z in a rectangular cavity, Eys 5 2 Find Hxs. npy jvm mp ppz mpx bHo sin a b cos a a b b sin a c a a b h2 12.41 For a rectangular cavity, show that Hxs 5 12_Sadiku_Ch12.indd 687 npy jve np ppz mpx b cos a a bEo sin a b b cos a c a b h2 b for propagation from the TM mode to z. Determine Eys. 15/11/17 5:19 PM 688   CHAPTER 12   WAVEGUIDES 12.42 In a rectangular resonant cavity, identify the dominant made when (a)   a , b , c (b)   a . b . c (c)   a 5 c . b 12.43 A rectangular cavity has dimensions a 5 3 cm, b 54 cm, and c 5 6 cm. The cavity is filled with a lossless dielectric with «r 5 4.6. Calculate: (a) the resonant frequency of the dominant mode, (b) the quality factor.
12.44 An air-filled waveguide has dimension 3 cm 2.5 cm. The guide is 4 cm long. It is shorted at each end, forming a cavity. Determine the lowest three resonance frequency. 12.45 A rectangular cavity has dimension a 5 1 cm, b 5 2 cm, c 5 3 cm. If it is filled with polyethylene (e 5 2.5eo), find the first five resonant frequencies. 12.46 For a cubical cavity (a 5 b 5 c) in the TE101 mode, show that Q5 a 3d where d is the skin depth. 12.47 An air-filled cavity has dimensions 20 mm 3 8 mm 3 10 mm.
If the walls are silver-plated, find (a) dominant resonant frequency, (b) Q for the TE101mode. 12.48 Design an air-filled cubical cavity to have its dominant resonant frequency at 3 GHz. 12.49 Design a cubical resonant cavity with a dominant frequency of 5.6 GHz. Assume that the cavity is filled with (a) air, (b) Teflon having «r = 2.05. 12.50 An air-filled cubical cavity of size 10 cm has E 5 200 sin(30x) sin(30y) cos(6 3 109t)az V/m (a) Find H. (b) Show that E H 5 0. 12.51 (a) Determine the size of an air-filled cubical cavity made of copper that it will give a dominant resonant frequency of 12 GHz. (b) Calculate the quality factor Q at that frequency. 12.52 Shielded rooms act as resonant cavities. We must avoid operating equipment in any such room at a resonant frequency of the cavity. If an air-filled shielded room has the dimensions 10.2 m by 8.7 m by 3.6 m, find all resonant frequencies below 50 MHz. Section 12.9—Application Note—Optical Fiber 12.53 The speed of light in a given medium is measured as 2.1 3 108 m/s. Find its refractive index. 12.54 Determine the numerical aperture of an optical fiber which has n1 5 1.51 and n2 5 1.45. 12_Sadiku_Ch12.indd 688 25/09/17 5:28 PM Problems   689 12.55 A glass fiber has a core diameter of 50 mm, a core refractive index of 1.62, and a cladding with a refractive index of 1.604. If light having a wavelength of 1300 nm is used, find (a) The numerical aperture (b)   The acceptance angle (c)   The number of transmission modes 12.56 A silicon fiber has a core index of 1.48 and a cladding index of 1.46. If the core radius is 5 mm, find the number of propagating modes for the source wavelength of 1300 nm. 12.57 An optical fiber with an attenuation of 0.4 dB/km is 5 km long. The fiber has n1 5 1.53, n2 5 1.45, and a diameter of 50 mm. Find: (a)   The maximum angle at which rays will enter the fiber and be trapped (b)   The percentage of input power received 12.58 A laser diode is capable of coupling 10 mW into a fiber with attenuation of 0.5 dB/km. If the fiber is 850 m long, calculate the power received at the end of the fiber.
12.59 Attenuation a10 in Chapter 10 is in nepers per meter (Np/m), whereas attenuation a12 in this chapter is in decibels per kilometer (dB/km).
What is the relationship between the two? 12.60 A power of 1.25 mW is launched into an optical fiber that has a 0.4 dB/km attenuation. Determine the fiber length such that a power of 1 mW is received at the other end of the fiber. 12.61 A lightwave system uses a 30 km fiber link with a loss of 0.4 dB/km. If the system requires at least 0.2 mW at the receiver, calculate the minimum power that must be launched into the fiber. 12.62 (a) Discuss the advantages derived from using a fiber-optic cable. 12_Sadiku_Ch12.indd 689 (b)   What is pulse dispersion? 25/09/17 5:28 PM SMART ANTENNAS Just as we hear better with two ears than with one, a communications system with two or more antennas can outperform a system with a single antenna. Smart antennas (also known as adaptive antennas) basically consist of an antenna array combined with signal processing in both time and space. They are different from common antennas in that they have adaptive (nonfixed) lobe patterns. They exploit the fact that interferers and users rarely have the same location. There are basically two types of smart antenna: switched beam (a finite number of fixed, predefined patterns) and adaptive array (an infinite number of patterns that are adjusted in real time). The switched-beam type is the simplest technique. It is simply a controlled RF switch connected to many fixed antennas. It employs a grid of beams and usually chooses the beam that gives the best signal-to-noise ratio. It is easily deployed but has low gain between beams. The wireless personal communications market, especially the cellular telephone segment, has been growing exponentially for years and will continue to grow. But there are some challenges along the way.
These include quality of service, traffic capacity, and cost of service. The smart antenna technology is a promising approach to these problems, offering increased capacity, extended range, better link quality, and longer battery life in mobile units. Smart antenna systems enable operators of PCs, cellular phones, wireless local-area networks, and wireless local-loop networks to realize significant increases in channel capacity, signal quality, spectrum efficiency, and coverage. Although smart antennas make wireless systems more complex, they provide real improvements in areas that are critical for making wireless service more universal and reliable. It has been rightly argued that the performance requirements of future wireless systems cannot be met without the use of smart antennas. To know about smart antennas, one should take a class on antennas to acquire general background information.
Unfortunately, most electrical engineering departments do not offer courses on antennas at the undergraduate level. Source: Adapted with permission from M. Chryssomallis, “Smart a ntennas,” IEEE Antennas and Propagation Magazine, vol. 42, no. 3, June 2000, pp. 129–136. 13_Sadiku_Ch13.indd 690 17/10/17 12:05 PM CHAPTER 13 ANTENNAS A committee is a group of the unwilling, chosen from the unfit, to do the unnecessary. —ANONYMOUS 13.1 INTRODUCTION Up until now, we have not asked ourselves how EM waves are produced. Recall that electric charges are the sources of EM fields. If the sources are time varying, EM waves propagate away from the sources and radiation is said to have taken place. Radiation may be thought of as the process of transmitting electric energy. The radiation or launching of the waves into space is efficiently accomplished with the aid of conducting or dielectric structures called antennas. Theoretically, any structure can radiate EM waves, but not all structures can serve as efficient radiation mechanisms. An antenna may also be viewed as a transducer used in matching the transmission line or as a waveguide (used in guiding the wave to be launched) to the surrounding medium, or vice versa. Figure 13.1 shows how an antenna is used to accomplish a match between the line or guide and the medium. The antenna is needed for two main reasons: for efficient radiation and for matching wave impedances to minimize reflection. The antenna uses voltage and current from the transmission line (or the EM fields from the waveguide) to launch an EM wave into the medium. An antenna may be used for either transmitting or receiving EM energy. Typical antennas are illustrated in Figure 13.2. The dipole antenna in Figure 13.2(a) consists of two straight wires lying along the same axis. The loop antenna, exemplified in Figure 13.2(b), consists of one or more turns of wire. The helical antenna in Figure 13.2(c) consists of a wire in the form of a helix backed by a ground plane. Antennas in Figure 13.2(a–c) are called wire antennas; they are used in automobiles, buildings, aircraft, ships, and so on.
The horn antenna in Figure 13.2(d), an example of an aperture antenna, is a tapered section of waveguide providing a transition between a waveguide and the surroundings. Since it is conveniently flush mounted, it is useful in various applications such as aircraft communications. The parabolic dish reflector in Figure 13.2(e) utilizes the fact that EM waves are reflected by a conducting sheet. When used as a transmitting antenna, a feed antenna such as a dipole or horn is placed at the focal point. The radiation from the source is reflected by the dish (acting like a mirror), and a parallel beam results. Parabolic dish antennas are used in communications, radar, and astronomy. 691 13_Sadiku_Ch13.indd 691 17/10/17 12:05 PM 692   CHAPTER 13   ANTENNAS FIGURE 13.1   An antenna as a matching device between the guiding structure and the surrounding medium.
D L H P P FIGURE 13.2   Typical antennas. 13_Sadiku_Ch13.indd 692 17/10/17 12:05 PM 13.2 Hertzian Dipole   693 The phenomenon of radiation is rather complicated, so we have intentionally delayed its discussion until this chapter. We will not attempt a broad coverage of antenna theory; our discussion will be limited to the basic types of antennas such as the Hertzian dipole, the half-wave dipole, the quarter-wave monopole, and the small loop. For each of these types, we will determine the radiation fields by taking the following steps: 1. Select an appropriate coordinate system and determine the magnetic vector potential A. 2. Find H from B 5 mH 5 = 3 A. 'E 3. Determine E from = 3 H 5 e or E 5 hH 3 ak assuming a lossless medium 't 1s 5 02.
4. Find the far field and determine the time-average power radiated by using Prad 5 3 ave # dS where ave 5 1 Re 1 Es 3 H*s 2 2 Note that Prad throughout this chapter is the same as Pave in eq. (10.80). We will consider antenna arrays which produce particular directional properties of the radiated field.
We will derive the Friis transmission equation for coupling between two antennas. Finally, we will consider the problem of electromagnetic interference (EMI). 13.2   HERTZIAN DIPOLE By “Hertzian dipole” we mean an infinitesimal current element I dl, where dl # l /10. Although such a current element does not exist in real life, it serves as a building block from which the field of a practical antenna can be calculated by integration. Consider the Hertzian dipole shown in Figure 13.3. We assume that it is located at the origin of a coordinate system and that it carries a uniform current (constant throughout the FIGURE 13.3   A Hertzian dipole carrying current I 5 Io cos vt .
13_Sadiku_Ch13.indd 693 17/10/17 12:05 PM 694   CHAPTER 13   ANTENNAS dipole), I 5 Io cos vt. From eq. (9.54), the retarded magnetic vector potential at the field point P, due to the dipole, is given by A5 where [I] is the retarded current given by m 3 I 4 dl a (13.1) 4p r z r 3 I 4 5 Io cos v at 2 b 5 Io cos 1 vt 2 br 2 u 5 Re 3 Ioe j1vt2br2 4 (13.2) where b 5 v/u 5 2p/l, and u 5 1/ !me. The current is said to be retarded at point P because there is a propagation time delay r/u or phase delay br from O to P.
By substituting eq. (13.2) into eq. (13.1), we may write A in phasor form as Azs 5 mIodl 2jbr e (13.3) 4pr Transforming this vector from Cartesian to spherical coordinates yields As 5 1 Ars, Aus, Afs 2 where Ars 5 Azs cos u, Aus 5 2Azs sin u, Afs 5 0(13.4) Since Bs 5 mHs 5 = 3 As, we obtain the H field as jb Iodl 1 sin u c 1 2 d e2jbr(13.5a) r 4p r Hfs 5 Hrs 5 0 5 Hus (13.5b) We find the E field by using = 3 H 5 e 'E/'t or = 3 Hs 5 jveEs, j hIodl 1 cos u c 2 2 3 d e2jbr 2p br r Ers 5 Eus 5 Efs 5 0 (13.6a) jb j hIodl 1 sin u c 1 2 2 3 d e2jbr (13.6b) r 4p br r (13.6c) where h5 13_Sadiku_Ch13.indd 694 m b 5 ve Å e 17/10/17 12:05 PM 13.2 Hertzian Dipole   695 A close observation of the field equations in eqs. (13.5) and (13.6) reveals that we have terms varying as 1/r3, 1/r2, and 1/r. The 1/r3 term is called the electrostatic field, since it corresponds to the field of an electric dipole [see eq. (4.82)]. This term dominates other terms in a region very close to the Hertzian dipole.
The 1/r2 term is called the inductive field, and it is predictable from the Biot–Savart law [see eq. (7.3)]. The term is important only at near field, that is, at distances close to the current element. The 1/r term is called the far field or radiation field because it is the only term that remains at the far zone, that is, at a point very far from the current element. Here, we are mainly concerned with the far field or radiation zone 1 br W 1 or 2pr W l 2 , where the terms in 1/r3 and 1/r2 can be neglected in favor of the 1/r term.
Also note that near-zone and far-zone fields are determined, respectively, to be the inequalities br V 1 and br W 1. More specifically, we define the boundary between the near and the far zones by the value of r given by r5 2d2 (13.7) l where d is the largest dimension of the antenna. Thus at far field, Hfs 5 jIobdl 4pr sin u e2jbr, Eus 5 h Hfs(13.8a) Hrs 5 Hus 5 Ers 5 Efs 5 0(13.8b) Note from eq. (13.8a) that the radiation terms of Hfs and Eus are in time phase and orthogonal just as the fields of a uniform plane wave. The time-average power density is obtained as 1 1 Re 1 Es 3 H*s 2 5 Re 1 Eus H*fsar 2 2 2 1 5 h 0 Hfs 0 2 ar 2 ave 5 (13.9) Substituting eq. (13.8a) into eq. (13.9) yields the time-average radiated power as Prad 5 3 ave # dS 53 2p f50 5 But c3 I2ohb2 dl2 2 2 2 2 sin u r sin u du d df u50 32p r p (13.10) p I2ohb2 dl2 3 2p 3 sin u du 32p2 0 3 2 3 sin u du 5 3 1 1 2 cos u 2 d 1 2cos u 2 p 0 p 0 p cos3 u 4 5 2 cos u ` 5 3 3 0 13_Sadiku_Ch13.indd 695 17/10/17 12:05 PM 696   CHAPTER 13   ANTENNAS and b2 5 4p2 /l2. Hence eq. (13.10) becomes Prad 5 I2o ph dl 2 c d (13.11a) 3 l If free space is the medium of propagation, h 5 120p and Prad 5 40p2 c dl 2 2 d Io(13.11b) l This power is equivalent to the power dissipated in a fictitious resistance Rrad by current I 5 Io cos vt; that is, Prad 5 I2rms Rrad or Prad 5 1 2 Io Rrad(13.12) 2 where Irms is the root-mean-square value of I.
From eqs. (13.11) and (13.12), we obtain Rrad 5 2Prad (13.13a) I2o or Rrad 5 80p2 c dl 2 d (13.13b) l The resistance Rrad is a characteristic property of the Hertzian dipole antenna and is called its radiation resistance. From eqs. (13.12) and (13.13), we observe antennas with large radiation resistances are required to deliver large amounts of power to space. For example, if dl 5 l/20, Rrad . 2 V, which is small in that it can deliver relatively small amounts of power. It should be noted that Rrad in eq. (13.13b) is for a Hertzian dipole in free space. If the dipole is in a different, lossless medium, h 5 "m/e is substituted in eq. (13.11a) and Rrad is determined by using eq. (13.13a). Note that the Hertzian dipole is assumed to be infinitesimally small 1 b dl V 1 or dl # l/10 2 . Consequently, its radiation resistance is very small, and it is in practice difficult to match it with a real transmission line.
We have also assumed that the dipole has a uniform current; this requires that the current be nonzero at the end points of the dipole. This is practically impossible because the surrounding medium is not conducting. However, our analysis will serve as a useful, valid approximation for an antenna with dl # l/10. A more practical (and perhaps the most important) antenna is the half-wave dipole, considered in the next section. 13_Sadiku_Ch13.indd 696 17/10/17 12:05 PM 13.3 Half-Wave Dipole Antenna   697 13.3   HALF-WAVE DIPOLE ANTENNA The half-wave dipole derives its name from the fact that its length is half a wavelength 1 , 5 l/2 2 . As shown in Figure 13.4(a), it consists of a thin wire fed or excited at the midpoint by a voltage source connected to the antenna via a transmission line (e.g., a two-wire line). The field due to the dipole can be easily obtained if we consider it as consisting of a chain of Hertzian dipoles.
The magnetic vector potential at P due to a differential length dl 1 5 dz 2 of the dipole [see Figure. 13.4(b)] carrying a phasor current Is 5 Io cos bz is dAzs 5 mIo cos bz dz 2jbrr e (13.14) 4prr Notice that to obtain eq. (13.14), we have assumed a sinusoidal current distribution for two reasons. First, the sinusoidal current assumption is based on the transmission line model of the dipole. Second, the current must vanish at the ends of the dipole. A triangular current distribution is also possible (see Problem 13.5) but would give less accurate results. The actual current distribution on the antenna is not precisely known. It is determined by solving Maxwell’s equations subject to the boundary conditions on the antenna, but FIGURE 13.4   (a) A half-wave dipole. (b) Geometry for calculating the fields. 13_Sadiku_Ch13.indd 697 17/10/17 12:05 PM 698   CHAPTER 13   ANTENNAS the procedure is mathematically complex. However, the sinusoidal current assumption approximates the distribution obtained by solving the boundary-value problem and is commonly used in antenna theory. If r W ,, as explained in Section 4.9 on electric dipoles (see Figure 4.20), then r 2 rr 5 z cos u or rr 5 r 2 z cos u Thus we may substitute rr . r in the denominator of eq. (13.14), where the magnitude of the distance is needed. For the phase term in the numerator of eq. (13.14), the difference between br and br is significant, so we replace r by r 2 z cos u and not r. In other words, we maintain the cosine term in the exponent while neglecting it in the denominator because the exponent involves the phase constant while the denominator does not. Thus, mIo l/4 2jb1r2z cos u 2 Azs 5 e cos bz dz 4pr 32l/4 mIo 2jbr l/4 jbz cos u 5 e e cos bz dz 3 4pr 2l/4 (13.15) From the integral tables of Appendix A.8, az 3 e cos bz dz 5 Applying this to eq. (13.15) gives Azs 5 eaz 1 a cos bz 1 b sin bz 2 1c a 2 1 b2 mIoe2jbre jbz cos u 1 jb cos u cos bz 1 b sin bz 2 l/4 (13.16) ` 4pr 2b2 b2 cos2 u 1 b2 2l/4 Since b 5 2p/l or b l/4 5 p/2 and 2cos2 u 1 1 5 sin2 u, eq. (13.16) becomes Azs 5 mIoe2jbr 3 e j1p/22 cos u 1 0 1 b 2 2 e2j1p/22 cos u 1 0 2 b 2 4 (13.17) 4prb2 sin2 u Using the identity e jx 1 e2jx 5 2 cos x, we obtain Azs 5 p cos ub 2 (13.18) 2prb sin2 u mIoe2jbr cos a We use eq. (13.4) in conjunction with the fact that Bs 5 mHs 5 = 3 As and = 3 Hs 5jveEs to obtain the magnetic and electric fields at far zone (discarding the 1/r3 and 1/r2 terms) as 13_Sadiku_Ch13.indd 698 Hfs 5 p cos ub 2 , 2pr sin u jIoe2jbr cos a Eus 5 hHfs(13.19) 17/10/17 12:05 PM 13.3 Half-Wave Dipole Antenna   699 Notice again that the radiation terms Hfs and Es are in time phase and orthogonal. By using eqs. (13.9) and (13.19), we obtain the time-average power density as 1 h 0 Hfs 0 2 ar 2 p hI2o cos2 a cos ub 2 5 ar 2 2 8p r sin2 u 3ave 5 (13.20) The time-average radiated power can be determined as Prad 5 3 ave # dS S 53 ≥3 2p f50 5 hI2o 2 8p p u50 2p 3 8p p u50 2p 3 p 0 5 30 I2o 3 p 0 p cos ub 2 r2 sin u du¥ df 8p2r2 sin2 u hI2o cos2 a p cos ub 2 du sin u cos2 a (13.21) p cos ub 2 du sin u cos2 a where h 5 120p has been substituted assuming free space as the medium of propagation. 3 p/2 0 p 2 p cos ub cos ub p cos a 2 2 du 5 3 du sin u sin u p/2 cos2 a This is easily illustrated by a rough sketch of the variation of the integrand with . Hence Prad 5 60I2o 3 p/2 0 cos2 a p cos ub du 2 (13.22) sin u Changing variables, u 5 cos u, and using partial fraction reduces eq. (13.22) to 1 cos2 pu 2 5 60I2o 3 du 2 u2 0 1 Prad 13_Sadiku_Ch13.indd 699 2 1 2 1 pu pu 1 cos 1 cos 2 2 § 2 £ 5 30Io 3 du 1 3 du 1 1 u 1 2 u 0 0 (13.23) 17/10/17 12:05 PM 700   CHAPTER 13   ANTENNAS Replacing 1 1 u with v in the first integrand and 1 2 u with v in the second results in 1 2 2 1 sin2 pv pv 2 sin 2 2 § 2 £ 5 30Io 3 dv 1 3 dv v v 0 1 1 Prad 2 1 pv 2 sin 2 2 5 30Io 3 dv v 0 (13.24) Changing variables, w 5 pv, yields Prad 5 30I2o 3 2p 5 15I2o 3 2p 5 15I2o 3 2p 1 w 2 dw w sin2 0 0 0 1 1 2 cos w 2 dw w c (13.25) w w3 w5 w7 . . . 2 1 2 1 d dw 2! 4! 6! 8! w2 w4 w6 w8 . . . 1 2 1 2 . Integrating eq. (13.25) term by term and 2! 4! 6! 8! evaluating at the limit leads to since cos w 5 1 2 Prad 5 15I2o c . 36.56 1 2p 2 2 1 2p 2 4 1 2p 2 6 1 2p 2 8 2 1 2 1 .
. .d 2 1 2! 2 4 1 4! 2 6 1 6! 2 8 1 8! 2 I2o (13.26) The radiation resistance Rrad for the half-wave dipole antenna is readily obtained from eqs. (13.12) and (13.26) as Rrad 5 2Prad . 73 V(13.27) I2o Note the significant increase in the radiation resistance of the half-wave dipole over that of the Hertzian dipole. Thus the half-wave dipole is capable of delivering greater amounts of power to space than the Hertzian dipole. The total input impedance Zin of the antenna is the impedance seen at the terminals of the antenna and is given by 13_Sadiku_Ch13.indd 700 Zin 5 Rin 1 jXin(13.28) 17/10/17 12:05 PM 13.4 Quarter-Wave Monopole Antenna   701 where Rin 5 Rrad for a lossless antenna. Deriving the value of the reactance Xin involves a complicated procedure beyond the scope of this text. It is found that Xin 5 42.5 V, so Zin 5 73 1 j42.5 V for a dipole length , 5 l/2. The inductive reactance drops rapidly to zero as the length of the dipole is slightly reduced.
For , 5 0.485 l, the dipole is resonant, with Xin 5 0. Thus in practice, a l/2 dipole is designed such that Xin approaches zero and Zin . 73 V. This value of the radiation resistance of the l/2 dipole antenna is the reason for the standard 75 V coaxial cable. Also, the value is easy to match to transmission lines. These factors in addition to the resonance property are the reasons for the dipole antenna’s popularity and its extensive use. 13.4   QUARTER-WAVE MONOPOLE ANTENNA Basically, the quarter-wave monopole antenna consists of half of a half-wave dipole antenna located on a conducting ground plane, as in Figure 13.5. The monopole antenna is perpendicular to the plane, which is usually assumed to be infinite and perfectly conducting.
It is fed by a coaxial cable connected to its base. Using image theory of Section 6.6, we replace the infinite, perfectly conducting ground plane with the image of the monopole. The field produced in the region above the ground plane due to the l/4 monopole with its image is the same as the field due to a l/2 wave dipole. Thus eq. (13.19) holds for the

l/4 monopole. However, the integration in eq. (13.21) is only over the hemispherical surface above the ground plane (i.e., 0 # u # p/2) because the monopole radiates only through that surface. Hence, the monopole radiates only half as much power as the dipole with the same current. Thus for a l/4 monopole, Prad . 18.28 I2o(13.29) and Rrad 5 2Prad I2o or Rrad . 36.5 V(13.30) By the same token, the total input impedance for a l/4 monopole is Zin 5 36.5 1 j21.25 V. FIGURE 13.5 The monopole antenna. 13_Sadiku_Ch13.indd 701 17/10/17 12:05 PM 702 CHAPTER 13 ANTENNAS 13.5 SMALL-LOOP ANTENNA The loop antenna is of practical importance. It is used as a directional finder (or search loop) in radiation detection and as a TV antenna for ultrahigh frequencies. The term “small” implies that the dimensions (such as ro) of the loop are much smaller than l. Consider a small filamentary circular loop of radius ro carrying a uniform current, Io cos vt, as in Figure 13.6. The loop may be regarded as an elemental magnetic dipole. The magnetic vector potential at the field point P due to the loop is m 3 I 4 dl A5 C (13.31) L 4prr where 3 I 4 5 Io cos 1 vt 2 brr 2 5 Re 3 Ioe j1vt2brr2 4 . Substituting [I] into eq. (13.31), we obtain A in phasor form as As 5 mIo e2jbrr dl(13.32) 4p CL rr Evaluating this integral requires a lengthy procedure. It can be shown that for a small loop 1 ro V l 2 , r can be replaced by r in the denominator of eq. (13.32) and As has only a f-component given by Afs 5 mIoS 1 1 jbr 2 e2jbr sin u(13.33) 4pr2 where S 5 pr2o 5 loop area. For a loop with N turns, S 5 Npr2o. Using the fact that Bs 5 mHs 5 = 3 As and = 3 Hs 5 jveEs, we obtain the electric and magnetic fields from eq. (13.33) as 2jvmIoS Efs 5 Hrs 5 Hus 5 Ers 5 Eus 5 Hfs 5 0 4p jvmIoS 2ph jvmIoS 4ph sin u c cos u c sin u c jb 1 1 2 d e2jbr r r j 1 d e2jbr 2 2 br3 r (13.34a) (13.34b) jb j 1 1 2 2 3 d e2jbr (13.34c) r br r (13.34d) Comparing eqs. (13.5) and (13.6) with eq. (13.34), we observe the dual nature of the field due to an electric dipole of Figure 13.3 and the elemental magnetic dipole of Figure 13.6 (see Table 8.2 also). At far field, only the 1/r term (the radiation term) in eq. (13.34) remains. Thus at far field, 13_Sadiku_Ch13.indd 702 17/10/17 12:05 PM 13.5 Small-Loop Antenna 703 FIGURE 13.6 The small-loop antenna. Efs 5 5 vmIoS b sin u e2jbr 4pr hpIoS sin u e2jbr rl2 or Efs 5 120p2Io S sin u e2jbr, r l2 Hus 5 2 Efs h (13.35a) Ers 5 Eus 5 Hrs 5 Hfs 5 0(13.35b) where h 5 120p for free space has been assumed. Though the far-field expressions in eqs. (13.35) are obtained for a small circular loop, they can be used for a small square loop with one turn 1 S 5 a2 2 or with N turns 1 S 5 Na2 2 , or for any small loop, provided the loop dimensions are small (d # l/10, where d is the largest dimension of the loop). It is left as an exercise to show that using eqs. (13.13a) and (13.35) gives the radiation resistance of a small loop antenna as EXAMPLE 13.1 Rrad 5 320 p4S2 (13.36) l4 A magnetic field strength of 5 mA/m is required at a point on u 5 p/2, which is 2 km from an antenna in air. Neglecting ohmic loss, how much power must the antenna transmit if it is (a) A Hertzian dipole of length /25? (b) A half-wave dipole? 13_Sadiku_Ch13.indd 703 17/10/17 12:05 PM 704 CHAPTER 13 ANTENNAS (c) A quarter-wave monopole? (d) A 10-turn loop antenna of radius ro 5 l/20? Solution: (a) For a Hertzian dipole, 0 Hfs 0 5 Iob dl sin u 4pr 2p # l 2p 5 . Hence, l 25 25 where dl 5 l/25 or b dl 5 5 3 1026 or 2p 112 Io # Io 25 5 3 5 1 2 4p 2 3 10 105 Io 5 0.5 A Prad 5 40p2 c 40p2 1 0.5 2 2 dl 2 2 d Io 5 1 25 2 2 l 5 158 mW (b) For a l/2 dipole, 0 Hfs 0 5 5 3 1026 5 or Prad p cos ub 2 2pr sin u Io cos a Io # 1 2p 1 2 3 103 2 # 1 1 2 Io 5 20p mA 1 2 1 5 Io Rrad 5 1 20p 2 2 3 1026 1 73 2 2 2 5 144 mW (c) For a l/4 monopole, Io 5 20p mA as in part (b). 1 2 1 Io Rrad 5 1 20p 2 2 3 1026 1 36.56 2 2 2 5 72 mW Prad 5 13_Sadiku_Ch13.indd 704 17/10/17 12:05 PM 13.5 Small-Loop Antenna 705 (d) For a loop antenna, 0 Hus 0 5 pIo S sin u r l2 For a single turn, S 5 pr2o. For N-turn, S 5 Npr2o. Hence, 5 3 1026 5 or Io 5 pIo10p ro 2 c d 2 3 103 l 10 l 2 202 23 5 2 3 1023 2 c r d 3 10 10p p o 5 40.53 mA Rrad 5 ro 4 320 p4 S2 5 320 p6 N2 c d 4 l l 5 320 p6 3 100 c Prad 5 1 4 d 5 192.3 V 20 1 2 1 Io Rrad 5 1 40.53 2 2 3 1026 1 192.3 2 2 2 5 158 mW PRACTICE EXERCISE 13.1 A Hertzian dipole of length l/100 is located at the origin in free space and fed with a current of 0.25 sin 108t A. Determine the magnetic field at (a) r 5 l/5, u 5 30 (b) r 5 200l, u 5 60 Answer: (a) 0.2119 sin 1 108t 2 20.5° 2 af mA/m, (b) 0.2871 sin 1 108t 1 90° 2 af mA/m. EXAMPLE 13.2 13_Sadiku_Ch13.indd 705 An electric field strength of 10 mV/m is to be measured at an observation point u 5 p/2, 500 km from a half-wave (resonant) dipole antenna operating in air at 50 MHz. (a) What is the length of the dipole? (b) Calculate the current that must be fed to the antenna. (c) Find the average power radiated by the antenna. (d) If a transmission line with Zo 5 75 V is connected to the antenna, determine the standing wave ratio. 17/10/17 12:05 PM 706 CHAPTER 13 ANTENNAS Solution: c 3 3 108 5 5 6 m. f 50 3 106 l Hence, the length of the half-dipole is , 5 5 3 m. 2 (b) From eq. (13.19), (a) The wavelength l 5 0 Eus 0 5 or Io 5 5 p cos ub 2 2pr sin u hoIo cos a 0 Eus 0 2pr sin u p ho cos a cos ub 2 10 3 1026 2p 1 500 3 103 2 # 1 1 2 120p 1 1 2 5 83.33 mA (c) (d) Rrad .

73 V 1 1 Prad 5 I2o Rrad 5 1 83.33 2 2 3 1026 3 73 2 2 5 253.5 mW G5 5 5 s5 ZL 2 Zo 1 ZL 5 Zin in this case 2 ZL 1 Zo 73 1 j42.5 2 75 73 1 j42.5 1 75 42.55l92.69° 153.98l16.02° 5 22 1 j42.5 148 1 j42.5 5 0.2763l76.67° iii 1 1 0G0 1 1 0.2763 5 5 1.763 1 2 0.2763 1 2 0G0 PRACTICE EXERCISE 13.2 Repeat Example 13.2 with the dipole antenna replaced by a l/4 monopole.

Answer: (a) 1.5 m, (b) 83.33 mA, (c) 126.8 mW, (d) 2.265. 13_Sadiku_Ch13.indd 706 17/10/17 12:05 PM 13.6 Antenna Characteristics 707 13.6 ANTENNA CHARACTERISTICS Having considered the basic elementary antenna types, we now discuss some important characteristics of antennas as radiators of electromagnetic energy.

These characteristics include (a) antenna patterns, (b) radiation intensity, (c) directive gain, (d) power gain. A. Antenna Patterns When the amplitude of a specified component of the E field is plotted, it is called the field pattern or voltage pattern. When the square of the amplitude of E is plotted, it is called the power pattern. A three-dimensional plot of an antenna pattern is avoided by plotting separately the normalized 0 Es 0 versus for a constant f (this is called an E-plane pattern or vertical pattern) and the normalized 0 Es 0 versus f for u 5 p/2 (called the H-plane pattern or horizontal pattern). The normalization of 0 Es 0 is with respect to the maximum value of the 0 Es 0 so that the maximum value of the normalized 0 Es 0 is unity. For the Hertzian dipole, for example, the normalized 0 Es 0 is obtained from eq. (13.8a) as f 1 u 2 5 0 sin u 0 (13.37) which is independent of f. From eq. (13.37), we obtain the E-plane pattern as the polar plot of f() with varying from 0° to 180°.

The result is shown in Figure 13.7(a). Note that the plot is symmetric about the z-axis 1 u 5 0 2 . For the H-plane pattern, we set u 5 p/2 so that f 1 u 2 5 1, which is circle of radius 1 as shown in Figure 13.7(b). When the two FIGURE 13.7 Field patterns of the Hertzian dipole: (a) normalized E-plane or vertical pattern (f 5 constant 5 0), (b) normalized H-plane or horizontal pattern (5 p/2), (c) three-dimensional pattern. 13_Sadiku_Ch13.indd 707 17/10/17 12:05 PM 708 CHAPTER 13 ANTENNAS FIGURE 13.8 Power patterns of the Hertzian dipole: (a) (f 5 constant 5 0), (b) 5 constant 5 p/2. plots of Figure 13.7(a) and (b) are combined, we have the three-dimensional field pattern of Figure 13.7(c), which has the shape of a doughnut. A plot of the time-average power, |ave| 5 ave, for a fixed distance r is the power pattern of the antenna. It is obtained by plotting separately ave versus for constant f and ave versus f for constant . For the Hertzian dipole, the normalized power pattern is easily obtained from eqs. (13.37) or (13.9) as f 2 1 u 2 5 sin2 u(13.38) which is sketched in Figure 13.8. Notice that Figures 13.7(b) and 13.8(b) show circles because f() is independent of f and that the value of OP in Figure 13.8(a) is the relative average power for that particular . Thus, at point Q 1 u 5 45° 2 , the average power is half the maximum average power (the maximum average power is at u 5 p/2). An antenna pattern (or radiation pattern) is a three-dimensional plot of its radiation at far field. B. Radiation Intensity The radiation intensity of an antenna is defined as U 1 u, f 2 5 r2 ave 13_Sadiku_Ch13.indd 708 (13.39) 17/10/17 12:05 PM 13.6 Antenna Characteristics 709 From eq. (13.39), the total average power radiated can be expressed as Prad 5 C ave dS 5 C ave r2 sin u du df S S 5 3 U 1 u, f 2 sin u du df S 53 2p 3 (13.40) p f50 u50 U 1 u, f 2 dV where dV 5 sin u du df is the differential solid angle in steradian (sr). Hence the radiation intensity U 1 u, f 2 is measured in watts per steradian (W/sr). The average value of U 1 u, f 2 is the total radiated power divided by 4p sr; that is, Uave 5 Prad (13.41) 4p C. Directive Gain Besides the antenna patterns just described, we are often interested in measurable quantities such as gain and directivity to determine the radiation characteristics of an antenna. The directive gain Gd(,) of an antenna is a measure of the concentration of the radiated power in a particular direction (,). It may be regarded as the ability of the antenna to direct radiated power in a given direction. It is usually obtained as the ratio of radiation intensity in a given direction 1 u, f 2 to the average radiation intensity, that is, Gd 1 u, f 2 5 U 1 u, f 2 4p U 1 u, f 2 5 (13.42) Uave Prad By substituting eq. (13.39) into eq. (13.42), ave may be expressed in terms of directive gain as ave 5 Gd Prad 4pr2 (13.43) The directive gain Gd 1 u, f 2 depends on antenna pattern. For the Hertzian dipole (as well as for l/2 dipole and l/4 monopole), we notice from Figure 13.8 that ave is maximum at u 5 p/2 and minimum (zero) at u 5 0 or p. Thus the Hertzian dipole radiates power in a direction broadside to its length. For an isotropic antenna (one that radiates equally in all directions), Gd 5 1. However, such an antenna is not a physicality but an ideality.

The directivity D of an antenna is the ratio of the maximum radiation intensity to the average radiation intensity. 13_Sadiku_Ch13.indd 709 17/10/17 12:05 PM 710 CHAPTER 13 ANTENNAS Obviously, D is the maximum directive gain Gd max. Thus D5 Umax 5 Gd max(13.44a) Uave or, from eq. (13.41), D5 4p Umax (13.44b) Prad For an isotropic antenna, D 5 1; this is the smallest value D can have. For the Hertzian d ipole, For the l/2 dipole, Gd 1 u, f 2 5 1.5 sin2 u, Gd 1 u, f 2 5 where h 5 120p, Rrad .

73 V, and h f 21u 2, pRrad f1u 2 5 D 5 1.5(13.45) D 5 1.64(13.46) p cos ub 2 (13.47) sin u cos a D. Power Gain Our definition of the directive gain in eq. (13.42) does not account for the ohmic power loss P, of the antenna. This power loss P, occurs because the antenna is made of a conductor with finite conductivity. As illustrated in Figure 13.9, if Pin is the total input power to the antenna, Pin 5 P, 1 Prad 5 1 0 I 0 2 1 R, 1 Rrad 2 2 in (13.48) where Iin is the current at the input terminals and R, is the loss or ohmic resistance of the antenna. In other words, Pin is the power accepted by the antenna at its terminals during the radiation process, and Prad is the power radiated by the antenna; the difference between the two powers is P,, the power dissipated within the antenna. FIGURE 13.9 Relating Pin, P, and Prad. in 13_Sadiku_Ch13.indd 710 rad 17/10/17 12:05 PM 13.6 Antenna Characteristics 711 We define the power gain Gp 1 u, f 2 of the antenna as Gp 1 u, f 2 5 4p U 1 u, f 2 (13.49) Pin The ratio of the power gain in any specified direction (, f) to the directive gain in that direction is referred to as the radiation efficiency hr of the antenna; that is, hr 5 Prad GP 5 Gd Pin Introducing eq. (13.48) leads to hr 5 Prad Rrad 5 (13.50) Pin Rrad 1 R, For many antennas, hr is close to 100% so that GP . Gd. It is customary to express directivity and gain in decibels. Thus D 1 dB 2 5 10 log10 D(13.51a) G 1 dB 2 5 10 log10 G (13.51b) It should be mentioned at this point that the radiation patterns of an antenna are measured in the far-field region. The far-field region of an antenna is commonly taken to exist at a distance r $ rmin, where rmin 5 2d2 (13.52) l and d is the largest dimension of the antenna. For example, d 5 , for the electric dipole antenna and d 5 2ro for the small-loop antenna.

EXAMPLE 13.3 Show that the directive gain of the Hertzian dipole is and that of the half-wave dipole is Gd 1 u, f 2 5 1.5 sin2 u Gd 1 u, f 2 5 1.64 13_Sadiku_Ch13.indd 711 p cos ub 2 sin2 u cos2 a 17/10/17 12:05 PM 712 CHAPTER 13 ANTENNAS Solution: Starting from eq. (13.42) and introducing the expressions for U(, f) and Prad, we obtain Gd 1 u, f 2 5 (a) For the Hertzian dipole, Gd 1 u, f 2 5 3 2p 3 4p f 2 1 u 2 2 3 f 1 u 2 dV 4p sin2 u p sin3 u du df f50 u50 5 4p sin2 u 2p 1 4/3 2 5 1.5 sin2 u as required. (b) For the half-wave dipole, p cos ub 2 sin2 u Gd 1 u, f 2 5 2 p cos ub du df 2p p cos a 2 3 3 sin u f50 u50 4p cos2 a From eq. (13.26), the integral in the denominator gives 2p 1 1.2188 2 . Hence, Gd 1 u, f 2 5 p cos ub 2 1 # sin2 u 2p 1 1.2188 2 4p cos2 a 5 1.64 p cos ub 2 sin2 u cos2 a as required. PRACTICE EXERCISE 13.3 Calculate the directivity of (a) The Hertzian monopole (b) The quarter-wave monopole Answer: (a) 3, (b) 3.28. 13_Sadiku_Ch13.indd 712 17/10/17 12:05 PM 13.6 Antenna Characteristics 713 EXAMPLE 13.4 Determine the electric field intensity at a distance of 10 km from an antenna having a directive gain of 5 dB and radiating a total power of 20 kW. Solution: 5 5 Gd 1 dB 2 5 10 log10 Gd or 0.5 5 log10 Gd S Gd 5 100.5 5 3.162 From eq. (13.43), ave 5 GdPrad 4pr2 ave 5 0 Es 0 2 But 2h Hence, 0 Es 0 2 5 hGdPrad 120p 1 3.162 2 1 20 3 103 2 5 2pr2 2p 3 10 3 103 4 2 0 Es 0 5 0.1948 V/m PRACTICE EXERCISE 13.4 A certain antenna with an efficiency of 95% has maximum radiation intensity of 0.5 W/sr. Calculate its directivity when (a) The input power is 0.4 W (b) The radiated power is 0.3 W Answer: (a) 16.53, (b) 20.94. EXAMPLE 13.5 The radiation intensity of a certain antenna is U 1 u, f 2 5 e 2 sin u sin3 f, 0, 0 # u # p, 0 # f # p elsewhere Determine the directivity of the antenna. 13_Sadiku_Ch13.indd 713 17/10/17 12:05 PM 714 CHAPTER 13 ANTENNAS Solution: The directivity is defined as D5 Umax Uave From the given U, Umax 5 2 From eqs. (13.40) and (13.41), we get the expression for the average radiated intensity. Uave 5 1 U 1 u, f 2 dV 4p 3 1 2 sin u sin3 f sin u du df 4p 3f50 3u50 p 5 p 1 5 sin2 u du 3 sin3 f df 2p 30 0 p p 1 1 1 1 2 cos 2u 2 du 3 1 1 2 cos2 f 2 d 1 2cos f 2 3 2p 0 2 0 p 5 5 5 Hence p p 1 1 sin 2u p cos3 f 2 cos fb ` au 2 b` a 2p 2 2 3 0 0 1 p 4 1 a ba b5 2p 2 3 3 D5 2 5 6 1 1/3 2 PRACTICE EXERCISE 13.5 Evaluate the directivity of an antenna with normalized radiation intensity Answer: 2.546. 13_Sadiku_Ch13.indd 714 U 1 u, f 2 5 e sin u, 0, 0 # u # p/2, 0 # f # 2p otherwise 17/10/17 12:05 PM 13.7 Antenna Arrays 715 13.7 ANTENNA ARRAYS In many practical applications (e.g., in an AM broadcast station), it is necessary to design antennas with more energy radiated in some particular directions and less in other directions. This is tantamount to requiring that the radiation pattern be concentrated in the direction of interest. This is hardly achievable with a single antenna element. An antenna array is used to obtain greater directivity than can be obtained with a single antenna element. An antenna array is a group of radiating elements arranged to produce particular radiation characteristics.

It is practical and convenient that the array consists of identical elements, but this is not fundamentally required. We shall consider the simplest case of a two-element array and extend our results to the more complicated, general case of an N-element array. Consider an antenna consisting of two Hertzian dipoles placed in free space along the z-axis but oriented parallel to the x-axis as depicted in Figure 13.10. We assume that the dipole at 1 0, 0, d/2 2 carries current I1s 5 Iola and the one at 1 0, 0, 2d/2 2 carries current I2s 5 Iol0, where a is the phase difference between the two currents. By varying the spacing d and phase difference a, the fields from the array can be made to interfere constructively (add) in certain directions of interest and interfere destructively (cancel) in other directions. The total electric field at point P is the vector sum of the fields due to the individual elements. If P is in the far-field zone, we obtain the total electric field at P from eq. (13.8a) as Es 5 E1s 1 E2s jhbIodl e2jbr1 ja e2jbr2 5 e au1 1 cos u 2 a d ccos u 1 r1 r2 u2 4p (13.53) Note that sin in eq. (13.8a) has been replaced by cos because the element of Figure 13.3 is z-directed, whereas those in Figure 13.10 are x-directed. Since P is far from the array, u 1 . u . u 2 and au1 . au . au2. In the amplitude, we can set r1 . r < r2 but in the phase, we use r1 . r 2 d cos u(13.54a) 2 FIGURE 13.10 A two-element array.

13_Sadiku_Ch13.indd 715 17/10/17 12:05 PM 716 CHAPTER 13 ANTENNAS r2 . r 1 d cos u (13.54b) 2 Thus eq. (13.53) becomes Es 5 5 jhbIo dl 4p r jhbIo dl 4p r cos u e2jbre ja/2 3 e j1bd cos u 2/2e ja/2 1 e2j1bd cos u 2/2e2ja/2 4 au cos u e2jbre ja/22 cos c 1 1 bd cos u 1 a 2 d au 2 (13.55) Comparing this with eq. (13.8a) shows that the total field of an array is equal to the field of single element located at the origin multiplied by an array factor given by AF 5 2 cos c 1 1 bd cos u 1 a 2 d e ja/2(13.56) 2 Thus, in general, the far field due to a two-element array is given by E 1 total 2 5 1 E due to single element at origin 2 3 1 array factor 2 (13.57) Also, from eq. (13.55), note that 0 cos u 0 is the radiation pattern due to a single element, whereas the normalized array factor, 0 cos 3 1/2 1 bd cos u 1 a 2 4 0 , is the radiation pattern the array would have if the elements were isotropic. These may be regarded as “unit pattern” and “group pattern,” respectively. Thus the “resultant pattern” is the product of the unit pattern and the group pattern; that is, resultant pattern 5 unit pattern 3 group pattern(13.58) This is known as pattern multiplication, and it can be used to sketch, almost by inspection, the pattern of an array. Therefore, pattern multiplication is a useful tool in the design of an array. We should note that while the unit pattern depends on the type of elements comprising the array, the group pattern is independent of the element type as long as the spacing d, the phase difference a, and the orientation of the elements remain the same. Let us now extend the results on the two-element array to the general case of an N-element array shown in Figure 13.11. We assume that the array is linear in that the elements are spaced equally along a straight line and lie along the z-axis. Also, we assume that the array is uniform so that each element is fed with current of the same magnitude but of progressive phase shift a; that is, I1s 5 Iol0 , I2s 5 Iola , I3s 5 Iol2a , and so on. i the far field i can easilyi We are mainly interested in finding the array factor; be found from eq. (13.57) once the array factor is known.

For the uniform linear array, the array factor is the sum of the contributions by all the elements. Thus, 13_Sadiku_Ch13.indd 716 AF 5 1 1 e jc 1 e j2c 1 e j3c 1 . . . 1 e j1N212c(13.59) 17/10/17 12:05 PM 13.7 Antenna Arrays 717 FIGURE 13.11 An N-element uniform linear array. where c 5 bd cos u 1 a(13.60) In eq. (13.60), b 5 2p/l, and d and a are, respectively, the spacing, and the interelement phase shift. Notice that the right-hand side of eq. (13.59) is a geometric series of the form 1 1 x 1 x2 1 x3 1 . . . 1 xN21 5 1 2 xN (13.61) 12x Hence eq. (13.59) becomes AF 5 1 2 e jNc (13.62) 1 2 e jc which can be written as AF 5 e jNc 2 1 e jNc/2 e jNc/2 2 e2jNc/2 5 e jc 2 1 e jc/2 e jc/2 2 e2jc/2 5e j 1 N21 2 c/2 sin 1 Nc/2 2 sin 1 c/2 2 (13.63) The phase factor e j1N212c/2 would not be present if the array were centered about the origin. Neglecting this unimportant term, we have Nc 2 0 AF 0 5 4 4, c sin 2 sin 13_Sadiku_Ch13.indd 717 c 5 bd cos u 1 a(13.64) 17/10/17 12:05 PM 718 CHAPTER 13 ANTENNAS Note that this equation reduces to eq. (13.56) when 0 AF 0 is considered and N 5 2 as expected. Also, note the following: 1. Since 0 AF 0 has the maximum value of N, the normalized 0 AF 0 is obtained by dividing 0 AF 0 by N. The principal maximum occurs when c 5 0; that is, 0 5 bd cos u 1 a or cos u 5 2 a (13.65) bd 2. When 0 AF 0 5 0, 0 AF 0 has nulls (or zeros); that is, Nc 5 6kp, 2 k 5 1, 2, 3, . .

. (13.66) where k is not a multiple of N. 3. A broadside array has its maximum radiation directed normal to the axis of the array; that is, c 5 0, u 5 90º so that a 5 0. 13_Sadiku_Ch13.indd 718 | | | | | | FIGURE 13.12 Array factors for uniform linear arrays.

17/10/17 12:05 PM 13.7 Antenna Arrays 719 4. An end-fire array has its maximum radiation directed along the axis of the array, 0 2bd that is, c 5 0, u 5 c so that a 5 c p . bd These points are helpful in plotting 0 AF 0 . For N 5 2, 3, and 4, the plots of 0 AF 0 are sketched in Figure 13.12. EXAMPLE 13.6 For the two-element antenna array of Figure 13.10, sketch the normalized field pattern when the currents are: (a) Fed in phase 1 a 5 0 2 , d 5 l/2 (b) Fed 90° out of phase 1 a 5 p/2 2 , d 5 l/4 Solution: The normalized field of the array is obtained from eqs.

(13.55) to (13.57) as f 1 u 2 5 ` cos u cos c (a) If a 5 0, d 5 l/2, bd 5 1 1 bd cos u 1 a 2 d ` 2 2p l 5 p. Hence, l 2 f1u 2 5 0 cos u 0 T T resultant 5 unit 3 pattern pattern p 2 1 cos u 2 ` T group pattern ` cos The sketch of the unit pattern is straightforward.

It is merely a rotated version of that in Figure 13.7(a) for the Hertzian dipole and is shown in Figure 13.13(a).

To sketch a group pattern, we must first determine nulls and maxima. For the nulls (or zeros), or cos a p p p cos ub 5 0 S cos u 5 6 2 2 2 u 5 0°, 180° For the maxima, cos a 13_Sadiku_Ch13.indd 719 p cos ub 5 1 S cos u 5 0 2 17/10/17 12:05 PM 720 CHAPTER 13 ANTENNAS U G R FIGURE 13.13 For part (a) of Example 13.6: field patterns in the plane containing the axes of the elements.

or u 5 90° The group pattern, shown in Figure 13.12(b), is the polar plot obtained by sketching p ` cos a cos ub ` for u 5 0, 5°, 10°, 15°, . . . , 180° and incorporating the nulls and maxima 2 at u 5 0°, 180° and u 5 90°, respectively. Multiplying Figure 13.13(a) with Figure 13.13 (b) gives the resultant pattern in Figure 13.13(c). MATLAB can easily be used to do this. It should be observed that the field patterns in Figure 13.13 are in the plane containing the axes of the elements. Note the following: (1) In the yz-plane, which is normal to the axes of the elements, the unit pattern 1 5 1 2 is a circle [see Figure 13.7(b)] while the group pattern remains as in Figure 13.13(b); therefore, the resultant pattern is the same as the group pattern in this case. (2) In the xy-plane, u 5 p/2, so the unit pattern vanishes while the group pattern 1 5 1 2 is a circle. 2p l p (b) If a 5 p/2, d 5 l/4, and bd 5 5 l 4 2 f1u 2 5 0 cos u 0 T T resultant 5 unit 3 pattern pattern ` cos p ` 4 1 cos u 1 1 2 T group pattern The unit pattern remains as in Figure 13.13(a). For the group pattern, the null occurs when cos or p p p 1 1 1 cos u 2 5 0 S 1 1 1 cos u 2 5 4 4 2 cos u 5 1 S u 5 0 13_Sadiku_Ch13.indd 720 14/11/17 2:16 PM 13.7 Antenna Arrays 721 U G R FIGURE 13.14 For part (b) of Example 13.6; field patterns in the plane containing the axes of the elements. The maxima and minima occur when d p p c cos 1 1 1 cos u 2 d 5 0 S sin u sin 1 1 1 cos u 2 5 0 du 4 4 sin u 5 0 S u 5 0°, 180° and sin p 1 1 1 cos u 2 5 0 S cos u 5 21 or u 5 180° 4 Each field pattern is obtained by varying u 5 0°, 5°, 10°, 15°, . . .

, 180°. Note that u 5 180° corresponds to the maximum value of 0 AF 0 , whereas u 5 0° corresponds to the null. Thus the unit, group, and resultant patterns in the plane containing the axes of the elements are shown in Figure 13.14.

Observe from the group patterns that the broadside array 1 a 5 0 2 in Figure 13.13 is bidirectional, while the end-fire array 1 a 5 bd 2 in Figure 13.14 is unidirectional. PRACTICE EXERCISE 13.6 Repeat Example 13.6 for the following cases: (a) a 5 p, d 5 l/2, (b) a 5 2p/2, d 5 l/4. Answer: See Figure 13.15. 13_Sadiku_Ch13.indd 721 17/10/17 12:05 PM 722 CHAPTER 13 ANTENNAS FIGURE 13.15 For Practice Exercise 13.6. EXAMPLE 13.7 Consider a three-element array that has current ratios 121 as in Figure 13.16(a). Sketch the group pattern in the plane containing the axes of the elements. Solution: For the purpose of analysis, we split the middle element in Figure 13.16(a) carrying current 2Il0° into two elements each carrying current Il0° .

This results in four elements ithree, as shown in Figure 13.16(b). If we consider i elements 1 and 2 as a group instead of and elements 3 and 4 as another group, we have a two-element array of Figure 13.16(c). Each group is a two-element array with d 5 l/2, a 5 0, so that the group pattern of the two-element array (or the unit pattern for the three-element array) is as shown in Figure 13.13(b). The two groups form a two-element array similar to Example 13.6(a) with d 5 l/2, a 5 0, so that the group pattern is the same as that in Figure 13.13(b). Thus, in this case, both the unit and group patterns are the same pattern in Figure 13.13(b). The resultant group pattern is obtained in Figure 13.17(c).

We should note that the pattern in Figure 13.17(c) is not the resultant pattern but the group pattern of the three-element array. The resultant group pattern of the array is Figure 13.17(c) multiplied by the field pattern of the element type. 13_Sadiku_Ch13.indd 722 17/10/17 12:05 PM 13.7 Antenna Arrays 723 FIGURE 13.16 For Example 13.7: (a) a three-element array with current ratios 121; (b) and (c) equivalent two-element arrays. An alternative method of obtaining the resultant group pattern of the three-element array of Figure 13.16 is by following steps similar to those taken to obtain eq. (13.59). We obtain the normalized array factor (or the group pattern) as 1 0 1 1 2ejc 1 e j2c 0 4 1 5 0 e jc 0 0 2 1 e2jc 1 e jc 0 4 c2 1 5 0 1 1 cos c 0 5 ` cos ` 2 2 1 AF 2 n 5 where c 5 bd cos u 1 a if the elements are placed along the z-axis but oriented parallel to 2p # l the x-axis. Since a 5 0, d 5 l/2, bd 5 5 p, and l 2 1 AF 2 n 5 ` cos a 1 AF 2 n T resultant group pattern 2 p cos ub ` 2 p p cos ub ` ` cos a cos ub ` 2 2 T T unit 3 group pattern pattern 5 ` cos a The sketch of these patterns is exactly what is in Figure 13.17.

13_Sadiku_Ch13.indd 723 17/10/17 12:05 PM 724 CHAPTER 13 ANTENNAS FIGURE 13.17 For Example 13.7; obtaining the resultant group pattern of the three-element array of Figure 13.16(a). U G R FIGURE 13.18 For Example 13.7 and Practice Exercise 13.7: four-element array with current ratios 1331. If two three-element arrays in Figure 13.16(a) are displaced by l/2, we obtain a four-element array with current ratios 1331 as in Figure 13.18. Two of such four-element arrays, displaced by l/2, give a five-element array with current ratios 14641. Continuing this process results in an N-element array, spaced l/2 and 1 N 2 1 2 l/2 long, whose current ratios are the binomial coefficients. Such an array is called a linear binomial array. PRACTICE EXERCISE 13.7 (a) Sketch the resultant group pattern for the four-element array with current ratios 1331 shown in Figure 13.18. (b) Derive an expression for the group pattern of a linear binomial array of N elements. Assume that the elements are placed along the z-axis, oriented parallel to the x-axis with spacing d and interelement phase shift a. Answer: (a) See Figure 13.19, (b) ` cos c N21 ` , where c 5 bd cos u 1 a. 2 FIGURE 13.19 For part (a) of Practice Exercise 13.7. 13_Sadiku_Ch13.indd 724 17/10/17 12:05 PM 13.8 Effective Area and the Friis Equation 725 † 13.8 EFFECTIVE AREA AND THE FRIIS EQUATION When the incoming EM wave is normal to the entire surface of a receiving antenna, the power received is Pr 5 3 ave # dS 5 aveS (13.67) S But in most cases, the incoming EM wave is not normal to the entire surface of the antenna. This necessitates the idea of the effective area of a receiving antenna. The concept of effective area or effective aperture (receiving cross section of an antenna) is usually employed in the anaysis of receiving antennas. The effective area Ae of a receiving antenna is the ratio of the time-average power received Pr (or delivered to the load, to be strict) to the time-average power density ave of the incident wave at the antenna. That is, Ae 5 Pr (13.68) ave From eq. (13.68), we notice that the effective area is a measure of the ability of the antenna to extract energy from a passing EM wave. Let us derive the formula for calculating the effective area of the Hertzian dipole acting as a receiving antenna. The Thévenin equivalent circuit for the receiving antenna is shown in Figure 13.20, where Voc is the open-circuit voltage induced on the antenna terminals by a remote transmitter, Zin 5 Rrad 1 jXin is the antenna impedance, and ZL 5 RL 1 jXL is the external load impedance, which might be the input impedance to the transmission * and XL 5 2Xin. The line feeding the antenna. For maximum power transfer, ZL 5 Zin time-average power delivered to the matched load is therefore Pr 5 5 1 0 Voc 0 2 c d Rrad 2 2Rrad 0 Voc 0 2 8 Rrad (13.69) FIGURE 13.20 Thévenin equivalent of a receiving antenna. oc 13_Sadiku_Ch13.indd 725 17/10/17 12:05 PM 726 CHAPTER 13 ANTENNAS For the Hertzian dipole, eq.

(13.13b) gives Rrad 5 80p2 1 dl/l 2 2 and Voc 5 E dl, where E is the effective field strength parallel to the dipole axis. Hence, eq. (13.69) becomes Pr 5 E2l2 (13.70) 640p2 The time-average power at the antenna is ave 5 E2 E2 5 2h 240p (13.71) Inserting eqs. (13.70) and (13.71) in eq. (13.68) gives Ae 5 3l2 l2 5 1.5 8p 4p or Ae 5 l2 D(13.72) 4p where D 5 1.5 is the directivity of the Hertzian dipole. Although eq. (13.72) was derived for the Hertzian dipole, it holds for any antenna if D is replaced by Gd 1 u, f 2 . Thus, in general Ae 5 l2 G 1 u, f 2 (13.73) 4p d Now suppose we have two antennas separated by distance r in free space as shown in Figure 13.21. The transmitting antenna has effective area Aet and directive gain Gdt and transmits a total power Pt 1 5 Prad 2 . The receiving antenna has effective area of Aer and directive gain Gdr and receives a total power of Pr. At the transmitter, Gdt 5 4pr2ave 4pU 5 Pt Pt or ave 5 Pt Gdt 4pr2 (13.74) FIGURE 13.21 Transmitting and receiving antennas in free space. 13_Sadiku_Ch13.indd 726 17/10/17 12:05 PM 13.8 Effective Area and the Friis Equation 727 By applying eqs. (13.68) and (13.73), we obtain the time-average power received as l2 G 4p dr ave Pr 5 ave Aer 5 (13.75) Substituting eq.

(13.74) into eq. (13.75) results in Pr 5 GdrGdt c l 2 d Pt 4pr (13.76) This is referred to as the Friis transmission formula. It relates the power received by one antenna to the power transmitted by the other, provided the two antennas are separated by r $ 2d2 /l, where d is the largest dimension of either antenna [see eq. (13.52)]. Therefore, to apply the Friis equation, we must make sure that each antenna is in the far field of the other. EXAMPLE 13.8 Find the maximum effective area of a l/2 wire dipole operating at 30 MHz. How much power is received with an incident plane wave of strength 2 mV/m? Solution: Ae 5 l5 l2 G 1 u, f 2 4p d c 3 3 108 5 5 10 m f 30 3 106 Gd 1 u, f 2 5 h 120p 2 f 21u 2 5 f 1 u 2 5 1.64f 2 1 u 2 pRrad 73p Gd max 5 1.64 Ae max 5 102 1 1.64 2 5 13.05 m2 4p Pr 5 ave Ae 5 5 E20 A 2h e 1 2 3 1023 2 2 13.05 5 69.23 nW 240p PRACTICE EXERCISE 13.8 Determine the maximum effective area of a Hertzian dipole of length 10 cm operating at 100 MHz. If the antenna receives 3 mW of power, what is the power density of the incident wave? Answer: 1.074 m2, 2.793 mW/m2. 13_Sadiku_Ch13.indd 727 17/10/17 12:05 PM 728 CHAPTER 13 ANTENNAS EXAMPLE 13.9 The transmitting and receiving antennas are separated by a distance of 200 l and have directive gains of 25 dB and 18 dB, respectively. If 5 mW of power is to be received, calculate the minimum transmitted power. Solution: Given that Gdt 1 dB 2 5 25 dB 5 10 log10 Gdt, Gdt 5 102.5 5 316.23 Similarly, Gdr 1 dB 2 5 18 dB or Gdr 5 101.8 5 63.1 Using the Friis equation, we have Pr 5 GdrGdt c or Pt 5 Pr c l 2 d Pt 4pr 4pr 2 1 d l GdrGdt 5 5 3 1023 c 5 1.583 W 4p 3 200 l 2 1 d 1 2 1 l 63.1 316.23 2 PRACTICE EXERCISE 13.9 An antenna in air radiates a total power of 100 kW so that a maximum radiated electric field strength of 12 mV/m is measured 20 km from the antenna.

Find (a) its directivity in decibels, (b) its maximum power gain if hr 5 98%.

Answer: (a) 20.18 dB, (b) 9.408 103. † 13.9 THE RADAR EQUATION Radars are electromagnetic devices used for detection and location of objects. The term radar is derived from the phrase radio detection and ranging. In a typical radar system, as shown in Figure 13.22(a), pulses of EM energy are transmitted to a distant object. The same antenna is used for transmitting and receiving, so the time interval between the transmitted and reflected pulses is used to determine the distance of the target. If r is the distance 13_Sadiku_Ch13.indd 728 17/10/17 12:05 PM 13.9 The Radar Equation 729 between the radar and target and c is the speed of light, the elapsed time between the transmitted and received pulse is 2r/c. By measuring the elapsed time, we determine r. The ability of a target to scatter (or reflect) energy is characterized by its scattering cross section (also called the radar cross section). The scattering cross section has the units of area and can be measured experimentally. The scattering cross section is the equivalent area intercepting the amount of power that, when scattering isotropically, produces at the radar a power density that is equal to that scattered (or reflected) by the actual target. That is, si ps 5 lim c d r S ` 4pr2 or s s 5 lim 4pr2 i rS` (13.77) where i is the incident power density at the target T while s is the scattered power density at the transceiver O as in Figure 13.22(b). From eq. (13.43), the incident power density i at the target T is i 5 ave 5 Gd Prad 4pr2 (13.78) The power received at transreceiver O is Pr 5 Aer s FIGURE 13.22 (a) Typical radar system. (b) Simplification of the system in (a) for calculating the target cross section s. 13_Sadiku_Ch13.indd 729 17/10/17 12:05 PM 730 CHAPTER 13 ANTENNAS or s 5 Pr Aer (13.79) Note that i and s are the time-average power densities in watts per square meter, and Prad and Pr are the total time-average powers in watts. Since Gdr 5 Gdt 5 Gd and Aer 5 Aet 5 Ae, substituting eqs. (13.78) and (13.79) into eq. (13.77) gives or s 5 1 4pr2 2 2 Pr 5 From eq. (13.73), Ae 5 l2Gd /4p. Hence, Pr 5 Pr 1 (13.80a) Prad Ae Gd AesGdPrad (13.80b) 1 4pr2 2 2 1 lGd 2 2sPrad (13.81) 1 4p 2 3r4 This is the radar transmission equation for free space. It is the basis for measurement of scattering cross section of a target. Solving for r in eq. (13.81) results in r5 c l2 G2ds # Prad 1/4 d (13.82) 1 4p 2 3 Pr Equation (13.82) is called the radar range equation. Given the minimum detectable power of the receiver, the equation determines the maximum range for a radar. It is also useful for obtaining engineering information concerning the effects of the various parameters on the performance of a radar system. The radar considered so far is the monostatic type because of the predominance of this type of radar in practical applications.

A bistatic radar is one in which the transmitter and receiver are separated.

If the transmitting and receiving antennas are at distances r1 and r2 from the target and Gdr 2 Gdt, eq. (13.81) for bistatic radar becomes Pr 5 2 GdtGdr l c d sPrad(13.83) 4p 4pr1r2 Radar transmission frequencies range from 25 to 70,000 MHz. Table 13.1 shows radar frequencies and their designations as commonly used by radar engineers.

13_Sadiku_Ch13.indd 730 17/10/17 12:05 PM 13.9 The Radar Equation 731 TABLE 13.1 Designations of Radar Frequencies EXAMPLE 13.10 Designation Frequency UHF L S C X Ku K Millimeter 300–1000 MHz 1000–2000 MHz 2000–4000 MHz 4000–8000 MHz 8000–12,500 MHz 12.5–18 GHz 18–26.5 GHz .35 GHz An S-band radar transmitting at 3 GHz radiates 200 kW. Determine the signal power density at ranges 100 and 400 nautical miles if the effective area of the radar antenna is 9 m2. With a 20 m2 target at 300 nautical miles, calculate the power of the reflected signal at the radar. Solution: The nautical mile is a common unit in radar communications.

1 nautical mile 1 nm 2 5 1852 m l5 Gdt 5 c 3 3 108 5 5 0.1 m f 3 3 109 4p 4p 9 5 3600p 2 Aet 5 1 l 0.1 2 2 For r 5 100 nm 5 1.852 3 105 m 5 GdtPrad 3600p 3 200 3 103 2 5 4pr 4p 1 1.852 3 105 2 2 5 5.248 mW/m2 For r 5 400 nm 5 4 1 1.852 3 105 2 m 5 5.248 5 0.328 mW/m2 1422 Using eq. (13.80b) Pr 5 13_Sadiku_Ch13.indd 731 Aes Gd Prad 3 4pr2 4 2 17/10/17 12:05 PM 732 CHAPTER 13 ANTENNAS where r 5 300 nm 5 5.556 3 105 m Pr 5 9 3 20 3 3600p 3 200 3 103 5 2.706 3 10214 W 3 4p 3 5.5562 4 2 3 1020 The same result can be obtained by using eq. (13.81). PRACTICE EXERCISE 13.10 A C-band radar with an antenna 1.8 m in radius transmits 60 kW at a frequency of 6000 MHz. If the minimum detectable power is 0.26 mW, for a target cross section of 5 m2, calculate the maximum range in nautical miles and the signal power density at half this range.

Assume unity efficiency and that the effective area of the antenna is 70% of the actual area. Answer: 0.031 nm, 501 W/m2. † 13.10 APPLICATION NOTE—ELECTROMAGNETIC INTERFERENCE AND COMPATIBILITY Every electronic device is a source of radiated electromagnetic fields called radiated emissions. These are often an accidental by-product of the design. Electromagnetic interference (EMI) is the degradation in the performance of a device due to the fields making up the electromagnetic environment.

The electromagnetic environment consists of various apparatuses such as radio and TV broadcast stations, radar, and navigational aids that radiate EM energy as they operate. Every electronic device is susceptible to EMI. Its influence can be seen all around us. The results include “ghosts” in TV picture reception, taxicab radio interference with police radio systems, power line transient interference with personal computers, and self- oscillation of a radio receiver or transmitter circuit. Electromagnetic compatibility (EMC) is achieved when a device functions satisfactorily without introducing intolerable disturbances to the electromagnetic environment or to other devices in its neighborhood. EMC1 is achieved when electronic devices coexist in harmony, such that each device functions according to its intended purpose in the presence of, and in spite of, the others. EMI 1 For an in-depth treatment of EMC, see C. R. Paul, Introduction to Electromagnetic Compatibility, 2nd ed. Hoboken, NJ: John Wiley & Sons, 2006. 13_Sadiku_Ch13.indd 732 17/10/17 12:05 PM 13.10 Application Note—Electromagnetic Interference and Compatibility 733 is the problem that occurs when unwanted voltages or currents are present to influence the performance of a device, while EMC is the solution to the problem. The goal of EMC is system or subsystem compatibility, and this is achieved by applying proven design techniques, the use of which ensures a system relatively free of EMI problems.

EMC is a growing field because of the ever-increasing density of electronic circuits in modern systems for computation, communication, control, and so on. It is a concern not only to electrical and computer engineers, but also to automotive engineers. The increasing application of automotive electronic systems to improve fuel economy, reduce - exhaust emissions, ensure vehicle safety, and provide assistance to the driver has resulted in a growing need to ensure compatibility during normal operation. We will consider the sources and characteristics of EMI. Later, we will examine EMI control techniques. A. Source and Characteristics of EMI First, let us classify EMI in terms of its causes and sources. The classification will facilitate recognition of sources and assist in determining means of control.

As mentioned earlier, any electronic device may be the source of EMI, although this is not the intention of the designer. The cause of the EMI problem may be either within the system, in which case it is termed an intrasystem problem, or from the outside, in which case it is called an intersystem problem. Figure 13.23 shows intersystem EMI problems. The term “emitter” is commonly used to denote the source of EMI, while the term “susceptor” is used to designate a victim device. Tables 13.2 and 13.3 present typical causes of intrasystem and intersystem problems. Both intrasystem and intersystem EMI generally can be controlled by the system design engineer by following some design guidelines and techniques. For intrasystem EMI problems, for example, the design engineer may apply proper grounding and wiring arrangements, shielding of circuits and devices, and filtering.

The sources of EMI can be classified as natural or artificial (manmade). The origins of EMI are basically undesired conducted emissions (voltages and/or currents) or radiated emissions (electric and/or magnetic fields). Conducted emissions are currents that are carried by metallic paths (the unit’s power cord) and placed on the common power network, where they may cause interference with other devices that are connected to the network. Radiated emissions concern the electric fields radiated by the device that may be received by other electronic devices causing interference in those devices. Figure 13.24 illustrates the conceptual difference between conducted and radiated paths. No single operating agency has jurisdiction over all systems to dictate actions necessary to achieve EMC. Thus, EMC is usually achieved by industrial association, voluntary regulation, government-enforced regulation, and negotiated agreements between the affected parties. Frequency plays a significant role in EMC. Frequency allocations and - assignments are made according to the constraints established by international treaties. The Radio Regulations resulting from such international treaties are published by the International Telecommunication Union (ITU). The Federal Communications Commission (FCC) has the authority over radio and wire communications in the United States. The FCC has set limits on the radiated and conducted emissions of electronic devices including calculators, televisions, printers, modems, and personal computers. It is illegal to market an electronic device in the United States unless its radiated and conducted emissions have been measured and do not exceed the limits of FCC regulations.

Therefore, any electronic device designed today that is designed without incorporating EMC design principles will probably fail to comply with the FCC limits.

13_Sadiku_Ch13.indd 733 17/10/17 12:05 PM 734 CHAPTER 13 ANTENNAS Aircraft Microwave relay link E E E S Ship S S E S E Radar Power lines E S FM/TV broadcast Mobile TX Telecommunications E S E Radio S Handy talkie E = Emitters of interference S = Susceptible equipment FIGURE 13.23 Typical examples of intersystem EMI problems. Source: J. I. N. Violette et al., Electromagnetic Compatibility Handbook.

New York: Van Nostrand Reinhold, 1987, p.

4. TABLE 13.2 Intrasystem EMI Causes Emitters Susceptors Power supplies Radar transmitters Mobile radio transmitters Fluorescent lights Car ignition systems Relays Radar receivers Mobile radio receivers Ordnance Car radio receivers TABLE 13.3 Intersystem EMI Causes Emitters Susceptors Lightning strokes Computers Power lines Radar transmitters Police radio transmitters Fluorescent lights Aircraft transmitters 13_Sadiku_Ch13.indd 734 Radio receivers TV sets Heart pacers Aircraft navigation systems Taxicab radio receivers Industrial controls Ship receivers 17/10/17 12:05 PM 13.10 Application Note—Electromagnetic Interference and Compatibility 735 FIGURE 13.24 Differences between conducted and radiated emissions. Radiated interference Receiver Transmitter Common ground Conducted interference Common power source Power cables B. EMI Control Techniques The three common design approaches used to control or suppress EMI are grounding, shielding, and filtering. Although each technique has a distinct role in system design, proper grounding may sometimes minimize the need for shielding and filtering; also proper shielding may minimize the need for filtering. Therefore, we discuss the three techniques, grounding, shielding, and filtering, in that order. Grounding Grounding is the establishment of an electrically conductive path between two points to connect electrical and electronic elements of a system to one another or to some reference point, which may be designated the ground. An ideal ground plane is a zero-potential, zeroimpedance body that can be used as a reference for all signals in associated circuitry and to which any undesired current can be transferred for the elimination of its effects. The purpose of the floating ground is to isolate circuits or equipment electrically from a common ground plane. This type of grounding technique may cause a hazard. Singlepoint grounding is used to minimize the effects of facility ground currents. Multiple-point grounding minimizes ground lead lengths. The ground plane might be a ground wire that is carried throughout the system or a large conductive body. Bonding is the establishment of a low-impedance path between two metal surfaces. Grounding is a circuit concept, while bonding denotes the physical implementation of that concept. The purpose of a bond is to make a structure homogeneous with respect to the flow of electrical currents, thus avoiding the development of potentials between the metallic parts, since such potentials may result in EMI. Bonds provide protection from electrical shock, power circuit current return paths, and antenna ground plane connections and also minimize the potential difference between the devices. They have the ability to carry large fault current.

There are two types of bonds: direct and indirect. The direct bond is a metal-to-metal contact between the elements connected, while the indirect bond is a contact through the use of conductive jumpers. 13_Sadiku_Ch13.indd 735 17/10/17 12:05 PM 736 CHAPTER 13 ANTENNAS The dc resistance Rdc of a bond is often used as an indication of bond quality. It is given by Rdc 5 , (13.84) sS where is the length of the bond, s is its conductivity, and S is its cross-sectional area. As frequency increases, the bond resistance increases due to skin effect. Thus the ac resistance Rac is given as Rac 5 , (13.85) sdw where w is the width of the bond and d is the skin depth. Bonding effectiveness can be expressed as the difference (in dB) between the induced voltages on an equipment case with and without the bond trap. Shielding The purpose of shielding is to confine radiated energy to a specific region or to prevent radiated energy from entering a specific region. Shields may be in the form of partitions and boxes as well as in the form of cable and connector shields. Shield types include solid, nonsolid (e.g., screen), and braid, as is used on cables. In all cases, a shield can be characterized by its shielding effectiveness.

The shielding effectiveness (SE) is defined as SE 5 10 log10 incident power density transmitted power density (13.86) where the incident power density is the power density at a measuring point before a shield is installed and the transmitted power is the power density at the same point after the shield is in place. In terms of the field strengths, the shielding effectiveness may also be defined as the ratio of the field Et transmitted through to the inside to the incident field Ei. Thus, SE is given by SE 5 20 log10 Ei (13.87) Et SE 5 20 log10 Hi (13.88) Ht For magnetic fields, For example, aluminum has s 5 3.5 3 107 S/m, e 5 eo, m 5 mo. An aluminum sheet at 100 MHz has an SE of 100 dB at a thickness of 0.01 mm. Since an aluminum sheet for a computer cabinet is much thicker than this, an aluminum case is considered a highly effective shield. A cabinet that effectively shields the circuits inside from external fields is also highly effective in preventing radiation from those circuits to the external world. Because of the effective shield, radiated emission from the computer system is caused by openings in the cabinet such as cracks or holes from disk drives and from wires that penetrate the cabinet such as power cords and cables to external devices. 13_Sadiku_Ch13.indd 736 17/10/17 12:05 PM 13.11 Application Note—Textile Antennas and Sensors 737 Filtering An electrical filter is a network of lumped or distributed constant resistors, inductors, and capacitors that offers comparatively little opposition to certain frequencies, while blocking the passage of other frequencies. A filter provides the means whereby levels of conducted interference are substantially reduced. The most significant characteristic of a filter is the insertion loss it provides as a function of frequency. Insertion loss (IL) is defined as IL 5 20 log10 V1 (13.89) V2 where V1 is the output voltage of a signal source with the filter in the circuit, and V2 is the output voltage of the signal source without the use of the filter. Low-pass filters are commonly used in EMC work. The insertion loss for the low-pass filters is given by IL 5 10 log10 1 1 1 F2 2 dB (13.90) where and f is the frequency. F5 e pfRC, pfL/R, for a capacitive filter (13.91) for an inductive filter 13.11 APPLICATION NOTE—TEXTILE ANTENNAS AND SENSORS Antennas for body-centric communication (Figure 13.25) have been introduced in recent years. Antennas in this new class can be sewn directly onto clothes. Weaving antennas and other electronic sensors into textiles heralds a new era for the apparel industry.

The garments of the future will not only cover the human body and protect against the extremes of nature, but also collect and transmit crucial information about the wearer’s vital signs and current environment. These capabilities will be achieved by seamlessly tailoring biomedical and environmental monitoring systems into fabric. Researchers at the Ohio State University created a prototype using plastic film and metallic thread. Some of the novel body-worn antennas and medical sensors they have developed are based on embroidered conductive polymer fibers called e-fibers on textiles. The flexible conductors are constructed from silver-coated p-phenylene-2,6-benzobisoxazole (PBO) fibers. The e-fibers are composed of high-strength and flexible polymer cores that incorporate conductive metallic coatings. They are readily embroidered onto regular textiles and can also be laminated onto polymer dielectric substrates. The e-fiber textiles exhibit an insertion loss of only 0.07 dB/cm at 1 GHz and 0.15 dB/ cm at 2 GHz. They provide inherent mechanical strength that is due to their polymer core, together with high electrical conductivity resulting from the silver coating. These e-fibers are twisted together to improve their conductivity. For instance, the 332-strand e-fibers have a low resistivity of only 0.8 /m.

More importantly, e-fibers are suitable for automatic 13_Sadiku_Ch13.indd 737 17/10/17 12:05 PM 738 CHAPTER 13 ANTENNAS FIGURE 13.25 Textile antenna. embroidery onto textiles to realize various antenna and circuit designs. The embroidered e-fiber textile electronics exhibit both mechanical and electrical advantages. Also, because of their high conductivity, e-fibers provide much better antenna performance than other textile antennas utilizing less conductive materials or embedded metal wires. The e-fibers are sewn onto textiles via computerized embroidery processes to form antennas or RF circuits. Because the fibers are so thin, they can be bundled to form much thicker threads (664 strands per thread) for improved conductivity. During embroidery, an “assistant” yarn is used to couch the e-fibers onto one side of the textile’s surface. This procedure avoids abrasion damage to the silver coatings on the e-fiber’s polymer core.

The antenna and sensor designs are translated into embroidery software, followed by digitizing stitches of the assistant yarn. As the sewing machine carries out each stitch, the e-fibers are firmly and precisely placed onto the textile. To improve surface conductivity by minimizing physical discontinuities and thread gaps, a second layer can be embroidered right on top of the first (double embroidery). It is recommended that the resultant e-fiber surface discontinuities be kept to less than λ/20, where λ is the free-space wavelength of the operational frequency. This precaution is critical to realizing high-performance antennas and circuits.

Although primarily designed for military use, the e-fiber technology could potentially be applied to the manufacture of gear for police officers, firefighters, and astronauts— anybody who needs to keep the hands free for important work. The European Integrated Project Proetex aims at developing such wearable textile systems chiefly for professional firefighters and other first responders. A variety of sensors are being sewn inside and outside the firefighter’s outfit, and the signals from them are processed in a wearable electronic unit and transmitted to a base station. Suitable antennas that combine flexibility with robustness and reliability are needed for this purpose.

In other applications substrates at least 2 mm thick are used to print the antennas. But since the clothes are usually thinner, a flexible protective foam available in a variety of thicknesses and easily layered with garments such as firefighter suits is employed in the design. Proetex also contemplates designing wearable textile systems for civilian victims of natural and other disasters. Textile antennas find applications not only for continuous monitoring, but also for therapeutic regimes. For instance, they can be made to produce hyperthermia for the treatment of tumors and to monitor various physiological parameters. In addition to medical applications, textile antennas serve as part of a biotelemetry system to establish wireless communication links between implantable devices and exterior instruments. 13_Sadiku_Ch13.indd 738 17/10/17 12:05 PM 13.12 Application Note—Fractal Antennas 739 13.12 APPLICATION NOTE—FRACTAL ANTENNAS Historically, antennas were called aerials; but they were so named because of their resemblance to insect antennae. Standard antennas come in various shapes, almost all of which have mathematical description. Many shapes occurring in nature cannot be described in terms of euclidean geometry. For example, a fern leaf, a thin snowflake, the shoreline, and a class of crustaceans possess self-similarity, a property that the whole is deterministically or statistically similar to a part thereof. These are characterized by fractal geometry. A fractal is an iteratively generated geometry that has fractal dimensions. The underlying notions were conceived by Benoit B. Mandelbrot. He investigated the relationship between these iterated function systems and the nature around us using previous contributions of Gaston Julia, Pierre Fatou, and Felix Hausdorff. He depicted many fractals existing in nature and was able to accurately model certain phenomena. Also, he introduced new fractal models for more complex structures, including trees, clouds, and mountains, that possess an inherent self-similarity and self-affnity by way of geometrical continuation in terms of non-euclidean elements. Antennas whose shape is inherited from fractals are called fractal antennas. The widespread use of wireless communication systems posed the necessity for the design of wideband, or multiband, low-profile, small antennas. Their role became important in personal communication systems, satellite communication terminals, RFID, unmanned aerial vehicles, and so on. The central idea in their design involves optimal appropriation of the physical space either in planar or in 3-dimension.

This accomplishes greater bandwidth of operation from a low quality factor. Fractal concepts have been applied to many branches of science and engineering, including fractal electrodynamics for radiation, propagation, and scattering. They have been extended to antenna theory and design resulting in the implementations of different fractal antenna elements and arrays. There are broadly two categories of fractals: deterministic and random. Deterministic fractals are generated from several scaled-down and rotated copies of themselves. Examples are the von Koch snowflake and the Sierpinski gaskets. Recursive algorithms are used to generate such fractals. Random fractals evince some degree of randomness such as is found in natural phenomena. Fractal geometries can best be characterized and generated using an iterative process suitable for self-similar and self-affinity structures.

Figure 13.26 illustrates this iterative process in generating few iterations of a Koch loop and a Koch loop antenna at a chosen iteration. In a similar fashion, other shapes can be generated. Figure 13.27 shows the stages of fractal tree dipole antenna. Figures 13.28 and 13.29 depict a fern leaf and the Sierpinski triangle, which can also be rendered as antennas. A useful feature of fractal antennas arises from their space-filling property that helps in miniaturization while increasing the length. This permits low values of Q factors and higher bandwidths. At resonance, the impedance is higher compared to that of traditional antennas. A small circular loop of quarter wavelength perimeter has low radiation resistance, but a Koch loop occupying similar space has 35 times higher resistance. Higher iterative geometries cause longer electrical lengths and exhibit lower resonant frequencies. The meanderings of fractal contour impart distributive loading. For example, in the Koch fractal loop inductance adds in a distributive way. The increase in inductance 13_Sadiku_Ch13.indd 739 17/10/17 12:05 PM 740 CHAPTER 13 ANTENNAS allows the monopole to be smaller than the corresponding linear monopole and still be resonant.

Most of the miniaturization benefits of the fractal dipoles occur within the first five iterations, with meager marginal changes in the characteristics at greater complexities. Similar to several fractal antennas, Sierpinski gaskets possess desirable radiation 0 1 3 2 4 Koch loop antenna FIGURE 13.26 Generating Koch loop antenna. FIGURE 13.27 Fractal tree dipole antenna iteration after iteration. 13_Sadiku_Ch13.indd 740 17/10/17 12:05 PM 13.12 Application Note—Fractal Antennas 741 and impedance characteristics and can be used as monopoles and dipoles with similar cross-sectional contours. Although fractal antennas have several advantages, they do not necessarily outperform other classical categories. FIGURE 13.28 Fern leaf. FIGURE 13.29 Sierpinski triangle.

13_Sadiku_Ch13.indd 741 17/10/17 12:06 PM 742 CHAPTER 13 ANTENNAS 13.13 APPLICATION NOTE—RFID N.R.Solutions, 2302 Bernwood Dr, Erie PA 16510 Radio-frequency identification (RFID) is a relatively new technology for tracking and access applications that have been under development since the 1980s.

For example, with RFID, one can use electromagnetic signals to read data from electronic labels without actual contact. The label to be read does not have be located in a line-of-sight region. This technology is very attractive for tracking and tagging purposes (Figure 13.30). For a long time, the universal product code (UPC), also known as the bar code, has been the foremost means of identifying products. Designed to provide an open standard for product labeling, bar codes not only reduced costs but also increased efficiency and favored innovations that benefited consumers, manufacturers, wholesalers, and retailers alike. However, bar codes suffer drawbacks such as the need for line-of-sight from the scanner to the bar code. They are also limited in their capacity for storing data and thus are employed only to track product categories. The Electronic Product Code (EPC) is one of the industrial standards for global RFID usage, and a core element of the EPCglobal network, an architecture of open standards developed by the GS1 EPCglobal community. Most currently deployed EPC RFID tags comply with ISO/IEC 18000-6C for the RFID air interface standard. In ambient conditions that do not permit bar code labels as effective identifiers, RFID tags could serve the purpose. Thanks to the technology's ability to track moving objects, use of RFID has become an established practice in a wide range of markets. FIGURE 13.30 Schematic view of an RFID tag in use. 13_Sadiku_Ch13.indd 742 15/11/17 9:29 PM 13.13 Application Note—RFID 743 RFID also greatly facilitates identification of and automated data collection from livestock, moving trucks, or other nonstationary objects.

Pharmaceuticals can be tracked through warehouses. Livestock and pets may have tags injected, allowing positive identification of the animals. Essentially, an RFID system works as follows. Two-way radio transmitter–receivers called interrogators or readers send a signal to the tag and read its response. The readers generally transmit their observations to a computer system running RFID software or RFID middleware. RFID systems typically come in three configurations. One is a passive reader/active tag (PRAT) system in which a passive reader does no more than receive radio signals from active tags that are battery operated and do no more than transmit. The reception range of a PRAT system reader can be adjusted from 1 to 2000 feet, thereby allowing for great flexibility in applications such as asset protection and supervision. Another configuration is an active reader/passive tag (ARPT) system, in which an active reader transmits interrogator signals and also receives authentication replies from passive tags. Finally, there is the active reader/active tag (ARAT) system in which active tags are awakened with an interrogator signal from the active reader. A variation of this system could also use a battery-assisted passive (BAP) tag, which acts like a passive tag but has a small battery to power the tag’s return reporting signal. RFID tags can be passive, active, or battery-assisted passive. An active tag has an onboard battery and periodically transmits its ID signal. A BAP tag has a small battery onboard and is activated in the presence of an RFID reader.

A passive tag is cheaper and smaller because it has no battery. Instead, the tag uses the radio energy transmitted by the reader as its energy source.

The interrogator must be close for the RF field to be strong enough to transfer sufficient power to the tag. Since tags have individual serial numbers, the RFID system design can discriminate several tags that might be within the range of the RFID reader and read them simultaneously. The tag’s information is stored electronically in a nonvolatile memory. The RFID tag includes a small RF transmitter and receiver. An RFID reader transmits an encoded radio signal to interrogate the tag. The tag receives the message and responds with its identification information. This may be only a unique tag serial number or perhaps such product-related information as a stock number, lot or batch number, production date, or other specific information. RFID tags contain at least two parts: an integrated circuit (for storing and processing information, modulating and demodulating an RF signal, collecting dc power from the incident reader signal, and other specialized functions) and an antenna (for receiving and transmitting the signal). Fixed readers are set up to create a specific interrogation zone, which can be tightly controlled. This allows a highly defined reading area to accommodate movement of the tags in and out of the interrogation zone.

Mobile readers may be handheld or mounted on carts or vehicles. Printed antenna technology, electronic circuitry, the middleware for data collection and filtering, and other areas of innovation are constantly evolving. Some of the complexities are not understood by the general public. Since RFID tags can be attached to clothing, possessions, or even implanted within people, the possibility of reading personally linked information without consent has raised privacy concerns. 13_Sadiku_Ch13.indd 743 15/11/17 5:36 PM 744 CHAPTER 13 ANTENNAS MATLAB 13.1 % % % % % % % % This script allows the user to calculate and plot the far-field radiation pattern for an array of isotropic point sources, placed anywhere around the "origin". The main assumption is that the array elements are clustered together relative to the far-field, so that the origin is roughly in the center of the cluster. The user can still enter [x y] coordinates and magnitude/phase for each point source. Also the calculation assumes free space. % Prompt user for number of antennas and frequency n=input('Enter number of point sources > '); f=input('Enter frequency > '); % Begin loop prompting the user to enter each coefficient and each % coordinate for i=1:n disp('Enter source coefficient for point ') disp(sprintf(' %d source coefficient (a*exp(j*b) format)',i)) A(i)=input('> '); % store complex coefficients in A disp('Enter the (x,y) coordinate for point ') disp(sprintf(' %d source coefficient (in format [x y])',i)) P(i,:)=input('> '); % store coordinates in P, where i is % the ith element (row) % and column 1 contains the x values, column 2 the y values end Beta = 2*pi*f/3e8; % solve for free space phi = 0:2*pi/1000:2*pi; % create a vector of 1001 points around % a circle Etotal=zeros(1,length(phi)); % the total electric field is also % a vector of 1001 points % loop through the antennas for i=1:n % first determine how much the point source is offset from the % phi = 0 % axis (x-axis) phi_offset(i) = atan2(P(i,2), P(i,1)); % now find the distance from the ith point source to the origin rho(i) = sqrt(P(i,1)^2+P(i,2)^2); end % loop through the phi array and calculate the far field as a % function of phi for phi_index = 1:length(phi) for i=1:n % need to add % The apparent angle between a point in the far field is the % difference between phi and the offset of the antenna from the % x-axis 13_Sadiku_Ch13.indd 744 15/11/17 8:01 PM Summary 745 end end phi_apparent = phi(phi_index) - phi_offset(i); Etotal(phi_index)=Etotal(phi_index) + A(i)*... exp(j*Beta*rho(i)*cos(phi_apparent)); % Polar plot of far-field field pattern polar(phi, abs(Etotal)) SUMMARY 1. We have discussed the fundamental ideas and definitions in antenna theory. The basic types of antenna considered were the Hertzian (or differential length) dipole, the halfwave dipole, the quarter-wave monopole, and the small loop. 2. Theoretically, if we know the current distribution on an antenna, we can find the retarded magnetic vector potential A, and from it we can find the retarded electromagnetic fields H and E by using H5=3 A , m E 5 h H 3 ak The far-zone fields are obtained by retaining only 1/r terms.

3. The analysis of the Hertzian dipole serves as a stepping-stone for other antennas. The radiation resistance of the dipole is very small. This limits the practical usefulness of the Hertzian dipole. 4. The half-wave dipole has a length equal to l/2. It is more popular and of more practical use than the Hertzian dipole. Its input impedance is 73 1 j42.5 V. 5. The quarter-wave monopole is essentially half a half-wave dipole placed on a conducting plane. 6. The radiation patterns commonly used are the field intensity, power intensity, and radiation intensity patterns. The field pattern is usually a plot of 0 Es 0 or its normalized form f(). The power pattern is the plot of ave or its normalized form f 2(). 7. The directive gain is the ratio of U 1 u, f 2 to its average value. The directivity is the maximum value of the directive gain. 8. An antenna array is a group of radiating elements arranged to produce particular radiation characteristics. Its radiation pattern is obtained by multiplying the unit pattern (due to a single element in the group) with the group pattern, which is the plot of the normalized array factor. For an N-element linear uniform array, Nc b 2 0 AF 0 5 ∞ ∞ c sin a b 2 sin a where c 5 bd cos u 1 a, b 5 2p/l, d 5 spacing between the elements, and a 5 interelement phase shift. 9. The Friis transmission formula characterizes the coupling between two antennas in terms of their directive gains, separation distance, and frequency of operation. 13_Sadiku_Ch13.indd 745 15/11/17 8:01 PM 746 CHAPTER 13 ANTENNAS 10.

For a bistatic radar (one in which the transmitting and receiving antennas are separated), the power received is given by Pr 5 2 GdtGdr l c d s Prad 4p 4pr1r2 For a monostatic radar, r1 5 r2 5 r and Gdt 5 Gdr. 11. Electromagnetic compatibility (EMC) is the capability of electrical and electronic devices to operate in their intended electromagnetic environment without suffering or causing unacceptable degradation as a result of EMI. 12. Electromagnetic interference (EMI) is the disturbance generated by one electronic device that affects another device. It can be suppressed by grounding, shielding, and filtering. REVIEW QUESTIONS 13.1 An antenna located in a city is a source of radio waves. How much time does it take the waves to reach a town 12,000 km away?

(a) 36 s (b) 20 ms (c) 20 ms (d) 40 ms (e) None of the above 13.2 In eq. (13.34a–c), which term is the radiation term? (a) 1/r term (b) 1/r2 term (c) 1/r3 term (d) All of the above 13.3 A very small, thin wire of length l/100 has a radiation resistance of (a) . 0 V (b) 0.08 V (c) 7.9 V (d) 790 V 13.4 A quarter-wave monopole antenna operating in air at frequency 1 MHz must have an overall length of (a) , W l (c) 150 m (e) , V l (b) 300 m (d) 75 m 13.5 If a small single-turn loop antenna has a radiation resistance of 0.04 V, how many turns are needed to produce a radiation resistance of 1 V? (a) 150 (c) 50 (e) 5 (b) 125 (d) 25 13.6 At a distance of 8 km from a differential antenna, the field strength is 12 mV/m. The field strength at a location 20 km from the antenna is (a) 75 mV/m (b) 30 mV/m (c) 4.8 mV/m (d) 1.92 mV/m 13.7 An antenna has Umax 5 10 W/sr, Uave 5 4.5 W/sr, and hr 5 95%. The input power to the antenna is (a) 2.222 W (b) 12.11 W 13_Sadiku_Ch13.indd 746 (c) 55.55 W (d) 59.52 W 15/11/17 5:36 PM Problems 747 13.8 A receiving antenna in an airport has a maximum dimension of 3 m and operates at 100 MHz. An aircraft approaching the airport is 0.5 km from the antenna. The aircraft is in the far-field region of the antenna.

(a) True (b) False 13.9 A receiving antenna is located 100 m away from the transmitting antenna.

If the effective area of the receiving antenna is 500 cm2 and the power density at the receiving location is 2 mW/m2, the total power received is: (a) 10 nW (c) 1 mW (b) 100 nW (d) 10 mW (e) 100 mW 13.10 Let R be the maximum range of a monostatic radar.

If a target with radar cross section of 5 m2 exists at R/2, what should be the target cross section at 3R/2 to result in an equal signal strength at the radar? (a) 0.0617 m2 (c) 15 m2 (b) 0.555 m2 (d) 45 m2 (e) 405 m2 Answers: 13.1d, 13.2a, 13.3b, 13.4d, 13.5e, 13.6c, 13.7d, 13.8a, 13.9e, 13.10e. PROBLEMS Section 13.2—Hertzian Dipole 13.1 The magnetic vector potential at point P 1 r, u, f 2 due to a small antenna located at the origin is given by As 5 50 e2jbr ax r where r2 5 x2 1 y2 1 z2. Find E 1 r, u, f, t 2 and H 1 r, u, f, t 2 at the far field. 13.2 A dipole antenna has the following parameters : Antenna length < 0.02 Current magnitude Io 3 A Operating frequency f 400 MHz Radiation range r 60 m Determine the following: (a) The magnitude of the electric field intensity at 90 (b) The magnitude of the magnetic field intensity at 90 (c) The radiation resistance (d) The radiated power 13.3 A Hertzian antenna in free space is 10 cm long. It is fed by a current of 20 A at frequency of 50 MHz. Find the electric and magnetic fields at far zone. 13.4 Determine the current necessary for a 2 cm Hertzian dipole to radiate 12 W at 140 MHz. 13_Sadiku_Ch13.indd 747 15/11/17 5:36 PM 748 CHAPTER 13 ANTENNAS FIGURE 13.31 Short dipole antenna with triangular current distribution; for Problem 13.5. 13.5 (a) Instead of a constant current distribution assumed for the short dipole of Section 13.2, assume a triangular current distribution Is 5 Io a1 2 b shown in Figure 13.31.

, Show that , 2 Rrad 5 20 p2 c d l which is one-fourth of that in eq. (13.13). Thus Rrad depends on the current distribution. (b) Calculate the length of the dipole that will result in a radiation resistance of 0.5 V.

13.6 A short dipole antenna operates at the AM broadcast band at 1.2 MHz. To achieve a radiation resistance of 0.5 V, how long must the antenna be? Section 13.3—Half-Wave Dipole Antenna 13.7 A half-wave dipole antenna is driven by a 24 V, 200 MHz source having an internal impedance of 40 . Find the average power radiated by the antenna, given that Zin 73 j42 .

13.8 A car radio antenna 1 m long operates in the AM frequency band of 1.5 MHz. How much current is required to transmit 4 W of power? *13.9 (a) Show that the generated far-field expressions for a thin dipole of length carrying sinusoidal current Io cos bz are Hfs 5 jIoe 2br 2pr cos a b, b, cos ub 2 cos 2 2 , sin u Eus 5 hHfs [Hint: Use Figure 13.4 and start with eq. (13.14).] (b) On a polar coordinate sheet, plot f() in part (a) for , 5 l, 3l/2, and 2l. 13.10 An antenna engineer is asked to design a l/2 dipole antenna to operate at 450 MHz. 13_Sadiku_Ch13.indd 748 (a) Calculate the length of the antenna if it is located in free space. (b) Determine the length of the antenna if it is located under seawater (« 81«0, 5 0, 5 4 S/m). 15/11/17 5:36 PM Problems 749 Section 13.4—Quarter-Wave Monopole Antenna 13.11 Quarter-wavelength antennas are used to transmit: (a) AM signals at 1150 kHz (b) FM signals at 90 MHz (c) VHF-TV signals at 80 MHz (d) UHF-TV signals at 600 MHz Calculate the respective antenna lengths. 13.12 A dipole antenna (, l/8) operating at 400 MHz is used to send a message to a satellite in space.

Find the radiation resistance of the antenna.

Section 13.5—Small-Loop Antenna 13.13 A 100-turn loop antenna of radius 20 cm operating at 10 MHz in air is to give a 50 mV/m field strength at a distance 3 m from the loop. Determine (a) The current that must be fed to the antenna (b) The average power radiated by the antenna 13.14 A circular loop antenna has a mean radius of 1.2 cm and N turns.

If it operates at 80 MHz, find N that will produce a radiation resistance of 8 V. 13.15 A loop antenna with loop radius of 0.4 m is made of copper wire of radius 4 mm. If the loop radiates at 6 MHz and carries a current of 50 A, find (a) The radiation resistance of the loop (b) The power radiated (c) The radiation efficiency Section 13.6—Antenna Characteristics 13.16 Sketch the normalized E-field and H-field patterns for (a) A half-wave dipole (b) A quarter-wave monopole 13.17 The radiation efficiency is given by eq. (13.50) as hr 5 Prad Prad 1 Pohm where P rad is the radiation power and P ohm is the power loss due to ohmic resistance of the antenna. For a cylindrical conductor of length z that carries current I, 1 Pohm 5 I2 Rs Dz 2 13_Sadiku_Ch13.indd 749 1 and is the skin depth. A short antenna of length z has an effisd2pa ciency of 20%. Assume that the efficiency is improved by increasing the length to 2z, while keeping the same current.

What is the new efficiency? where Rs 5 15/11/17 5:36 PM 750 CHAPTER 13 ANTENNAS 13.18 Determine the radiation efficiency of a half-wave dipole operating at 6 MHz. The wire is made of copper (s = 58 MS/m) and is 1.2 mm in radius. 13.19 An antenna located at the origin has a far-zone electric field as Es 5 cos 2u 2jbr e au V/m r (a) Obtain the corresponding Hs field. (b) Determine the power radiated. (c) What fraction of the total power is radiated in the belt 60° , u , 120°?

13.20 In the far field of a particular antenna located at the origin, the magnetic field i ntensity is jbIo Hs 5 4pr 1 sin f au 1 cos u cos f af 2 where Io is the peak value of the input current. Show that the radiation resistance is given by Rrad = 20 2. *13.21 An antenna located on the surface of a flat earth transmits an average power of 200 kW. Assuming that all the power is radiated uniformly over the surface of a hemisphere with the antenna at the center, calculate (a) The time-average Poynting vector at 50 km (b) The maximum electric field at that location.

13.22 The radiated power density of an antenna at far-zone is given by 3ave 5 a sin2 u sin3 f af, r2 0 , u , p, 0 , f , p where a is a constant. Determine the directive gain and directivity. 13.23 For a one-and-a-half-wave antenna, show that the normalized field pattern is f 1u 2 5 Use MATLAB to plot f(). cos 1 1.5 p cos u 2 sin u 13.24 Divide the interval 0 , , 2p into 20 equal parts and use MATLAB to show that 3 2p 0 13_Sadiku_Ch13.indd 750 1 1 2 cos u 2 du 5 2.438 u 15/11/17 5:36 PM Problems 751 13.25 For a thin dipole l/16 long, find (a) The directive gain (c) The effective area (b) The directivity (d) The radiation resistance 13.26 Find the directive gain and directivity of the small loop antenna. 13.27 A quarter-wavelength monopole antenna is used at 1.2 MHz for AM transmission. The antenna is vertically placed above a conducting surface. Determine (a) The length of the antenna (b) The radiation resistance (c) The directivity of the antenna 13.28 Find Uave, Umax, and D if: (a) U 1 u, f 2 5 sin2 2u, 0 , u , p, 0 , f , 2p (b) U 1 u, f 2 5 4 csc2 u, p/3 , u , p/2, 0 , f , p (c) U 1 u, f 2 5 2 sin2 u sin2 f, 0 , u , p, 0 , f , p 13.29 For each of the following radiation intensities, calculate the directive gain and directivity.

(a) U(,) 10 sin sin2 , 0 , 0 2 (b) U(,) 2 sin2 sin3 , 0 , 0 (c) U(,) 5 (1sin2 sin2), 0 , 0 2 13.30 The radiation intensity of an antenna is given by U 1 u, f 2 5 e 4 sin2u sin f/2, 0 , f , p, 0 , u , p 0, otherwise Calculate the directivity of the antenna. 13.31 An antenna has a far-field electric field given by Es 5 Io 2jbr e sin u au r where Io is the maximum input current. Determine the value of Io to radiate a power of 50 mW. 13.32 Find the radiation intensity of a small loop antenna. 13.33 The field due to an isotropic (or omnidirectional) antenna is given by E5 al r where a is a constant. Determine the radiation resistance of the antenna. 13.34 Determine the fraction of the total power radiated by the elemental (Hertzian) antenna over 0 60º.

13_Sadiku_Ch13.indd 751 15/11/17 5:36 PM 752 CHAPTER 13 ANTENNAS Section 13.7—Antenna Array 13.35 Derive Es at far field due to the two-element array shown in Figure 13.32.

Assume that the Hertzian dipole elements are fed in phase with uniform current Io cos vt. 13.36 An array comprises two dipoles that are separated by one wavelength. If the dipoles are fed by currents of the same magnitude and phase, (a) Find the array factor. (b) Calculate the angles where the nulls of the pattern occur. (c) Determine the angles where the maxima of the pattern occur. (d) Sketch the group pattern in the plane containing the elements. 13.37 Sketch the group pattern in the xz-plane of the two-element array of Figure 13.10 with (a) d 5 l, a 5 p/2 (b) d 5 l/4, a 5 3p/4 (c) d 5 3l/4, a 5 0 13.38 A n antenna array consists of N identical Hertzian dipoles uniformly located along the z-axis and polarized in the z-direction. If the spacing between the dipoles is l/4, sketch the group pattern when (a) N 5 2, (b) N 5 4.

13.39 An array of isotropic elements has the group pattern F1c2 5 ∞ sin 1 2p cos c 2 p cos2 a cos cb 4 p 2 sina cos cb 2 Use MATLAB to plot F(c) for 0 c 180. FIGURE 13.32 Two-element array of Problem 13.35. 13_Sadiku_Ch13.indd 752 15/11/17 5:36 PM Problems 753 FIGURE 13.33 For Problem 13.40. 13.40 Sketch the resultant group patterns for the four-element arrays shown in Figure 13.33.

Section 13.8—Effective Area and the Friis Equation 13.41 A microwave antenna has a power gain of 20 dB at 10 GHz. Determine its effective area. 13.42 An antenna receives a power of 2 mW from a radio station. Calculate its effective area if the antenna is located in the far zone of the station where E 5 50 mV/m. 13.43 A telemetry transmitter situated on the moon transmits 120 mW at 200 MHz. The gain of the transmitting antenna is 15 dB. Calculate the gain (in dB) of the receiving antenna (situated on earth) in order to receive 4 nW. Assume that the moon is 238,857 miles away from the earth and that 1 mile 1.609 km. 13.44 In a communication system, suppose the transmitting and receiving antennas have gains 25 dB and 20 dB, respectively, and are 42 km apart. Find the minimum power that must be transmitted in order to deliver a minimum of 3 mW. The channel frequency is 3 GHz. 13.45 The power transmitted by a synchronous orbit satellite antenna is 320 W. If the antenna has a gain of 40 dB at 15 GHz, calculate the power received by another antenna with a gain of 32 dB at the range of 24,567 km. 13.46 A communication link uses a half-wave dipole antenna for transmission and another half-wave dipole antenna for reception. The link operates at 20 MHz and the two antennas are separated by a distance of 80 km. If the receiving antenna must receive an average power of 0.5 mW, determine the minimum required current of the transmitting antenna.

13.47 Two identical antennas in an anechoic chamber are separated by 12 m and are oriented for maximum directive gain. At a frequency of 5 GHz, the power received by one is 30 dB down from that transmitted by the other. Calculate the gain of the antennas in dB. 13_Sadiku_Ch13.indd 753 15/11/17 5:36 PM 754 CHAPTER 13 ANTENNAS Section 13.9—The Radar Equation 13.48 A n L-band pulse radar with a common transmitting and receiving antenna having a directive gain of 3500 operates at 1500 MHz and transmits 200 kW. If an object is 120 km from the radar and its scattering cross section is 8 m2, find (a) The magnitude of the incident electric field intensity of the object (b) The magnitude of the scattered electric field intensity at the radar (c) The amount of power captured by the object (d) The power absorbed by the antenna from the scattered wave 13.49 A monostatic radar operates at 4 GHz and has a directive gain of 30 dB. The radar is used to track a target 10 km away, and the radar cross section of the target is 12 m2. If the antenna of the radar transmits 80 kW, calculate the power intercepted by the target. 13.50 A 4 GHz radar antenna with effective area of 2 m2 transmits 60 kW.

A target with cross section of 5 m2 is located 160 km away. Calculate: (a) the round trip travel time, (b) the power received, (c) the maximum detectable range, assuming that the minimum detectable power is 8 pW.

13.51 It is required to double the range capacity of a radar. What percentage increase in transmitter power is necessary to achieve this? 13.52 In the bistatic radar system of Figure 13.34, the ground-based antennas are separated by 4 km and the 2.4 m2 target is at a height of 3 km. The system operates at 5 GHz. For Gdt of 36 dB and Gdr of 20 dB, determine the minimum necessary radiated power to obtain a return power of 8 3 10212 W. FIGURE 13.34 For Problem 13.52. 13_Sadiku_Ch13.indd 754 15/11/17 5:36 PM Problems 755 Section 13.10—Application Note—Electromagnetic Interference and Compatibility 13.53 (a) For an RL filter with L 5 50 nH and R 5 20 , find the insertion loss in dB at 300 MHz. (b) Repeat part (a) if an RC filter with C 5 60 pF and R 5 10 k is used instead. *13.54 The insertion loss of a filter circuit can be calculated in terms of its A, B, C, and D parameters when terminated by Zg and ZL as shown in Figure 13.35. Show that IL 5 20 log10 ` AZL 1 B 1 CZgZL 1 DZg Zg 1 ZL ` 13.55 A silver rod has rectangular cross section with height 0.8 cm and width 1.2 cm. Find (a) The dc resistance per kilometer of the conductor (b) The ac resistance per kilometer of the conductor at 6 MHz 13.56 Within a shielded enclosure, the electric field is 6 V/m. It is required that the electric field outside the shield be no more than 20 mV/m. Find the shielding effectiveness in dB. Zg A Vl Vg B VL C ZL D FIGURE 13.35 For Problem 13.54. 13_Sadiku_Ch13.indd 755 15/11/17 5:36 PM COMPUTATIONAL ELECTROMAGNETICS Until the 1940s, most electromagnetic problems were solved by using the classical m ethods of separation of variables and integral equations. Application of those methods, however, required a high degree of ingenuity, experience, and effort, and only a narrow range of p ractical problems could be investigated, owing to the complex geometries defining the problems. While theory and experiment remain the two conventional pillars of science and engineering, modeling and simulation represent the third pillar that complements them. Computational electromagnetics (CEM) is the theory and practice of solving EM field problems on digital computers. It offers the key to comprehensive solutions of Maxwell's equations. In this chapter, the common methods used in computational electromagnetics will be presented. CEM techniques can be used to model electromagnetic interaction phenomena in circuits, devices, and systems. Numerical modeling and simulation have revolutionized all aspects of engineering design to the extent that several software packages have been developed to aid designing and modeling. Widely used software packages for CEM include the Numerical Electromagnetics Code (NEC) based on the method of moments and developed at Lawrence Livermore National Laboratory, the High Frequency Structure Simulator (HFSS) based on the finite element method and developed by Ansoft, Microwave Office based on the method of moments and developed by Applied Wave Research, Sonnet developed by Sonnet, COMSOL based on the finite element method, and QuickField, which is another finite element modeling software. These software packages put powerful tools and techniques, previously available only to full-time theorists, into the hands of engineers not formally trained in CEM. The best method or software package to use depends on the particular problem you are trying to solve. 14_Sadiku_Ch14.indd 756 14/11/17 2:12 PM CHAPTER 14 NUMERICAL METHODS Young men think old men are fools, but old men know young men are fools. —GEORGE CHAPMAN 14.1 INTRODUCTION In the preceding chapters we considered various analytic techniques for solving EM problems and obtaining solutions in closed form.

A closed-form solution is one in the form of an explicit, algebraic equation in which values of the problem parameters can be substituted. Some of these analytic solutions were obtained assuming certain situations, thereby making the solutions applicable to those idealized situations. For example, in deriving the formula for calculating the capacitance of a parallel-plate capacitor, we assumed that the fringing effect was negligible and that the separation distance was very small compared with the width and length of the plates. Also, our application of Laplace's equation in Chapter 6 was restricted to problems with boundaries coinciding with coordinate surfaces. Analytic solutions have an inherent advantage of being exact. They also make it easy to observe the behavior of the solution when there is variation in the problem parameters. However, analytic solutions are available only for problems with simple configurations. When the complexities of theoretical formulas make analytic solution intractable, we resort to nonanalytic methods, which include (1) graphical methods, (2) experimental methods, (3) analog methods, and (4) numerical methods. Graphical, experimental, and analog methods are applicable to solving relatively few problems. Numerical methods have come into prominence and have become more attractive with the advent of fast digital computers. The three most commonly used simple numerical techniques in EM are the moment method, the finite difference method, and the finite element method. Most EM problems involve either partial differential equations or integral equations. Partial differential equations are usually solved by using the finite difference method or the finite element method; integral equations are solved conveniently by using the moment method. Although numerical methods give approximate solutions, the solutions are sufficiently accurate for engineering purposes. We should not get the impression that analytic techniques are outdated because of numerical methods; rather, they are complementary. As will be observed later, every numerical method involves analytic simplification until the method can be easily applied.

The MATLAB codes developed for computer implementation of the concepts developed in this chapter are simplified and self-explanatory for instructional purposes. (Appendix C provides a short tutorial on MATLAB.) The notations used in the programs 757 14_Sadiku_Ch14.indd 757 14/11/17 2:12 PM 758 CHAPTER 14 NUMERICAL METHODS are as close as possible to those used in the main text; some are defined wherever necessary. These programs are by no means unique; there are several ways of writing a computer program. Therefore, users may decide to modify the programs to suit their objectives. † 14.2 FIELD PLOTTING In Section 4.9, we used field lines and equipotential surfaces for visualizing an electrostatic field. However, the graphical representations in Figure 4.21 for electrostatic fields and in Figures 7.8(b) and 7.16 for magnetostatic fields are very simple, trivial, and qualitative. Accurate pictures of more complicated charge distributions would be more helpful. This section presents a numerical technique that may be developed into an interactive computer program. It generates data points for electric field lines and equipotential lines for arbitrary configuration of point sources. Electric field lines and equipotential lines can be plotted for coplanar point sources with simple programs.

Suppose we have N point charges located at position vectors r1, r2, . . . , rN, the electric field intensity E and potential V at position vector r are given, respectively, by and N Qk 1 r 2 rk 2 E5 a 3 (14.1) k51 4pe 0 r 2 rk 0 N Qk V5 a (14.2) k51 4pe 0 r 2 rk 0 If the charges are on the same plane 1 z 5 constant 2 , eqs. (14.1) and (14.2) become N Qk 3 1 x 2 xk 2 ax 1 1 y 2 yk 2 ay 4 E5 a 2 2 3/2 (14.3) k51 4pe 3 1 x 2 xk 2 1 1 y 2 yk 2 4 N Qk V5 a 2 2 1/2 (14.4) k51 4pe 3 1 x 2 xk 2 1 1 y 2 yk 2 4 To plot the electric field lines, follow these steps: 1. Choose a starting point on the field line. 2. Calculate Ex and Ey at that point using eq. (14.3). 3. Take a small step along the field line to a new point in the plane. As shown in Figure 14.1, a movement D along the field line corresponds to movements Dx and Dy in the x- and y-directions, respectively. From Figure 14.1, it is evident that Ex Ex Dx 5 5 2 3 Ex 1 E2y 4 1/2 D, E 14_Sadiku_Ch14.indd 758 14/11/17 2:12 PM 14.2 Field Plotting 759 FIGURE 14.1 A small displacement on a field line. F N O or Dx 5 Similarly, Dy 5 D, # Ex (14.5) 3 E2x 1 E2y 4 1/2 D, # Ey 3 E2x 1 E2y 4 1/2 (14.6) Move along the field line from the old point 1 x, y 2 to a new point xr 5 x 1 Dx, yr 5 y 1 Dy. 4. Go back to steps 2 and 3 and repeat the calculations. Continue to generate new points until a line is completed within a given range of coordinates. On c ompleting the line, go back to step 1 and choose another starting point. Note that since there are an infinite number of field lines, any starting point is likely to be on a field line. The points generated can be plotted by a plotter as illustrated in Figure 14.2. To plot the equipotential lines, follow these steps: 1. Choose a starting point. 2. Calculate the electric field 1 Ex, Ey 2 at that point by using eq. (14.3). 3. Move a small step along the line perpendicular to the E-field line at that point. Utilize the fact that if a line has slope m, a perpendicular line must have slope 21/m.

Since an E-field line and an equipotential line meeting at a given point are mutually orthogonal there, Dx 5 Dy 5 14_Sadiku_Ch14.indd 759 2D, # Ey 3 E2x 1 E2y 4 1/2 (14.7) D, # Ex (14.8) 1 E2y 4 1/2 3 E2x move along the equipotential line from the old point 1 x, y 2 to a new point 1 x 1 Dx, y 1 Dy 2 . As a way of checking the new point, calculate the potential at 14/11/17 2:12 PM 760 CHAPTER 14 NUMERICAL METHODS FIGURE 14.2 Generated points on E-field lines (shown thick) and equipotential lines (dotted). the new and old points using eq. (14.4); the potentials must be equal because the points are on the same equipotential line.

4. Go back to steps 2 and 3 and repeat the calculations. Continue to generate new points until a line is completed within the given range of x and y. After completing the line, go back to step 1 and choose another starting point. Join the points generated by a plotter as illustrated in Figure 14.2. By following the same reasoning, we can use the Biot–Savart law to plot the magnetic field line due to various current distributions. Programs for determining the magnetic field line due to line current, a current loop, a Helmholtz pair, and a solenoid can be developed. Programs for drawing the electric and magnetic field lines inside a rectangular waveguide or the power radiation pattern produced by a linear array of vertical half-wave electric dipole antennas can also be written. EXAMPLE 14.1 Write a program to plot the electric field and equipotential lines due to: (a) Two point charges Q and 24Q, located at 1 x, y 2 5 1 21, 0 2 and 1 1, 0 2 , respectively. (b) Four point charges Q, 2Q, Q, and 2Q, located at 1 x, y 2 5 1 21, 21 2 , 1 1, 21 2 , 1 1, 1 2 , and 1 21, 1 2 , respectively. Take Q/4pe 5 1 and D, 5 0.1. Consider the range 25 , x , 5, 2 5 , y , 5. Solution: Based on the steps given in Section 14.2, the program in Figure 14.3 was developed. Enough comments are inserted to make the program as self-explanatory as possible. For example, to use the program to generate the plot in Figure 14.4(a), load program plotit in your MATLAB directory.

At the command prompt in MATLAB, type plotit 1 3 1 24 4 , 3 21 0; 1 0 4 , 1, 1, 0.1, 0.01, 8, 2, 5) where the numbers have meanings provided in the program. Further explanation of the program is provided in the following paragraphs. 14_Sadiku_Ch14.indd 760 14/11/17 2:12 PM 14.2 Field Plotting 761 Since the E-field lines emanate from positive charges and terminate on negative charges, it seems reasonable to generate starting points 1 xs, ys 2 for the E-field lines on small circles centered at charge locations 1 xQ, yQ 2 ; that is, xs 5 xQ 1 r cos u(14.1.1a) ys 5 yQ 1 r sin u(14.1.1b) function plotit(charges,location,ckEField,ckEq,DLE,DLV,NLE,NLV,PTS) figure; hold on; % Program for plotting the electric field lines % and equipotential lines due to coplanar point charges % the plot is to be within the range -5=5)) break; end if (sum(abs(XE-XQ) < .05 & abs(YE-YQ) < .05) >0) break; end JJ=JJ+1; if (~mod(JJ,PTS)) plot (XE,YE); end end % while loop end % I =1:NLE end % K = 1:NQ end % if % NEXT, DETERMINE THE EQUIPOTENTIAL LINES % FOR CONVENIENCE, THE STARTING POINTS (XS,YS) ARE % CHOSEN LIKE THOSE FOR THE E-FIELD LINES if (ckEq) FIGURE 14.3 (Continued) 14_Sadiku_Ch14.indd 762 14/11/17 2:12 PM 14.2 Field Plotting 763 JJ=1; DELTA = .2; ANGLE = 45*pi/180; for K =1:NQ FACTOR = .5; for KK = 1:NLV XS = XQ(K) + FACTOR*cos(ANGLE); YS = YQ(K) + FACTOR*sin(ANGLE); if (abs(XS) >= 5 | abs(YS) >=5) break; end DIR = 1; XV = XS; YV = YS; JJ=JJ+1; if (~mod(JJ,PTS)) plot(XV,YV); end % FIND INCREMENT AND NEW POINT (XV,YV) N=1; while (1) EX = 0; EY = 0; for J = 1:NQ R = sqrt((XV-XQ(J))^2 + (YV-YQ(J))^2); EX = EX + Q(J)*(XV-XQ(J))/(R^3); EY = EY + Q(J)*(YV-YQ(J))/(R^3); end E=sqrt(EX^2 + EY^2); if (E 5 | abs(YV) > 5) DIR = DIR -2; XV = XS; FIGURE 14.3 (Continued) 14_Sadiku_Ch14.indd 763 14/11/17 2:12 PM 764 CHAPTER 14 NUMERICAL METHODS else YV = YS; if (abs(DIR) > 1) FACTOR = 2*FACTOR; break; end if (sum(abs(XV-XQ) < .005 & abs(YV-YQ) < .005) >0) break; end end JJ=JJ+1; if (~mod(JJ,PTS)) N=N+1; plot(XV,YV); end end % WHILE loop end % KK end % K end % if FIGURE 14.4 For Example 14.1; plots of E-field lines and equipotential lines due to (a) two point charges and (b) four point charges (a two-dimensional quadrupole). (a) (b) 14_Sadiku_Ch14.indd 764 14/11/17 2:12 PM 14.2 Field Plotting 765 where r is the radius of the small circle 1 e.g., r 5 0.1 or 0.05 2 , and u is a prescribed angle chosen for each E-field line. The starting points for the equipotential lines can be generated in different ways: along the x- and y-axes, along line y 5 x, and so on. However, to make the program as general as possible, the starting points should depend on the charge locations like those for the E-field lines. They could be chosen by using eq. (14.1.1) but with fixed u (e.g., 45°) and variable r (e.g., 0.5, 1.0, 2.0, . . .). The value of incremental length D is crucial for accurate plots. Although the smaller the value of D, the more accurate the plots, we must keep in mind that the smaller the value of D, the more points we generate, and memory storage may be a problem. For example, a line may consist of more than 1000 generated points. In view of the large number of points to be plotted, the points are usually stored in a data file and a graphics routine is used to plot the data. For both the E-field and equipotential lines, different checks are inserted in the program in Figure 14.3: (a) (b) (c) (d) Check for singular point (E = 0?). Check whether the point generated is too close to a charge location. Check whether the point is within the given range of 25 , x , 5, 2 5 , y , 5, Check whether the (equipotential) line loops back to the starting point. The plot of the points generated for the cases of two point charges and four point charges are shown in Figure 14.4(a) and (b), respectively. PRACTICE EXERCISE 14.1 Write a complete program for plotting the electric field lines and equipotential lines due to coplanar point charges. Run the program for N 5 3; that is, there are three point charges 2Q, 1Q, and 2Q, located at 1 x, y 2 5 1 21, 0 2 , 1 0, 1 2 , and 1 1, 0 2 ,-respectively. Take Q/4pe 5 1, D, 5 0.1 or 0.01 for greater accuracy and limit your plot to 25 , x , 5, 2 5 , y , 5. Answer: See Figure 14.5. FIGURE 14.5 For Practice Exercise 14.1. 14_Sadiku_Ch14.indd 765 14/11/17 2:12 PM 766 CHAPTER 14 NUMERICAL METHODS 14.3 THE FINITE DIFFERENCE METHOD The finite difference method1 (FDM) is a simple numerical technique used in solving problems like those solved analytically in Chapter 6. A problem is uniquely defined by three things: 1. A partial differential equation such as Laplace's or Poisson's equation 2. A solution region 3. Boundary and/or initial conditions A finite difference solution to Poisson's or Laplace's equation, for example, proceeds in three steps: (1) dividing the solution region into a grid of nodes, (2) approximating the differential equation and boundary conditions by a set of linear algebraic equations (called difference equations) on grid points within the solution region, and (3) solving this set of algebraic equations. Step 1: Suppose we intend to apply the finite difference method to determine the electric potential in a region shown in Figure 14.6(a). The solution region is divided into rectangular meshes with grid points or nodes as in Figure 14.6(a). A node on the boundary of the region where the potential is specified is called a fixed node (fixed by the problem), and interior points in the region are called free points (free in that the potential is unknown). Step 2: Our objective is to obtain the finite difference approximation to Poisson's equation and use this to determine the potentials at all the free points. We recall that Poisson's equation is given by rv =2V 5 2 (14.9a) e FIGURE 14.6 Finite difference solution pattern: (a) division of the solution into grid points, (b) finite difference five-node molecule. 1 For an extensive treatment of the finite difference method, see G. D. Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods, 3rd ed.

Oxford: Oxford Univ. Press, 1985. 14_Sadiku_Ch14.indd 766 14/11/17 2:12 PM 14.3 The Finite Difference Method 767 For two-dimensional solution region such as in Figure 14.6(a), '2V 5 0, so 'z2 rv '2V '2V 1 2 2 5 2 e (14.9b) 'x 'y From the definition of the derivative of V 1 x, y 2 at point 1 xo, yo 2 , Vr 5 V 1 xo 1 Dx, yo 2 2 V 1 xo 2 Dx, yo 2 'V . ` 'x x5xo 2Dx 5 Vi11, j 2 Vi21, j 2 Dx (14.10) where Dx is a sufficiently small increment along x. For the second derivative, which is the derivative of the first derivative V, Vs 5 Vr 1 xo 1 Dx/2, yo 2 2 Vr 1 xo 2 Dx/2, yo 2 '2V 'Vr 5 . 2 ` 'x Dx 'x x5xo 5 5 V 1 xo 1 Dx, yo 2 2 2V 1 xo, yo 2 1 V 1 xo 2 Dx, yo 2 1 Dx 2 2 Vi11, j 2 2Vi, j 1 Vi21, j 1 Dx 2 2 (14.11) Equations (14.10) and (14.11) are the finite difference approximations for the first and second partial derivatives of V with respect to x, evaluated at x 5 xo.

The approximation in eq. (14.10) is associated with an error of the order of the Dx while that of eq. (14.11) has an associated error on the order of 1 Dx 2 2.

Similarly, V 1 xo, yo 1 Dy 2 2 2V 1 xo, yo 2 1 V 1 xo, yo 2 Dy 2 '2V . 2 ` 1 Dy 2 2 'y y5yo 5 Vi, j11 2 2Vi, j 1 Vi, j21 1 Dy 2 2 (14.12) Substituting eqs. (14.11) and (14.12) into eq.

(14.9b) and letting Dx 5 Dy 5 h gives Vi11, j 1 Vi21, j 1 Vi, j11 1 Vi, j21 2 4Vi, j 5 2 h2rv e or 14_Sadiku_Ch14.indd 767 Vi, j 5 h2rv 1 aVi11, j 1 Vi21, j 1 Vi, j11 1 Vi, j21 1 b(14.13) e 4 14/11/17 2:12 PM 768 CHAPTER 14 NUMERICAL METHODS where h is called the mesh size. Equation (14.13) is the finite difference approximation to Poisson's equation. If the solution region is charge free 1 rv 5 0 2 , eq. (14.9) becomes Laplace's equation: =2V 5 '2V '2V 1 5 0(14.14) 'x2 'y2 The finite difference approximation to this equation is obtained from eq. (14.13) by setting rv 5 0; that is, Vi, j 5 1 1V 1 Vi21, j 1 Vi, j11 1 Vi, j21 2 (14.15) 4 i11, j This equation is essentially a five-node finite difference approximation for the potential at the central point of a square mesh. Figure 14.6(b) illustrates what is called the finite difference five-node molecule.

The molecule in Figure 14.6(b) is taken out of Figure 14.6(a).

Thus eq. (14.15) applied to the molecule becomes Vo 5 1 1 V 1 V2 1 V3 1 V4 2 (14.16) 4 1 This equation clearly shows the average-value property of Laplace's equation. In other words, Laplace's equation can be interpreted as a differential means of stating the fact that the potential at a specific point is the average of the potentials at the neighboring points. Step 3: To apply eq. (14.16) [or eq. (14.13)] to a given problem, one of the following two methods is commonly used. A. Iteration Method We start by setting initial values of the potentials at the free nodes equal to zero or to any reasonable guessed value. Keeping the potentials at the fixed nodes unchanged at all times, we apply eq. (14.16) to every free node in turn until the potentials at all free nodes have been calculated. The potentials obtained at the end of this first iteration are just approximate. To increase the accuracy of the potentials, we repeat the calculation at every free node, using old values to determine new ones. The iterative or repeated modification of the potential at each free node is continued until a prescribed degree of accuracy is achieved or until the old and the new values at each node are satisfactorily close. B. Band Matrix Method Equation (14.16) applied to all free nodes results in a set of simultaneous equations of the form 3 A 4 3 V 4 5 3 B 4 (14.17) where [A] is a sparse matrix (i.e., one having many zero terms), [V] consists of the unknown potentials at the free nodes, and [B] is another column matrix formed by the 14_Sadiku_Ch14.indd 768 14/11/17 2:12 PM 14.3 The Finite Difference Method 769 known potentials at the fixed nodes. Matrix [A] is also banded in that its nonzero terms appear clustered near the main diagonal because only nearest neighboring nodes affect the potential at each node. The sparse, band matrix is easily inverted to determine [V]. Thus we obtain the potentials at the free nodes from matrix [V] as 3 V 4 5 3 A 4 21 3 B 4 (14.18) The finite difference method can be applied to solve time-varying problems.

For example, consider the one-dimensional wave equation of eq. (10.1), then u2 '2ϕ '2ϕ 2 5 'x 't2 (14.19) where u is the wave velocity and F is the E- or H-field component of the EM wave. The difference approximations of the derivatives at the 1 xo, to 2 or 1 i, j 2 th node shown in Figure 14.7 are ϕi21, j 2 2ϕi, j 1 ϕi11, j '2ϕ . 2 ` 1 Dx 2 2 'x x5xo (14.20) ϕi, j21 2 2ϕi, j 1 ϕi, j11 '2ϕ . 2 ` 1 Dt 2 2 't t5to (14.21) Where Dx and Dt are increments along x and t. Inserting eqs. (14.20) and (14.21) in eq. (14.19) and solving for ϕi,j11 gives where ϕi, j11 . a 1 ϕi21, j 1 ϕi11, j 2 1 2 1 1 2 a 2 ϕi, j 2 ϕi, j21 a5 c (14.22) u Dt 2 d (14.23) Dx FIGURE 14.7 Finite difference solution pattern for the wave equation: eq. (14.19). 14_Sadiku_Ch14.indd 769 14/11/17 2:12 PM 770 CHAPTER 14 NUMERICAL METHODS It can be shown that for the solution in eq. (14.22) to be stable, a # 1. To start the finite difference algorithm in eq. (14.22), we use the initial conditions. We assume that at t 5 0, 'ϕi,0 /'t 5 0 and use (central) difference approximation (see Review Question 14.2) to get 'ϕi,0 't . ϕi,1 2 ϕi,21 2Dt 50 or ϕi,1 5 ϕi,21 (14.24) Substituting eq. (14.24) into eq. (14.22) and taking j 5 0 1 t 5 0 2 , we obtain ϕi,1 . a 1 ϕi21,0 1 ϕi11,0 2 1 2 1 1 2 a 2 ϕi,0 2 ϕi,1 or 1 3 a 1 ϕi21,0 1 ϕi11,0 2 1 2 1 1 2 a 2 ϕi,0 4 (14.25) 2 With eq. (14.25) as the "starting" formula, the value of F at any point on the grid can be obtained directly from eq. (14.22). Note that the three methods discussed for solving eq. (14.16) do not apply to eq. (14.22) because eq. (14.22) can be used directly with eq. (14.25) as the starting formula. In other words, we do not have a set of simultaneous equations; eq.

(14.22) is an explicit formula. The FDM concept can be extended to Poisson's, Laplace's, or wave equations in other coordinate systems. The accuracy of the method depends on the fineness of the grid and the amount of time spent in refining the potentials. We can reduce computer time and increase the accuracy and convergence rate by the method of successive overrelaxation, by making reasonable guesses at initial values, by taking advantage of symmetry if possible, by making the mesh size as small as possible, and by using more complex finite difference molecules. One limitation of the finite difference method is that interpolation of some kind must be used to determine solutions at points not on the grid. One obvious way to overcome this is to use a finer grid, but this requires a greater number of computations and a larger amount of computer storage. ϕi,1 . EXAMPLE 14.2 Solve the one-dimensional boundary-value problem 2ϕs 5 x2, 0 # x # 1, subject to ϕ 1 0 2 5 0 5 ϕ 1 1 2 . Use the finite difference method. Solution: First, we obtain the finite difference approximation to the differential equation ϕs 5 2x2, which is Poisson's equation in one dimension. Next, we divide the entire domain 0 # x # 1 into N equal segments each of length h 1 5 1/N 2 as in Figure 14.8(a) so that there are 1 N 1 1 2 nodes. 2x2o 5 14_Sadiku_Ch14.indd 770 ϕ 1 xo 1 h 2 2 2ϕ 1 xo 2 1 ϕ 1 xo 2 h 2 d2ϕ . 2 ` dx x5xo h2 14/11/17 2:12 PM 14.3 The Finite Difference Method 771 FIGURE 14.8 For Example 14.2. or 2x2j 5 ϕj11 2 2ϕj 1 ϕ21 h2 Thus 22ϕj 5 22j h2 2 ϕj11 2 ϕj21 or 1 ϕj 5 1 ϕj11 1 ϕj21 1 x2j h2 2 2 Using this finite difference scheme, we obtain an approximate solution for various values of N. The MATLAB code is shown in Figure 14.9. The number of iterations ni depends on the degree of accuracy desired. For a one-dimensional problem such as this, ni 5 50 may suffice. For two- or three-dimensional problems, larger values of ni would be required (see later: Table 14.1). It should be noted that the values of F at end points (fixed nodes) are held fixed. The solutions for N 5 4 and 10 are shown in Figure 14.10. We may compare this with the exact solution obtained as follows. Given that d2ϕ/dx2 5 2x2, integrating twice gives ϕ52 x4 1 Ax 1 B 12 where A and B are integration constants. From the boundary conditions, ϕ102 5 0 S B 5 0 ϕ112 5 0 S 0 5 2 1 1A 12 or A 5 1 12 Hence, the exact solution is ϕ 5 x 1 1 2 x3 2 /12, which is calculated in Figure 14.9 and found to be very close to case N 5 10. 14_Sadiku_Ch14.indd 772 CHAPTER 14 NUMERICAL METHODS % % % % ONE-DIMENSIONAL PROBLEM OF EXAMPLE 14.2 SOLVED USING FINITE DIFFERENCE METHOD h = MESH SIZE ni = NO. OF ITERATIONS DESIRED P = [ ]; n=20; ni=500; 1=1.0; h = 1/n; phi=zeros(n+1,1); x=h*[0:n]'; x1=x(2:n); for k=1:ni phi([2:n])=[phi(3:n+1)+phi(1:n-1)+x1.^2*h^2]/2; end % CALCULATE THE EXACT VALUE ALSO phiex=x.*(1.0-x.^3)/12.0; diary a:test.out [[1:n+1]' phi phiex] diary off FIGURE 14.9 Computer program for Example 14.2. 0 0 0 0 0 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 FIGURE 14.10 For Example 14.2: plot of F(x). Continuous curve is for N 5 10; dashed curve is for N 5 4. 14_Sadiku_Ch14.indd 772 14/11/17 2:12 PM 14.3 The Finite Difference Method 773 PRACTICE EXERCISE 14.2 Solve the differential equation d2y/dx2 1 y 5 0 with the boundary conditions y 1 0 2 5 0, y 1 1 2 5 1 by using the finite difference method.

Take Dx 5 1/4. Answer: Compare your result with the exact solution y 1 x 2 5 EXAMPLE 14.3 sin 1 x 2 . sin 1 1 2 FIGURE 14.11 For Example 14.3. Determine the potential at the free nodes in the potential system of Figure 14.11 using the finite difference method. Solution: This problem will be solved by using the iteration method first, and then the band matrix method. Method 1 (Iteration Method): We first set the initial values of the potential at the free nodes equal to zero. We apply eq. (14.16) to each free node, using the newest surrounding potentials each time the potential at that node is calculated. For the first iteration: V1 5 1/4 1 0 1 20 1 0 1 0 2 5 5 V2 5 1/4 1 5 1 0 1 0 1 0 2 5 1.25 V3 5 1/4 1 5 1 20 1 0 1 0 2 5 6.25 V4 5 1/4 1 1.25 1 6.25 1 0 1 0 2 5 1.875 14_Sadiku_Ch14.indd 773 14/11/17 2:12 PM 774 CHAPTER 14 NUMERICAL METHODS and so on. To avoid confusion, each time a new value at a free node is calculated, we cross out the old value as shown in Figure 14.12. After V8 is calculated, we start the second iteration at node 1: V1 5 1/4 1 0 1 20 1 1.25 1 6.25 2 5 6.875 V2 5 1/4 1 6.875 1 0 1 0 1 1.875 2 5 2.187 and so on. If this process is continued, we obtain the uncrossed values shown in Figure 14.12 after five iterations. After 10 iterations (not shown in Figure 14.12), we obtain V1 5 10.04, V2 5 4.956, V3 5 15.22, V4 5 9.786 V5 5 21.05, V6 5 18.97, V7 5 15.06, V8 5 11.26 Method 2 (Band Matrix Method): This method reveals the sparse structure of the problem.

We apply eq. (14.16) to each free node and keep the known terms (prescribed FIGURE 14.12 For Example 14.3; the values not crossed out are the solutions after five iterations. 14_Sadiku_Ch14.indd 774 14/11/17 2:12 PM 14.3 The Finite Difference Method 775 potentials at the fixed nodes) on the right side; the unknown terms (potentials at free nodes) are on the left side of the resulting system of simultaneous equations, which will be expressed in matrix form as 3 A 4 3 V 4 5 3 B 4 . For node 1, 24V1 1 V2 1 V3 5 220 2 0 For node 2, V1 1 4V2 1 V4 5 20 2 0 For node 3, V1 2 4V3 1 V4 1 V5 5 220 For node 4, V2 1 V3 2 4V4 1 V6 5 20 For node 5, V3 2 4V5 1 V6 5 220 2 30 For node 6, V4 1 V5 2 4V6 1 V7 5 230 For node 7, V6 2 4V7 1 V8 5 230 2 0 For node 8, V7 2 4V8 5 20 2 0 2 30 Note that since we are using a five-node molecule, we have five terms at each node. The eight equations obtained are put in matrix form as 24 1 1 0 0 0 0 0 V1 220 1 24 0 1 0 0 0 0 V2 0 1 0 24 1 1 0 0 0 V3 220 0 1 1 24 0 1 0 0 V4 0 H XH X 5 H X 0 0 1 0 24 1 0 0 V5 250 0 0 0 1 1 24 1 0 V6 230 0 0 0 0 0 1 24 1 V7 230 0 0 0 0 0 0 1 24 V8 230 or 3A 4 3V 4 5 3B 4 14_Sadiku_Ch14.indd 775 14/11/17 2:12 PM 776 CHAPTER 14 NUMERICAL METHODS where [A] is the band, sparse matrix, [V] is the column matrix consisting of the unknown potentials at the free nodes, and [B] is the column matrix formed by the potential at the fixed nodes.

The "band" nature of [A] is shown by the dotted loop. Notice that matrix [A] could have been obtained directly from Figure 14.11 without writing down eq. (14.16) at each free node. To do this, we simply set the diagonal (or self) terms Aii 5 24 and set Aij 5 1 if i and j nodes are connected or Aij 5 0 if i and j nodes are not directly connected. For example, A23 5 A32 5 0 because nodes 2 and 3 are not connected, whereas A46 5 A64 5 1 because nodes 4 and 6 are connected. Similarly, matrix [B] is obtained directly from Figure 14.11 by setting Bi equal to minus the sum of the potentials at fixed nodes connected to node i. For example, B5 5 2 1 20 1 30 2 because node 5 is connected to two fixed nodes with potentials 20 V and 30 V. If node i is not connected to any fixed node, Bi 5 0. By using MATLAB to invert matrix [A], we obtain 3 V 4 5 3 A 4 21 3 B 4 or V1 5 10.04, V2 5 4.958, V3 5 15.22, V4 5 9.788 V5 5 21.05, V6 5 18.97, V7 5 15.06, V8 5 11.26 which compares well with the result obtained by means of the iteration method. PRACTICE EXERCISE 14.3 Use the iteration method to find the finite difference approximation to the potentials at points a and b of the system in Figure 14.13. Answer: Va = 10.135 V, Vb = 28.378 V. FIGURE 14.13 For Practice Exercise 14.3. 14_Sadiku_Ch14.indd 776 14/11/17 2:12 PM 14.3 The Finite Difference Method 777 FIGURE 14.14 For Example 14.4. EXAMPLE 14.4 Obtain the solution of Laplace's equation for an infinitely long trough whose rectangular cross section is shown in Figure 14.14. Let V1 5 10 V, V2 5 100 V, V3 5 40 V, and V4 5 0 V. Solution: We shall solve this problem by using the iteration method. In this case, the solution region has a regular boundary. We can easily write a program to determine the potentials at the grid points within the trough. We divide the region into square meshes. If we decide to use a 15 3 10 grid, the number of grid points along x is 15 1 1 5 16 and the number of grid points along y is 10 1 1 5 11. The mesh size h 5 1.5/15 5 0.1 m.

The 15 3 10 grid is illustrated in Figure 14.15. The grid points are numbered 1 i, j 2 starting from the lower left-hand corner of the trough. The computer program in Figure 14.16, for determining the potential at the free nodes, was developed by applying eq. (14.15) and using the iteration method. At points 1 x, y 2 5 1 0.5, 0.5 2 , 1 0.8, 0.8 2 , 1 1.0, 0.5 2 , and 1 0.8, 0.2 2 corresponding to 1 i, j 2 5 1 5, 5 2 , 1 8, 8 2 , 1 10, 5 2 , and 1 8, 2 2 , respectively, the potentials after 50, 100, and 200 iterations are shown in Table 14.1. The exact values obtained by using the method of separation of variables and a program similar to that of Figure 6.11 are also shown. It should be noted that the degree of accuracy depends on the mesh size h. It is always desirable to make h as small as possible. Also note that the potentials at the fixed nodes are held constant throughout the calculations. FIGURE 14.15 For Example 14.4; a 15 3 10 grid. 14_Sadiku_Ch14.indd 777 14/11/17 2:12 PM 778 CHAPTER 14 NUMERICAL METHODS % % % % % % % % USING FINITE DIFFERENCE (ITERATION) METHOD THIS PROGRAM SOLVES THE TWO-DIMENSIONAL BOUNDARY-VALUE PROBLEM (LAPLACE'S EQUATION) SHOWN IN FIG. 14.14. ni = NO. OF ITERATIONS nx = NO. OF X GRID POINTS ny = NO. OF Y GRID POINTS v(i,j) = POTENTIAL AT GRID POINT (i,j) OR (x,y) WITH NODE NUMBERING STARTING FROM THE LOWER LEFT-HAND CORNER OF THE TROUGH v1 = 10.0; v2 = 100.0; v3 = 40.0; v4 = 0.0; ni = 200; nx = 16; ny = 11; % SET INITIAL VALUES EQUAL TO ZEROES v = zeros(nx,ny); % FIX POTENTIALS ARE FIXED NODES for i=2:nx-1 v(i,1) = v1; v(i,ny) = v3; end for j=2:ny-1 v(1,j) = v4; v(nx,j) = v2; end v(1,1) = 0.5*(v1 + v4); v(nx,1) = 0.5*(v1 + v2); v(1,ny) = 0.5*(v3 + v4); v(nx,ny) = 0.5*(v2 + v3); % NOW FIND v(i,j) USING EQ.

(14.15) AFTER ni ITERATIONS for k=1:ni for i=2:nx-1 for j=2:ny-1 v(i,j) = 0.25*(v(i+1,j) + v(i-1,j) + v(i,j+1) + v(i,j-1)); end end end diary a:test1.out [v(6,6), v(9,9), v(11,6), v(9,3)] [[1:nx, 1:ny] v(i,j)] diary off FIGURE 14.16 Computer program for Example 14.4. 14_Sadiku_Ch14.indd 778 14/11/17 2:12 PM 14.4 The Moment Method 779 TABLE 14.1 Solution of Example 14.4 (Iteration Method) at Selected Points Coordinates (x, y) (0.5, 0.5) (0.8, 0.8) (1.0, 0.5) (0.8, 0.2) Number of Iterations 50 100 200 Exact Value 20.91 22.44 22.49 37.7 38.56 38.59 41.83 43.18 43.2 19.87 20.94 20.97 22.44 38.55 43.22 20.89 PRACTICE EXERCISE 14.4 Consider the trough of Figure 14.17. Use a five-node finite difference scheme to find the potential at the center of the trough using (a) a 4 3 8 grid, and (b) a 12 3 24 grid. Answer: (a) 31.08 V, (b) 42.86 V. FIGURE 14.17 For Practice Exercise 14.4. 14.4 THE MOMENT METHOD Like the finite difference method, the moment method,2 or the method of moments (MOM), has the advantage of being conceptually simple. While the finite difference method is used in solving differential equations, the moment method is commonly used in solving integral equations. 2 The term "moment method" was first used in the Western literature by Harrington. For further e xposition on the method, see R. F. Harrington, Field Computation by Moment Methods. New York: IEEE Press, 1993. 14_Sadiku_Ch14.indd 779 14/11/17 2:12 PM 780 CHAPTER 14 NUMERICAL METHODS For example, suppose we want to apply the moment method to solve Poisson's equation in eq.

(14.9a). It can be shown that an integral solution to Poisson's equation is V53 rv dv (14.26) v 4per We recall from Chapter 4 that eq. (14.26) can be derived from Coulomb's law. We also recall that given the charge distribution rv 1 x, y, z 2 , we can always find the potential V 1 x, y, z 2 , the electric field E 1 x, y, z 2 , and the total charge Q.

If, on the other hand, the potential V is known and the charge distribution is unknown, how do we determine rv from eq. (14.26)? In that situation, eq. (14.26) becomes what is called an integral equation. An integral equation is one involving the unknown function under the integral sign. It has the general form of V 1 x 2 5 3 K 1 x, t 2 r 1 t 2 dt(14.27) b a where the functions K 1 x, t 2 and V 1 t 2 and the limits a and b are known. The unknown function r 1 t 2 is to be determined; the function K 1 x, t 2 is called the kernel of the equation. The moment method is a common numerical technique used in solving integral equations such as in eq. (14.27). The method is probably best explained with an example. Consider a thin conducting wire of radius a, length L 1 L W a 2 located in free space as shown in Figure 14.18. Let the wire be maintained at a potential of Vo. Our goal is to determine the charge density rL along the wire by using the moment method. Once we have determined rL, related field quantities can be found. At any point on the wire, eq. (14.26) reduces to an integral equation of the form L rL dl Vo 5 3 (14.28) 0 4peor FIGURE 14.18 Thin conducting wire held at a constant potential. 14_Sadiku_Ch14.indd 780 14/11/17 2:12 PM 14.4 The Moment Method 781 Since eq. (14.28) applies for observation points everywhere on the wire, at a fixed point yk known as the match point. Vo 5 L rL 1 y 2 dy 1 (14.29) 3 0 4peo 0 yk 2 y 0 We recall from calculus that integration is essentially finding the area under a curve. If Dy is small, the integration of f(y) over 0 , y , L is given by . . . 1 f 1 y 2 Dy 3 f 1 y 2 dy . f 1 y1 2 Dy 1 f 1 y2 2 Dy 1 N L 0 5 a f 1 yk 2 Dy N (14.30) k51 where the interval L has been divided into N units, each having length Dy. With the wire divided into N segments of equal length D as shown in Figure 14.19, eq. (14.29) becomes 4peoVo .

r1 D r2 D rN D 1 1...1 (14.31) 0 yk 2 y1 0 0 yk 2 y2 0 0 yk 2 yN 0 where D 5 L/N 5 Dy. The assumption in eq. (14.31) is that the unknown charge density rk on the kth segment is constant on that segment. The kth term in eq.

(14.31) has |yk – yk| in the denominator and causes numerical problems. We shall soon circumvent this problem by modeling the line s egment by means of a cylindrical surface charge.

Thus in eq. (14.31), we have unknown constants r1, r2, . . .

, rN. Since eq. (14.31) must hold at all points on the wire, we obtain N similar equations by choosing N match points at y1, y2, . .

. , yk, . . . , yN on the wire. Thus we obtain 4peoVo 5 4peoVo 5 r1 D r2 D rND 1 1...1 (14.32a) 0 y1 2 y1 0 0 y1 2 y2 0 0 y1 2 yN 0 r1 D r2 D rND 1 1...1 (14.32b) 0 y2 2 y1 0 0 y2 2 y2 0 0 y2 2 yN 0 . . . r1 D r2 D rND 4peoVo 5 1 1...1 (14.32c) 0 yN 2 y1 0 0 yN 2 y2 0 0 yN 2 yN 0 The idea of matching the left-hand side of eq. (14.29) with the right-hand side of the equation at the match points is similar to the concept of taking moments in mechanics. Here lies the reason this technique is called the moment method. Notice from Figure 14.19 that the match points y1, y2, . . . , yN are placed at the center of each segment. Equation (14.32) can be put in matrix form as 14_Sadiku_Ch14.indd 781 3 B 4 5 3 A 4 3 r 4 (14.33) 14/11/17 2:12 PM 782 CHAPTER 14 NUMERICAL METHODS FIGURE 14.19 Division of the wire into N segments. where 1 1 . 3 B 4 5 4peoVo F V(14.34) . . 1 A11 A21 ...

... A1N A2N . V . (14.35a) . AN2 ... ANN . 3A 4 5 F . . AN1 A12 A22 Amn 5 D , 0 ym 2 yn 0 m 2 n (14.35b) r1 r2 . 3 r 4 5 F V (14.36) . . rN In eq. (14.33), [r] is the matrix whose elements are unknown. We can determine [r] from eq.

(14.33) by using Cramer's rule, matrix inversion, or the Gaussian elimination technique. With matrix inversion, 14_Sadiku_Ch14.indd 782 3 r 4 5 3 A 4 21 3 B 4 (14.37) 14/11/17 2:12 PM 14.4 The Moment Method 783 where 3 A 4 21 is the inverse of matrix [A]. In evaluating the diagonal elements (or self terms) of matrix [A] in eq. (14.35), caution must be exercised. Since the wire is conducting, a surface charge density rS is expected over the wire surface. Hence at the center of each segment, V 1 center 2 5 5 Assuming D W a, 2p D/2 rSa df dy 1 3 3 4peo 0 2D/2 3 a2 1 y2 4 1/2 2parS D/2 1 3 1 D/2 2 2 1 a2 4 1/2 ln e f 4peo 2D/2 1 3 1 D/2 2 2 1 a2 4 1/2 V 1 center 2 5 2parS D 2 ln a b a 4peo (14.38) 2rL D 5 ln a b a . 4peo where rL 5 2p arS. Thus, the self terms 1 m 5 n 2 are Equation (14.33) now becomes D 2 ln a b a D 0 y2 2 y1 0 I .

. . D 0 yN 2 y1 0 D Ann 5 2 ln a b(14.39) a D 0 y1 2 y2 0 ... D 2 ln a b a ... D 0 yN 2 y2 0 ... D 0 y1 2 yN 0 r1 1 D r2 1 0 y2 2 yN 0 . Y . . F V 5 4peoVo F V (14.40) . . . . . . rN 1 D 2 ln a b a Using eq. (14.37) with eq. (14.40) and letting Vo 5 1 V, L 5 1 m, a 5 1 mm, and N 5 20 1 D 5 L/N 2 , a MATLAB code such as in Figure 14.20 can be developed. The program in Figure 14.20 is self-explanatory. It inverts matrix [A] and plots rL against y. The plot is shown in Figure 14.21. The program also determines the total charge on the wire using 14_Sadiku_Ch14.indd 783 Q 5 3 rL dl(14.41) 14/11/17 2:12 PM 784 CHAPTER 14 NUMERICAL METHODS % % % % THIS PROGRAM DETERMINES THE CHARGE DISTRIBUTION ON A CONDUCTING THIN WIRE, OF RADIUS AA AND LENGTH L, MAINTAINED AT VO VOLT THE WIRE IS LOCATED AT 0 < Y < L ALL DIMENSIONS ARE IN S.I. UNITS % % % MOMENT METHOD IS USED N IS THE NO. OF SEGMENTS INTO WHICH THE WIRE IS DIVIDED RHO IS THE LINE CHARGE DENSITY, RHO = INV(A)*B % FIRST, SPECIFY PROBLEM PARAMETERS ER = 1.0; EO = 8.8541e-12; VO = 1.0; AA = 0.001; L = 1.0; N = 20; DELTA = L/N; % SECOND, CALCULATE THE ELEMENTS OF THE COEFFICIENT % MATRIX A I=1:N; Y=DELTA*(I-O.5); for i=1:N for j=1:N if(i ~=j) A(i,j)=DELTA/abs(Y(i)-Y(j)); else A(i,j)=2.0*log(DELTA/AA); end end end % NOW DETERMINE THE MATRIX OF CONSTANT VECTOR B % AND FIND Q B = 4.0*pi*EO*ER*VO*ones(N,1); C = inv(A); RHO = C*B; SUM = 0.0; for I=1:N SUM = SUM + RHO(I); end Q=SUM*DELTA; diary a:exam145a.out [EO,Q] [[1:N]' Y' RHO] diary off % FINALLY PLOT RHO AGAINST Y plot(Y,RHO) xlabel('y (m)'), ylabel('rho_L (pC/m)') FIGURE 14.20 MATLAB code for calculating the charge distribution on the wire in Figure 14.18. 14_Sadiku_Ch14.indd 784 15/11/17 8:07 PM 14.4 The Moment Method 785 which can be written in discrete form as Q 5 a rk D(14.42) N k51 With the chosen parameters, the value of the total charge was found to be Q 5 8.5793 pC. If desired, the electric field at any point can be calculated by using E53 rL dl aR(14.43) 4peoR2 which can be written as N rk D R E5 a 3 (14.44) k51 4peoR where R 5 0 R 0 and R 5 r 2 rk 5 1 x 2 xk 2 ax 1 1 y 2 yk 2 ay 1 1 z 2 zk 2 az r 5 1 x, y, z 2 is the position vector of the observation point, and rk 5 1 xk, yk, zk 2 is that of the source point. Notice that to obtain the charge distribution in Figure 14.21, we have taken N 5 20. It should be expected that a smaller value of N would give a less accurate result and a larger value of N would yield a more accurate result. However, if N is too large, we may have the computation problem of inverting the square matrix [A]. The capacity of the computing facilities at our disposal can limit the accuracy of the numerical experiment.

10.5 L (pC/m) 10 9.5 9 8.5 8 0 0.1 0.2 0.3 0.4 0.5 0.6 y (m) 0.7 0.8 0.9 1 FIGURE 14.21 Plot of rL against y.

14_Sadiku_Ch14.indd 785 14/11/17 2:12 PM 786 CHAPTER 14 NUMERICAL METHODS EXAMPLE 14.5 Use the moment method to find the capacitance of the parallel-plate capacitor of Figure 14.22. Take a 5 1 m, b 5 1 m, d 5 1 m, and er 5 1.0. Solution: Let the potential difference between the plates be Vo 5 2 V so that the top plate P1 is maintained at 11 V while the bottom plate P2 is at 21 V. We would like to determine the surface charge density rS on the plates so that the total charge on each plate can be found as Q 5 3 rS dS Once Q is known, we can calculate the capacitance as C5 Q Q 5 Vo 2 To determine rS by means of the moment method, we divide P1 into n subsections: DS1, DS2, . . . , DSn and P2 into n subsections: DSn11, DSn12, .

. . , DS2n.

The potential Vi at the center of a typical subsection DSi is 2n rj dS rS dS 1 . a 3 S 4peoR j51 4peo DSi Rij Vi 5 3 2n 1 dS 5 a rj 3 4pe o DSj Rij j51 It has been assumed that there is uniform charge distribution on each subsection. The last equation can be written as Vi 5 a rj Aij 2n j51 where Aij 5 1 dS 3 4peo DSi Rij FIGURE 14.22 Parallel-plate capacitor; for Example 14.5. 14_Sadiku_Ch14.indd 786 14/11/17 2:12 PM 14.4 The Moment Method 787 Thus V1 5 a rj A1j 5 1 2n j51 V2 5 a rj A2j 5 1 2n j51 . . . Vn 5 a rj Anj 5 1 2n j51 Vn11 5 a rj An11, j 5 21 2n j51 . . . V2n 5 a rj A2n, j 5 21 2n j51 yielding a set of 2n simultaneous equations with 2n unknown charge densities rj. In matrix form, A11

A12 A21 . G . . A22 ... ... A2n,1 A2n,2 ... A1,2n r1 1 A2,2n r2 1 . . .
. W G .
W 5 G .W . . . 21 A2n,2n r2n 21 or 3A 4 3r 4 5 3B 4 Hence, 3 r 4 5 3 A 4 21 3 B 4 where [B] is the column matrix defining the potentials and [A] is a square matrix containing elements Aij. To determine Aij, consider the two subsections i and j shown in Figure 14.23 where the subsections could be on different plates or on the same plate. 2 2 dx dy 1 3 3 4peo y5y1 x5x1 Rij y Aij 5 14_Sadiku_Ch14.indd 787 x 14/11/17 2:12 PM 788 CHAPTER 14 NUMERICAL METHODS FIGURE 14.23 Subsections i and j; for Example 14.5. where Rij 5 3 1 xj 2 xi 2 2 1 1 yj 2 yi 2 2 1 1 zj 2 zi 2 2 4 1/2 For the sake of convenience, if we assume that the subsections are squares, x2 2 x1 5 D, 5 y2 2 y1 it can be shown that Aij 5 and Aii 5 1 D, 2 2 DSi 5 4peoRij 4peoRij i2j D, D, 1 0.8814 2 ln 1 1 1 "2 2 5 peo peo With these formulas, the MATLAB code in Figure 14.24 was developed. With n 5 9, C = 26.52 pF, with n 5 16, C 5 27.27 pF, and with n 5 25, C 5 27.74 pF. % % % % % USING THE METHOD OF MOMENT, THIS PROGRAM DETERMINES THE CAPACITANCE OF A PARALLEL-PLATE CAPACITOR CONSISTING OF TWO CONDUCTING PLATES, EACH OF DIMENSION AA x BB, SEPARATED BY A DISTANCE D, AND MAINTAINED AT 1 VOLT AND -1 VOLT % % ONE PLATE IS LOCATED ON THE Z=0 PLANE WHILE THE OTHER IS LOCATED ON THE Z=D PLANE % ALL DIMENSIONS ARE IN S.I. UNITS % N IS THE NUMBER IS SUBSECTIONS INTO WHICH EACH PLATE IS DIVIDED FIGURE 14.24 MATLAB program for Example 14.5. 14_Sadiku_Ch14.indd 788 14/11/17 2:12 PM 14.4 The Moment Method 789 % FIRST, SPECIFY THE PARAMETERS ER = 1.0; EO = 8.8541e-12; AA = 1.0; BB = 1.0; D = 1.0; N = 9; NT = 2*N; M = sqrt(N); DX = AA/M; DY = BB/M; DL = DX; % SECOND, CALCULATE THE ELEMENTS OF THE COEFFICIENT % MATRIX A K = 0; for K1=1:2 for K2=1:M for K3=1:M K = K + 1; X(K) = DX*(K2 - 0.5); Y(K) = DY*(K3 - 0.5); end end end for K1=1:N Z(K1) = 0.0; Z(K1+N) = D; end for I=1:NT for J=1:NT if(I==J) A(I,J) = DL*0.8814/(pi*EO); else R = sqrt((X(I)-X(J))^2 + (Y(I)-Y(J))^2 + (Z(I)-Z(J))^2); A(I,J) = DL^2/(4.*pi*EO*R); end end end % NOW DETERMINE THE MATRIX OF CONSTANT VECTOR B for K=1:N B(K) = 1.0; B(K+N) = -1.0; end % INVERT A AND CALCULATE RHO CONSISTING OF % THE UNKNOWN ELEMENTS % ALSO CALCULATE THE TOTAL CHARGE Q AND CAPACITANCE C F = inv(A); FIGURE 14.24 (Continued) 14_Sadiku_Ch14.indd 789 14/11/17 2:12 PM 790 CHAPTER 14 NUMERICAL METHODS RHO = F*B'; SUM = 0.0; for I=1:N SUM = SUM + RHO(I); end Q = SUM*(DL^2); VO = 2.0; C = abs(Q)/VO; diary [C] [[1:NT]' X Y' Z' RHO] diary off PRACTICE EXERCISE 14.5 Use the moment method to write a program to determine the capacitance of two identical parallel conducting wires separated at a distance yo and displaced by xo as shown in Figure 14.25. If each wire is of length L and radius a, find the capacitance for cases xo 5 0, 0.2, 0.4, . . . , 1.0 m. Take yo 5 0.5 m, L 5 1 m, a 5 1 mm, er 5 1. Answer: For N 5 10 5 number of segments per wire, see Table 14.2. FIGURE 14.25 Parallel conducting wires for Practice Exercise 14.5. TABLE 14.2 Capacitance for Practice Exercise 14.5 14_Sadiku_Ch14.indd 790 xo (m) C (pF) 0.0 0.2 0.4 0.6 0.8 1.0 4.91 4.891 4.853 4.789 4.71 4.643 14/11/17 2:12 PM 14.5 The Finite Element Method 791 14.5 THE FINITE ELEMENT METHOD

The finite element method (FEM) has its origin in the field of structural analysis. The method was not applied to EM problems until 1968.3 Like the finite difference method, the finite element method is useful in solving differential equations.

As noticed in Section 14.3, the finite difference method represents the solution region by an array of grid points; its application becomes difficult with problems having irregularly shaped boundaries. Such problems can be handled more easily by using the finite element method.

The finite element analysis of any problem involves basically four steps: (a) discretizing the solution region into a finite number of subregions or elements, (b) deriving governing equations for a typical element, (c) assembling all the elements in the solution region, and (d) solving the system of equations obtained. A. Finite Element Discretization We divide the solution region into a number of finite elements as illustrated in Figure 14.26, where the region is subdivided into four nonoverlapping elements (two triangular and two quadrilateral) and seven nodes. We will assume only triangular elements in this section. We seek an approximation for the potential Ve within an element e and then interrelate the potential distributions in various elements such that the potential is continuous across interelement boundaries. The approximate solution for the whole region is V 1 x, y 2 . a Ve 1 x, y 2 (14.45) N e51 where N is the number of triangular or quadrilateral elements into which the solution region is divided. The most common form of approximation for Ve within an element is polynomial approximation, namely, for a triangular element and Ve 1 x, y 2 5 a 1 bx 1 cy(14.46) Ve 1 x, y 2 5 a 1 bx 1 cy 1 dxy(14.47) FIGURE 14.26 A typical finite element subdivision of an irregular domain. N E 3 See P. P. Silvester and R. L. Ferrari, Finite Elements for Electrical Engineers, 3rd ed. Cambridge, U.K.: Cambridge Univ. Press, 1996. 14_Sadiku_Ch14.indd 791 14/11/17 2:12 PM 792 CHAPTER 14 NUMERICAL METHODS for a quadrilateral element. The potential Ve in general is nonzero within element e but zero outside e.

It is difficult to approximate the boundary of the solution region with quadrilateral elements; such elements are useful for problems whose boundaries are sufficiently regular.

In view of this, we prefer to use triangular elements throughout our analysis in this section. Notice that our assumption of linear variation of potential within the triangular element as in eq. (14.46) is the same as assuming that the electric field is uniform within the element; that is, Ee 5 2=Ve 5 2 1 b ax 1 c ay 2 (14.48) B. Element-Governing Equations Consider a typical triangular element, as shown in Figure 14.27. The potential Ve1, Ve2, and Ve3 at nodes 1, 2, and 3, respectively, are obtained by using eq.

(14.46); that is, Ve1 1 x1 £ Ve2 § 5 £ 1 x2 Ve3 1 x3 y1 a y2 § £ b § (14.49) y3 c The coefficients a, b, and c are determined from eq. (14.49) as a 1 x1 £ b § 5 £ 1 x2 c 1 x3 y1 21 Ve1 y2 § £ Ve2 § (14.50) y3 Ve3 Substituting this into eq. (14.46) gives Ve 5 3 1 x y 4 or 1 x2y3 2 x3y2 2 1 £ 1 y2 2 y3 2 2A 1 x 3 2 x2 2 1 x3y1 2 x1y3 2 1 y3 2 y1 2 1 x1 2 x 3 2 1 x1y2 2 x2y1 2 Ve1 1 y1 2 y2 2 § £ Ve2 § 1 x2 2 x 1 2 Ve3 Ve 5 a ai 1 x, y 2 Vei(14.51) 3 i51 FIGURE 14.27 Typical triangular element; the local node numbering 1-2-3 must be counterclockwise as indicated by the arrow.

14_Sadiku_Ch14.indd 792 14/11/17 2:12 PM 14.5 The Finite Element Method 793 where a1 5 a2 5 a3 5 1 3 1 x2y3 2 x3y2 2 1 1 y2 2 y3 2 x 1 1 x3 2 x2 2 y 4 (14.52a) 2A 1 3 1 x3y1 2 x1y3 2 1 1 y3 2 y1 2 x 1 1 x1 2 x3 2 y 4 (14.52b) 2A 1 3 1 x1y2 2 x2y1 2 1 1 y1 2 y2 2 x 1 1 x2 2 x1 2 y 4 (14.52c) 2A and A is the area of the element e; that is, 1 x1 2A 5 † 1 x2 1 x3 or y1 y2 † y3 5 1 x1y2 2 x2y1 2 1 1 x3y1 2 x1y3 2 1 1 x2y3 2 x3y2 2 A 5 1/2 3 1 x2 2 x1 2 1 y3 2 y1 2 2 1 x3 2 x1 2 1 y2 2 y1 2 4 (14.53) The value of A is positive if the nodes are numbered counterclockwise (starting from any node) as shown by the arrow in Figure 14.27. Note that eq. (14.51) gives the potential at any point 1 x, y 2 within the element, provided the potentials at the vertices are known. This is unlike the situation in finite difference analysis, where the potential is known at the grid points only. Also note that ai are linear interpolation functions. They are called the element shape functions, and they have the following properties: ai 1 xj, yj 2 5 e 1, 0, i5j (14.54a) i2j a ai 1 x, y 2 5 1(14.54b) 3 i51 The shape functions a1 and a2, for example, are illustrated in Figure 14.28. FIGURE 14.28 Shape functions a1 and a2 for a triangular element. 14_Sadiku_Ch14.indd 793 14/11/17 2:12 PM 794 CHAPTER 14 NUMERICAL METHODS The energy per unit length associated with the element e is given by eq. (4.96); that is, We 5 1 1 e 0 E 0 2 dS 5 3 e 0 =Ve 0 2 dS(14.55) 2 3S 2 S where a two-dimensional solution region free of charge 1 rV 5 0 2 is assumed. But from eq. (14.51), =Ve 5 a Vei =ai(14.56) 3 i51 Substituting eq. (14.56) into eq. (14.55) gives We 5 1 3 3 # a eVei c 3 =ai =aj dSd Vej (14.57) 2 a S i51 j51 If we define the term in brackets as Cij1e2 5 3 =ai # =aj dS (14.58) S we may write eq. (14.57) in matrix form as We 5 1 e 3 Ve 4 T 3 C1e2 4 3 Ve 4 (14.59) 2 where the superscript T denotes the transpose of the matrix Ve1 3 Ve 4 5 £ Ve2 § (14.60a) Ve3 and 1e2 C11 1e2 C12 1e2 C13 1e2 C31 1e2 C32 1e2 C33 1e2 3 C1e2 4 5 D C21 1e2 C22 1e2 C23 T (14.60b) The matrix [C(e)] is usually called the element coefficient matrix. The matrix element Cij1e2 of the coefficient matrix may be regarded as the coupling between nodes i and j; its value is obtained from eqs. (14.52) and (14.58). For example, 1e2 C12 5 3 =a1 # =a2 dS 14_Sadiku_Ch14.indd 794 1 3 1 y2 2 y3 2 1 y3 2 y1 2 1 1 x3 2 x2 2 1 x1 2 x3 2 4 3 dS(14.61a) 4A2 S 1 3 1 y2 2 y3 2 1 y3 2 y1 2 1 1 x3 2 x2 2 1 x1 2 x3 2 4 5 4A 5 14/11/17 2:12 PM 14.5 The Finite Element Method 795 Similarly: 1e2 C11 5 1e2 C13 5 1e2 C22 5 1e2 C23 5 1e2 C33 5 Also 1 3 1 y2 2 y3 2 2 1 1 x3 2 x2 2 2 4 (14.61b) 4A 1 3 1 y1 2 y2 2 1 y3 2 y1 2 1 1 x2 2 x1 2 1 x1 2 x3 2 2 4 (14.61c) 4A 1 3 1 y3 2 y1 2 2 1 1 x1 2 x3 2 2 4 (14.61d) 4A 1 3 1 y2 2 y3 2 1 y1 2 y2 2 1 1 x1 2 x3 2 1 x2 2 x1 2 4 (14.61e) 4A 1 3 1 y1 2 y2 2 2 1 1 x2 2 x1 2 2 4 (14.61f) 4A 1e2 1e2 C21 5 C12 , 1e2 1e2 C31 5 C13 , 1e2 1e2 C32 5 C23 (14.61g) However, our calculations will be easier if we define P1 5 1 y2 2 y3 2 , Q 1 5 1 x 3 2 x2 2 , P2 5 1 y3 2 y1 2 , Q 2 5 1 x 1 2 x3 2 , P3 5 1 y1 2 y2 2 (14.62a) Q 3 5 1 x 2 2 x1 2 With Pi and Qi 1 i 5 1, 2, 3 are the local node numbers 2 , each term in the element coefficient matrix is found as Cij1e2 5 where A5 1 3 P P 1 QiQj 4 (14.62b) 4A i j 1 1 P Q 2 P3Q2 2 (14.62c) 2 2 3 Note that P1 1 P2 1 P3 5 0 5 Q1 1 Q2 1 Q3 and hence a Cij1e2 5 0 5 a Cij1e2. i51 j51 3 3 This may be used in checking our calculations. C. Assembling All the Elements Having considered a typical element, the next step is to assemble all such elements in the solution region. The energy associated with the assemblage of all elements in the mesh is 14_Sadiku_Ch14.indd 795 N 1 W 5 a We 5 e 3 V 4 T 3 C 4 3 V 4 (14.63) 2 e51 14/11/17 2:12 PM 796 CHAPTER 14 NUMERICAL METHODS where V1 V2 . 3 V 4 5 F V(14.64) . . Vn n is the number of nodes, N is the number of elements, and [C] is called the overall or global coefficient matrix, which is the assemblage of individual element coefficient matrices. The major problem now is obtaining [C] from 3 C1e2 4 . The process by which individual element coefficient matrices are assembled to obtain the global coefficient matrix is best illustrated with an example. Consider the finite element mesh consisting of three finite elements as shown in Figure 14.29. Observe the numberings of the nodes. The numbering of nodes as 1, 2, 3, 4, and 5 is called global numbering. The numbering i-j-k is called local numbering, and it corresponds with 1-2-3 of the element in Figure 14.27. For example, for element 3 in Figure 14.29, the global numbering 3-5-4 corresponds to local numbering 1-2-3 of the element in Figure 14.27. Note that the local numbering must be in counterclockwise sequence starting from any node of the element.

For element 3, for example, we could choose 4-3-5 or 5-4-3 instead of 3-5-4 to correspond with 1-2-3 of the element in Figure 14.27. Thus the numbering in Figure 14.29 is not unique. However, we obtain the same [C] whichever numbering is used. Assuming the particular numbering in Figure 14.29, the global coefficient matrix is expected to have the form C11 C21 3 C 4 5 EC31 C41 C51 C12 C22 C32 C42 C52 C13 C23 C33 C43 C53 C14 C24 C34 C44 C54 C15 C25 C35U(14.65) C45 C55 which is a 5 3 5 matrix, since five nodes 1 n 5 5 2 are involved. Again, Cij is the coupling between nodes i and j. We obtain Cij by utilizing the fact that the potential distribution FIGURE 14.29 Assembly of three elements: i-j-k corresponds to local numbering 1-2-3 of the element in Figure 14.27. 14_Sadiku_Ch14.indd 796 14/11/17 2:12 PM 14.5 The Finite Element Method 797 must be continuous across interelement boundaries. The contribution to the i, j position in [C] comes from all elements containing nodes i and j. To find C11, for example, we observe from Figure 14.29 that global node 1 belongs to elements 1 and 2 and it is local node 1 in both; hence, 112 122 C11 5 C11 1 C11 (14.66a) For C22, global node 2 belongs to element 1 only and is the same as local node 3; hence, 112 C22 5 C33 (14.66b) For C44, global node 4 is the same as local nodes 2, 3, and 3 in elements 1, 2, and 3, respectively; hence, 112 122 132 C44 5 C22 1 C33 1 C33 (14.66c) For C14, global link 14 is the same as the local links 12 and 13 in elements 1 and 2, respectively; hence, 112 122 C14 5 C12 1 C13 (14.66d) Since there is no coupling (or direct link) between nodes 2 and 3, C23 5 C32 5 0(14.66e) Continuing in this manner, we obtain all the terms in the global coefficient matrix by inspection of Figure 14.29 as 112 122 C11 1 C11 3C 4 5 112 C31 122 FC21 112 122 C21 1 C31 0 112 C13 122 C12 0 122 C22 112 C33 112 C23 0 112 122 C12 1 C13 0 1 132 C11 122 132 C32 1 C31 132 C21 122 C23 112 C32 1 132 C13 112 122 132 C22 1 C33 1 C33 132 C23 0 0 132 C12 V(14.67) 132 C32 132 C22 Note that element coefficient matrices overlap at nodes shared by elements and that there are 27 terms (nine for each of the three elements) in the global coefficient matrix [C]. Also note the following properties of the matrix [C]: 1. It is symmetric 1 Cij 5 Cji 2 just like the element coefficient matrix. 2. Since Cij 5 0 if no coupling exists between nodes i and j, it is evident that for a large number of elements [C] becomes sparse and banded. 3. It is singular. Although this is less obvious, it can be shown by using the element coefficient matrix of eq.

(14.60b). 14_Sadiku_Ch14.indd 797 14/11/17 2:12 PM 798 CHAPTER 14 NUMERICAL METHODS D. Solving the Resulting Equations From variational calculus, it is known that Laplace's (or Poisson's) equation is satisfied when the total energy in the solution region is minimum. Thus we require that the partial derivatives of W with respect to each nodal value of the potential be zero; that is, 'W 'W 'W 5...5 50 'V1 'V2 'Vn or 'W 5 0, 'Vk k 5 1, 2, . . . , n(14.68) For example, to get 'W/'V1 5 0 for the finite element mesh of Figure 14.29, we substitute eq. (14.65) into eq. (14.63) and take the partial derivative of W with respect to V1. We obtain 05 'W 5 2V1C11 1 V2C12 1 V3C13 1 V4C14 1 V5C15 'V1 1 V2C21 1 V3C31 1 V4C41 1 V5C51 or 0 5 V1C11 1 V2C12 1 V3C13 1 V4C14 1 V5C15(14.69) In general, 'W/'Vk 5 0 leads to 0 5 a Vi Cik(14.70) n i51 where n is the number of nodes in the mesh. By writing eq. (14.70) for all nodes k 5 1, 2, . . . , n, we obtain a set of simultaneous equations from which the solution of 3 V 4 T 5 3 V1, V2, . . . , Vn 4 can be found. This can be done in two ways similar to those used in solving finite difference equations obtained from Laplace's (or Poisson's) equation. Iteration Method The iterative approach is similar to that used in the finite difference method. Let us assume that node 1 in Figure 14.29, for example, is a free node. The potential at node 1 can be obtained from eq. (14.69) as V1 5 2 1 5 ViC1i(14.71) C11 a i52 In general, the potential at a free node k is obtained from eq. (14.70) as 14_Sadiku_Ch14.indd 798 Vk 5 2 n 1 a ViCik(14.72) Ckk i51,i2k 14/11/17 2:12 PM 14.5 The Finite Element Method 799 This is applied iteratively to all the free nodes in the mesh with n nodes. Since Cki 5 0 if node k is not directly connected to node i, only nodes that are directly linked to node k contribute to Vk in eq. (14.72). Thus if the potentials at nodes connected to node k are known, we can determine Vk by using eq. (14.72).

The iteration process begins by setting the potentials at the free nodes equal to zero or to the average potential. Vave 5 1 1 V 1 Vmax 2 (14.73) 2 min where Vmin and Vmax are the minimum and maximum values of the prescribed potentials at the fixed nodes. With those initial values, the potentials at the free nodes are calculated by using eq. (14.72).

At the end of the first iteration, when the new values have been calculated for all the free nodes, these values become the old values for the second iteration. The procedure is repeated until the change between subsequent iterations is negligible. Band Matrix Method If all free nodes are numbered first and the fixed nodes last, eq. (14.63) can be written such that W5 1 C e 3 Vf Vp 4 c ff Cpf 2 Cfp Vf d c d (14.74) Cpp Vp where subscripts f and p, respectively, refer to nodes with free and fixed (or prescribed) potentials. Since Vp is constant (it consists of known, fixed values), we differentiate only with respect to Vf , so that applying eq. (14.68) to eq. (14.74) yields CffVf 1 CfpVp 5 0 or This equation can be written as or 3 Cff 4 3 Vf 4 5 2 3 Cfp 4 3 Vp 4 (14.75) 3 A 4 3 V 4 5 3 B 4 (14.76a) 3 V 4 5 3 A 4 21 3 B 4 (14.76b) where 3 V 4 5 3 Vf 4 , 3 A 4 5 3 Cff 4 , and 3 B 4 5 2 3 Cfp 4 3 Vp 4 . Since [A] is, in general, nonsingular, the potential at the free nodes can be found by using eq. (14.75). We can solve for [V] in eq. (14.76a) by using the Gaussian elimination technique. We can also use matrix inversion to solve for [V] in eq. (14.76b) if the size of the matrix to be inverted is not large. 14_Sadiku_Ch14.indd 799 14/11/17 2:12 PM 800 CHAPTER 14 NUMERICAL METHODS Notice that as from eq. (14.55) onward, our solution has been restricted to a twodimensional problem involving Laplace's equation, =2V 5 0. The basic concepts developed in this section can be extended to finite element analysis of problems involving Poisson's equation 1 =2V 5 2rv /e, =2A 5 2mJ 2 or the wave equation 1 =2f 2 g2f 5 0 2 . A major problem associated with finite element analysis is the relatively large amount of computer memory required for storing the matrix elements, as well as the associated computational time. However, several algorithms have been developed to alleviate the problem to some degree. The finite element method has a number of advantages over the finite difference method and the method of moments.

First, the FEM can easily handle the complex solution region. Second, the generality of the FEM makes it possible to construct a generalpurpose program for solving a wide range of problems. A single program can be used to solve different problems (described by the same partial differential equations) with different solution regions and different boundary conditions; only the input data to the problem need be changed. However, the FEM has its own drawbacks. It is harder to understand and program than the other methods (FDM and MOM). It also requires preparing input data, a process that could be tedious. EXAMPLE 14.6 Consider the two-element mesh shown in Figure 14.30(a). Using the finite element method, determine the potentials within the mesh. FIGURE 14.30 For Example 14.6: (a) twoelement mesh, (b) local and global numbering of the elements. 14_Sadiku_Ch14.indd 800 14/11/17 2:12 PM 14.5 The Finite Element Method 801 Solution: The element coefficient matrices can be calculated by using one of the relations of eq. (14.62). For element 1, consisting of nodes 1-2-4 corresponding to the local numbering 1-2-3 as in Figure 14.30(b), P1 5 21.3, P2 5 0.9, P3 5 0.4 Q1 5 20.2, Q2 5 20.4, Q3 5 0.6 A 5 1/2 1 0.54 1 0.16 2 5 0.35 Substituting all these into eq. (14.62b) gives 1.236 20.7786 20.4571 3 C112 4 5 £ 20.7786 0.6929 0.0857 § (14.6.1) 20.4571 0.0857 0.3714 Similarly, for element 2, consisting of nodes 2-3-4 corresponding to local numbering 1-2-3, as in Figure 14.30(b), P1 5 20.6, P2 5 1.3, P3 5 20.7 Q1 5 20.9, Q2 5 0.2, Q3 5 0.7 A 5 1/2 1 0.91 1 0.14 2 5 0.525 Hence, 0.5571 3 C122 4 5 £ 20.4571 20.1 20.4571 20.1 0.8238 20.3667 § (14.6.2) 20.3667 0.4667 Applying eq. (14.75) gives c C22 C42 C21 C24 V2 d c d 5 2c C44 V4 C41 This can be written in a more convenient form as 1 0 ≥ 0 0 0 C22 0 C42 C23 V1 d c d (14.6.3) C43 V3 0 0 V1 1 0 C24 V2 2C21 ¥ ≥ ¥ 5 ≥ 1 0 V3 0 0 C44 V4 2C41 0 2C23 V ¥ c 1 d (14.6.4a) 1 V3 2C43 or 14_Sadiku_Ch14.indd 801 3 C 4 3 V 4 5 3 B 4 (14.6.4b) 14/11/17 2:12 PM 802 CHAPTER 14 NUMERICAL METHODS The terms of the global coefficient matrix are obtained as follows: 112 122 C22 5 C22 1 C11 5 0.6929 1 0.5571 5 1.25 112 122 C42 5 C24 5 C23 1 C13 5 0.0857 2 0.1 5 20.0143 112 122 C44 5 C33 1 C33 5 0.3714 1 0.4667 5 0.8381 112 C21 5 C21 5 20.7786 122 C23 5 C12 5 20.4571 112 C41 5 C31 5 20.4571 122 C43 5 C32 5 20.3667 Note that we follow local numbering for the element coefficient matrix and global numbering for the global coefficient matrix. Thus the square matrix [C] is obtained as 1 0 0 0 0 1.25 0 20.0143 3C 4 5 ≥ ¥ (14.6.5) 0 0 1 0 0 20.0143 0 0.8381 and the matrix [B] on the right-hand side of eq. (14.6.4a) is obtained as 0 4.571 3B 4 5 ≥ ¥ (14.6.6) 10.0 3.667 By inverting matrix [C] in eq. (14.6.5), we obtain 0 3.708 3 V 4 5 3 C 4 21 3 B 4 5 ≥ ¥ 10.0 4.438 Thus V1 5 0, V2 5 3.708, V3 5 10, and V4 5 4.438. Once the values of the potentials at the nodes are known, the potential at any point within the mesh can be determined by using eq.

(14.51). 14_Sadiku_Ch14.indd 802 14/11/17 2:12 PM 14.5 The Finite Element Method 803 PRACTICE EXERCISE 14.6 Calculate the global coefficient matrix for the two-element mesh shown in Figure 14.31 when (a) node 1 is linked with node 3 and the local numbering (i-j-k) is as indicated in Figure 14.31(a), (b) node 2 is linked with node 4 with local numbering as in Figure 14.31(b). 0.9964 0.05 0.05 0.7 Answer: (a) ≥ 20.2464 20.75 20.8 0.0 1.333 20.0777 (b) ≥ 0.0 21.056 20.7777 0.8192 20.98 0.2386 20.2464 20.75 1.5964 20.75 0.0 20.98 2.04 21.06 20.8 0.0 ¥.

20.75 1.4 21.056 0.2386 ¥. 21.06 1.877 FIGURE 14.31 For Practice Exercise 14.6. EXAMPLE 14.7 Write a program to solve Laplace's equation by means of the finite element method. Apply the program to the two-dimensional problem shown in Figure 14.32(a). Solution: The solution region is divided into 25 three-node triangular elements with the total number of nodes being 21, as shown in Figure 14.32(b). This step is necessary to have input data defining the geometry of the problem. Based on our discussions thus far, a general MATLAB program for solving problems involving Laplace's equation by using three-node triangular elements was developed as in Figure 14.33. The development of the program basically involves four steps indicated in the program and explained as follows. Step 1: This involves inputting the necessary data defining the problem. This is the only step that depends on the geometry of the problem at hand. Through a data file, we input the number of elements, the number of nodes, the number of fixed nodes, the prescribed values of the potentials at the free nodes, the x- and y-coordinates of all nodes, and a list identifying the 14_Sadiku_Ch14.indd 801 17/10/17 12:08 PM 804 CHAPTER 14 NUMERICAL METHODS FIGURE 14.32 For Example 14.7: (a) two-dimensional electrostatic problem, (b) solution region divided into 25 triangular elements. % % % % % % % % % FINITE ELEMENT SOLUTION OF LAPLACE'S EQUATION FOR TWO-DIMENSIONAL PROBLEMS TRIANGULAR ELEMENTS ARE USED ND = NO. OF NODES NE = NO. OF ELEMENTS NP = NO. OF FIXED NODES (WHERE POTENTIAL IS PRESCRIBED) NDP(I) = NODE NO. OF PRESCRIBED POTENTIAL, I=1,2,...,NP VAL(I) = VALUE OF PRESCRIBED POTENTIAL AT NODE NDP(I) NL(I,J) = LIST OF NODES FOR EACH ELEMENT I, WHERE FIGURE 14.33 Computer program for Example 14.7. 14_Sadiku_Ch14.indd 804 14/11/17 2:12 PM 14.5 The Finite Element Method 805 % % % % % % % % % J=1,2,3 REFERS TO THE LOCAL NODE NUMBER CE(I,J) = ELEMENT COEFFICIENT MATRIX C(I,J) = GLOBAL COEFFICIENT MATRIX B(I) = RIGHT-HAND SIDE MATRIX IN THE SYSTEM OF SIMULTANEOUS EQUATIONS; SEE EQ. (14.6.4) X(I), Y(I) = GLOBAL COORDINATES OF NODE I XL(J), YL(J) = LOCAL COORDINATES OF NODE J=1,2,3 V(I) = POTENTIAL AT NODE I MATRICES P(I) AND Q (I) ARE DEFINED IN EQ.

(14.62a) % % ** FIRST STEP - INPUT DATA DEFINING GEOMETRY AND BOUNDARY CONDITIONS ** % clear input('Name of input data file = ') % ** % SECOND STEP - EVALUATE COEFFICIENT MATRIX FOR EACH ELEMENT AND ASSEMBLE GLOBALLY % ** B = zeros(ND,1); C = zeros(ND,ND); for I=1:NE % FIND LOCAL COORDINATES XL(J), YL(J) FOR ELEMENT I K = NL(I,[1:3]); XL = X(K); YL = Y(K); P=zeros(3,1); Q=zeros(3,1); P(1) = YL(2) - YL(3); P(2) = YL(3) - YL(1); P(3) = YL(1) - YL(2); Q(1) = XL(3) - XL(2); Q(2) = XL(1) - XL(3); Q(3) = XL(2) - XL(1); AREA = 0.5*abs(P(2)*Q(3) - Q(2)*P(3)); % DETERMINE COEFFICIENT MATRIX FOR ELEMENT I CE=(P*P'+Q*Q')/(4.0*AREA); % ASSEMBLE GLOBALLY - FIND C(I,J) AND B(I) for J=1:3 IR = NL(I,J); IFLAG1=0; % CHECK IF ROW CORRESPONDS TO A FIXED NODE for K = 1:NP if (IR == NDP(K)) FIGURE 14.33 (Continued) 14_Sadiku_Ch14.indd 805 15/11/17 8:07 PM 806 CHAPTER 14 NUMERICAL METHODS C(IR,IR) = 1.0; B(IR) = VAL(K); IFLAG1=1; end end % end for K = 1:NP if(IFLAG1 == 0) for L = 1:3 IC = NL(I,L); IFLAG2=0; % CHECK IF COLUMN CORRESPONDS TO A FIXED NODE for K=1:NP if (IC == NDP(K)) , B(IR) = B(IR) - CE(J,L)*VAL(K); IFLAG2=1; end end % end for K=1:NP if(IFLAG2 == 0) C(IR,IC) = C(IR,IC) + CE(J,L); end end % end for L=1:3 end %end if(iflag1 == 0) end % end for J=1:3 end % end for I=1:NE % ** % THIRD STEP - SOLVE THE SYSTEM OF EQUATIONS % ** V = inv(C)*B; V=V'; % ** % FOURTH STEP - OUTPUT THE RESULTS % ** diary exam147.out [ND, NE, NP] [[1:ND]' X' Y' V'] diary off FIGURE 14.33 (Continued) nodes belonging to each element in the order of the local numbering 1-2-3. For the problem in Figure 14.32, the three sets of data for coordinates, the element–node relationship, and the prescribed potentials at fixed nodes are shown in Tables 14.3, 14.4, and 14.5, respectively. Step 2: This step entails finding the element coefficient matrix [C(e)] for each element and the global coefficient matrix [C]. The procedure explained in Example 14.6 is applied.

Equation (14.6.4) can be written in general form as c 14_Sadiku_Ch14.indd 806 1 0 0 VP 1 d c d 5 c d 3 Vp 4 Cff Vf 2Cfp 14/11/17 2:12 PM 14.5 The Finite Element Method 807 TABLE 14.3 Nodal Coordinates of the Finite Element Mesh of Figure 14.33 Node x y Node x 1 2 3 4 5 6 7 8 9 10 11 0.0 0.0 0.2 0.0 0.4 0.0 0.6 0.0 0.8 0.0 1.0 0.0 0.0 0.2 0.2 0.2 0.4 0.2 0.6 0.2 0.8 0.2 12 13 14 15 16 17 18 19 20 21 y 0.0 0.4 0.2 0.4 0.4 0.4 0.6 0.4 0.0 0.6 0.2 0.6 0.4 0.6 0.0 0.8 0.2 0.8 0.0 1.0 TABLE 14.4 Element–Node Identification Local Node No. Local Node No. Element No. 1 2 3 Element No. 1 2 3 1 2 3 4 5 6 7 8 9 10 11 12 13 1 2 7 2 8 7 2 3 8 3 9 8 3 4 9 4 10 9 4 5 10 5 11 10 5 6 11 7 8 12 8 13 12 8 9 13 9 14 13 14 9 10 14 10 15 14 16 10 11 15 17 12 13 16 18 13 17 16 19 13 14 17 20 14 18 17 21 14 15 18 22 16 17 19 23 17 20 19 24 17 18 20 25 19 20 21 TABLE 14.5 Prescribed Potentials at Fixed Nodes Node No. Prescribed Prescribed Potential Node No. Potential 1 0.0 18 100.0 2 0.0 20 100.0 3 0.0 21 50.0 4 0.0 19 0.0 5 0.0 16 0.0 6 50.0 12 0.0 11 100.0 7 0.0 15 100.0 14_Sadiku_Ch14.indd 807 14/11/17 2:12 PM 808 CHAPTER 14 NUMERICAL METHODS or 3C 4 3V 4 5 3B 4 Both the "global" matrix [C] and matrix [B] are calculated at this stage. Step 3: The global matrix obtained in step 2 is inverted. The values of the potentials at all nodes are obtained by matrix multiplication as in eq. (14.76b). Instead of inverting the global matrix, it is also possible to solve for the potentials at the nodes by using the Gaussian elimination technique. Step 4: Finally, the result of the computation is provided. The input and output data are presented in Tables 14.6 and 14.7, respectively. TABLE 14.6 Input Data for the Finite Element Program in Figure 14.33 NE ND NP NL = = = = 2 2 25; 21; 15; [1 2 7 8 7 3 8 3 9 8 3 4 9 4 10 9 4 5 10 5 11 10 5 6 11 7 8 12 8 13 12 8 9 13 9 14 13 14 9 10 14 10 15 14 10 15 16 11 15 17 12 13 16 18 13 17 16 13 14 17 14 18 17 14 15 18 16 17 19 17 19 17 20 19 17 18 20 19 20 21]; X = [0.0 0.2 0.6 0.2 0.4 0.0 0.4 0.6 0.2 0.6 0.8 0.4 0.8 0.0 0.0 1.0 0.2 0.2 0.0 ... 0.4 ... 0.0]; Y = [0.0 0.2 0.4 0.0 0.2 0.6 0.0 0.2 0.6 0.0 0.2 0.6 0.0 0.4 0.8 0.0 0.4 0.8 0.2 ... 0.4 ... 1.0]; NDP = [1 2 3 4 5 6 11 15 18 20 21 19 16 12 7]; VAL = [0.0 0.0 0.0 0.0 0.0 ... 50.0 100.0 100.0 100.0 100.0 50.0 0.0 0.0 0.0 0.0]; 14_Sadiku_Ch14.indd 808 14/11/17 2:12 PM 14.5 The Finite Element Method 809 TABLE 14.7 Output Data of the Program in Figure 14.33 Node 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 X Y Potential 0.00 0.00 0.000 0.20 0.00 0.000 0.40 0.00 0.000 0.60 0.00 0.000 0.80 0.00 0.000 1.00 0.00 50.000 0.00 0.20 0.000 0.20 0.20 18.182 0.40 0.20 36.364 0.60 0.20 59.091 0.80 0.20 100.000 0.00 0.40 0.000 0.20 0.40 36.364 0.40 0.40 68.182 0.60 0.40 100.000 0.00 0.60 0.000 0.20 0.60 59.091 0.40 0.60 100.000 0.00 0.80 0.000 0.20 0.80 100.000 0.00 1.00 50.000 FIGURE 14.34 For Practice Exercise 14.7. 14_Sadiku_Ch14.indd 809 14/11/17 2:12 PM 810 CHAPTER 14 NUMERICAL METHODS PRACTICE EXERCISE 14.7 Rework Example 14.3 using the finite element method. Divide the solution region into triangular elements as shown in Figure 14.34. Compare the solution with that obtained in Example 14.3 using the finite difference method. Answer: See Example 14.3. † 14.6 APPLICATION NOTE—MICROSTRIP LINES The numerical methods covered in this chapter have been applied successfully to solve many EM-related problems.

Besides the simple examples considered earlier in the chapter, the methods have been applied to diverse problems including transmission line problems, EM penetration and scattering problems, EM pulse (EMP) problems, EM e xploration of minerals, and EM energy deposition in human bodies. It is practically impossible to cover all these applications within the limited scope of this text. In this section, we use the finite difference method to consider the relatively easier problem of transmission lines.

The finite difference techniques are suited for computing the characteristic impedance, phase velocity, and attenuation of several transmission lines: polygonal lines, shielded strip lines, coupled strip lines, microstrip lines, coaxial lines, and rectangular lines. The knowledge of the basic parameters of these lines is of paramount importance in the design of microwave circuits. For concreteness, consider the microstrip line shown in Figure 14.35(a). The geometry in Figure 14.35(a) is deliberately selected to illustrate how one uses the finite difference technique to account for discrete inhomogeneities (i.e., homogeneous media separated by interfaces) and lines of symmetry. The techniques presented are equally applicable to other lines. Because the mode is TEM, having components of neither E nor H fields in the direction of propagation, the fields obey Laplace's equation over the line cross section. The TEM mode assumption provides good approximations if the line dimensions are much 2b y «1 = «o 2w a t 2a «2 = «o «r «1 = «o 2d Vd d «2 = «o «r 0 (a) 0V w b x (b) FIGURE 14.35 (a) Shielded double strip line with partial dielectric support. (b) Problem in (a) simplified by taking advantage of symmetry. 14_Sadiku_Ch14.indd 810 14/11/17 2:12 PM 14.6 Application Note—Microstrip Lines 811 smaller than half a wavelength, which means that the operating frequency is far below cutoff frequency for all higher-order modes. Also, owing to biaxial symmetry about the two axes, only one-quarter of the cross section need be considered, as shown in Figure 14.35(b). The finite difference approximation of Laplace's equation, =2V 5 0, was derived in eq. (14.15), namely 1 V 1 i,j 2 5 3 V 1 i 1 1,j 2 1 V 1 i 2 1,j 2 1 V 1 i,j 1 1 2 1 V 1 i,j 2 1 2 4 (14.77) 4 For the sake of conciseness, let us denote Vo V(i, j) V1 V(i, j 1) V2 V(i 1, j)(14.78) V3 V(i, j 1) V4 V(i 1, j) so that eq. (14.77) becomes 1 Vo 5 3 V1 1 V2 1 V3 1 V4 4 4 (14.79) with the computation molecule as shown in Figure 14.36. Equation (14.79) is the general formula to be applied to all free nodes in the free space and dielectric region of Figure 14.35(b). The only limitation on eq. (14.79) is that region is discretely homogeneous. On the dielectric boundary, the boundary condition D1n D2n (14.80) FIGURE 14.36 Computation molecule for Laplace's equation. V1 h V2 Vo h h h y V4 x V3 14_Sadiku_Ch14.indd 811 14/11/17 2:12 PM 812 CHAPTER 14 NUMERICAL METHODS must be imposed. We recall that this condition is based on Gauss's law for the electric field; that is, # # C Dn dl 5 C En dl 5 Qenc/m 5 0 (14.81) L L since no free charge is deliberately placed on the dielectric boundary. Substituting E 5 2,V in eq. (14.81) gives 'V 0 5 C e,V # dl 5 C e dl L L 'n (14.82) where ∂V/∂n denotes the derivative of V normal to the contour L. Applying eq. (14.82) to the interface in Figure 14.37 yields 0 5 e1 1 V1 2 Vo 2 1 V2 2 Vo 2 h 1 V2 2 Vo 2 h h 1 e1 1 e2 h h 2 h 2 1 e2 1 V3 2 Vo 2 1 V4 2 Vo 2 h 1 V4 2 Vo 2 h h 1 e2 1 e1 h h 2 h 2 Rearranging the terms, we have 2 1 e1 1 e2 2 Vo 5 e1V1 1 e2V3 1 or Vo 5 e1 e2 1 1 V1 1 V3 1 V2 1 V4 (14.83) 4 4 2 1 e1 1 e2 2 2 1 e1 1 e2 2 FIGURE 14.37 Interface between media of dielectric permittivities 1 and 2.

V1 «1 V2 1 e1 1 e2 2 1 V2 1 V4 2 2 L Vo V4 «2 V3 14_Sadiku_Ch14.indd 812 14/11/17 2:12 PM 14.6 Application Note—Microstrip Lines 813 This is the finite difference equivalent of the boundary condition in eq. (14.80). Notice that the discrete inhomogeneity does not affect points 2 and 4 on the boundary but affects points 1 and 3 in proportion to their corresponding permittivities. Also note that when e2 5 e1, eq. (14.83) reduces to eq. (14.79). On the line of symmetry, we impose the condition 'V 5 0 (14.84) 'n This implies that on the line of symmetry along the y-axis, (x 0 or i 0), 1 V4 2 V2 2 'V 5 5 0 or V2 V4 so that eq. (14.79) becomes 'x 2h Vo 5 or VA0, jB 5 1 3 V 1 V3 1 2V4 4 (14.85a) 4 1 1 3 VA0, j 1 1B 1 VA0, j 2 1B 1 2VA1, jB 4 1 V1 2 V3 2 'V On the line of symmetry along the x-axis (y 0 or j 0), 5 5 0, 'y 2h orV3 5 V1, so that Vo 5 or V 1 i,0 2 5 1 3 2V1 1 V2 1 V4 4 (14.86a) 4 1 3 2V1 1 i,1 2 1 V 1 i 2 1,0 2 1 V 1 i 1 1,0 2 4 (14.86b) 4 The computation molecules for eqs. (14.85) and (14.86) are displayed in Figure 14.38. FIGURE 14.38 Computation molecules used for satisfying symmetry conditions: (a) ∂V/dx 5 0 (b) ∂V/dy 5 0 y V1 y Vo V4 V1 V3 x (a) 14_Sadiku_Ch14.indd 813 V2 Vo V4 x (b) 14/11/17 2:12 PM 814 CHAPTER 14 NUMERICAL METHODS By setting the potential at the fixed nodes equal to their prescribed values and applying eqs. (14.79), (14.83), (14.85), and (14.86) to the free nodes according to the band m atrix or iterative methods discussed in Section 14.3, the potential at the free nodes can be determined. Once this is accomplished, the quantities of interest can be calculated. The characteristic impedance Zo and phase velocity u of the line are defined as Zo 5 u5 L (14.87a) ÅC 1 "LC (14.87b) where L and C are the inductance and capacitance per unit length, respectively. If the dielectric medium is nonmagnetic 1 m 5 mo 2 , the characteristic impedance Zoo and phase velocity uo with the dielectric removed (i.e., the line is air filled) are given by Zoo 5 uo 5 L (14.88a) Å Co 1 "LCo (14.88b) where Co is the capacitance per unit length without the dielectric. Combining eqs. (14.87) and (14.88) yields Zo 5 1 uo"CCo 5 1 (14.89a) uC uo Co 5 (14.89b) ÅC "eeff C eeff 5 (14.89c) Co u 5 uo where uo c 3 108 m/s, the speed of light in free space, and eff is the effective dielectric constant. Thus to find Zo and u for an inhomogeneous medium requires calculating the capacitance per unit length of the structure, with and without the dielectric substrate. If Vd is the potential difference between the inner and the outer conductor, C5 4Q (14.90) Vd so that the problem is reduced to finding the charge per unit length Q.

(The factor 4 is needed because we are working on only one-quarter of the cross section.) To find Q, we apply Gauss's law to a closed path L enclosing the inner conductor. We may select L as the rectangular path between two adjacent rectangles as shown in Figure 14.39. 14_Sadiku_Ch14.indd 814 14/11/17 2:12 PM 14.6 Application Note—Microstrip Lines 815 G H J h M K L P N l y x h FIGURE 14.39 The rectangular path used in calculating a charge enclosed. 'V Q 5 C D # dl 5 C e dl 'n L L (14.91) VG 2 VK VP 2 VN VM 2 VL VH 2 VL 5 ea bDy 1 ea bDy 1 ea bDx 1 ea bDx 1 c Dx Dx Dy Dy Since Dx 5 Dy 5 h, or Q 5 1 eVP 1 eVM 1 eVH 1 eVG 1 c2 2 1 eVN 1 2eVL 1 eVK 1 c2 (14.92) Q 5 eo 3 a eriVi for nodes i on external rectangle GHJMP with corners (such as J) not counted] 2eo 3 a eriVi for nodes i on inner rectangle KHL with corners (such as L) counted twice]. (14.93) The procedure is outlined as follows: 1. Calculate V (with the dielectric space replaced by free space), using eqs. (14.79), (14.83), (14.85), and (14.86). 2. Determine Q using eq. (14.93).

3. Find Co 5 4Qo /Vd. 4. Repeat steps 1 and 2 (with the dielectric space) and find C 4Q/Vd. 1 5. Finally, calculate Zo 5 , c 3 108 m/s. c"CCo 14_Sadiku_Ch14.indd 815 14/11/17 2:12 PM 816 CHAPTER 14 NUMERICAL METHODS EXAMPLE 14.8 Calculate Zo for the microstrip transmission line in Figure 14.35 with a b 2.5 cm, d 0.5 cm, w 1 cm, t 0.001 cm, 1 o, 2 2.35o. Solution: This problem is representative of the various problem types that can be solved by using the concepts developed in this section. A computer program shown in Figure 14.40 was developed based on the five-step procedure just outlined. By specifying the step size h and the number of iterations, the program first sets the potential at all nodes equal to zero. The potential on the outer conductor is also set equal to zero, while that on the inner conductor is set to 100 V so that Vd 100. The program finds Co when the dielectric slab is removed and C when the slab is in place and finally determines Zo. For a selected step size h, the number of iterations must be large enough and greater than the number of divisions along the x- or y-direction. Table 14.8 shows some typical results. % Using finite difference, % this programs finds the characteristic impedance of % a shielded microstrip line a=2.5; b=2.5; d=0.5; w=1; h=0.05; vd=100; ni=1000; nx=b/h; ny=a/h; nw=w/h; nd=d/h; er=2.35; eo=10^(-9)/(36*pi); e1=eo; e2=er*eo; u =3*10^8; % Initialization v=zeros(nx,ny); for i=1:nw % set the potential on the inner conductor to V_d v(i,nd)=vd; end % Calculate the potential everywhere p1=e1/(2.0*(e1+e2)); p2=e2/(2.0*(e1+e2)); for n=1:2 if n==1 er=1; else er=2.35; end FIGURE 14.40 MATLAB code for Example 14.8. 14_Sadiku_Ch14.indd 816 14/11/17 2:12 PM 14.6 Application Note—Microstrip Lines 817 for k=1:ni for i=2:nx-1 for j=2:nd-1 % below the interface v(i,j)=0.25*(v(i+1,j) + v(i-1,j) + v(i,j+1) + v(i,j-1)); end end for i=2:nx-1 for j=nd+1:ny-1 % above the interface v(i,j)=0.25*(v(i+1,j) + v(i-1,j) + v(i,j+1) + v(i,j-1)) ; end end j=nd; % on the interface for i=nw+1:nx-1 v(i,j)=0.25*(v(i+1,j) + v(i-1,j)) + p1*v(i,j+1) + p2*v(i,j-1); end % on the lines of symmetry for i=2:nx-1 v(i,1)=0.25*(v(i+1,1) + v(i-1,1) + 2*v(i,2)); end for j=2:nd-1 v(1,j)=0.25*(2*v(2,j) + v(1,j+1) + v(1,j-1)); end for j=nd+1:ny-1 v(1,j)=0.25*(2*v(2,j) + v(1,j+1) + v(1,j-1)); end end % Now calculate the charge enclosed % Select two adjacent paths sum1=0.0; sum2=0.0; nm=fix(nd + 0.5*(ny-nd)); nn=fix(nx/2); for i=2:nn sum1= sum1 + v(i,nm); sum2= sum2 + v(i,nm+1); end sum1 = sum1 + 0.5*v(1,nm); sum2 = sum2 + 0.5*v(1,nm+1); for j=2:nd-1 sum1 = sum1 + er*v(nn,j); sum2 = sum2 + er*v(nn+1,j); end sum1 = sum1 + 0.5*er*v(nn,1); sum2 = sum2 + 0.5*er*v(nn+1,1); for j=nd+1:nm FIGURE 14.40 (Continued) 14_Sadiku_Ch14.indd 817 14/11/17 2:12 PM 818 CHAPTER 14 NUMERICAL METHODS end sum1 = sum1 + v(nn,j); sum2 = sum2 + v(nn+1,j); end sum1 = sum1 + 0.5*(er+1)*v(nn,nd); sum2 = sum2 + 0.5*(er+1)*v(nn+1,nd); q(n)=eo*abs(sum2 - sum1); end % Calculate the characteristic impedance c1=4*q(1)/vd; c2=4*q(2)/vd; zo=1/(u*sqrt(c1*c2)) TABLE 14.8 Characteristic Impedance of a Microstrip Line; for Example 14.8 h Number of Iterations Zo 1 V 2 0.25 700 72.43 0.1 500 57.56 0.05 500 67.36 0.05 700 66.88 0.05 1000 66.53 MATLAB 14.1 % % % % % % % % % % % % This script allows the user to enter the dimensions and dielectric properties of a microstrip line and then use the finite-difference algorithm to solve Laplace's equation iteratively and obtain the potential as a function of space The microstrip problem is normally open-aired, meaning the upper and side boundaries exist at infinity, but due to the limitations of solution space in the numerical problem, we have added shielding walls (where E = 0) of perfect electric conductor, which should be sufficiently far from the microstrip structure a,b >> w,d for accurate simulation of free-space and boundaries at infinity. % The script creates a coarse rectangular grid and then solves % Prompt user for basic parameters Vstrip=input('Enter the voltage on the microstrip > '); a=input('Enter the horizontal span of space > '); b=input('Enter the vertical span of space > '); w=input('Enter the microstrip width > '); 14_Sadiku_Ch14.indd 818 15/11/17 8:07 PM 14.7 Application Note—Microstrip Lines 819 d=input('Enter the dielectric thickness > '); % the dielectric boundary disp('Enter the relative dielectric constant '); epstop=input(' above the microstrip > '); disp('Enter the relative dielectric constant '); epsbottom=input(' below the microstrip > '); epsave=(epstop+epsbottom)/2; % the average relative dielectric % along the boundary % Fill the potential solution space with zeros P=zeros(b,a); % set voltage on strip % the floor rounds odd numbers divided by 2 down to the % nearest integer for i=floor(b/2)-(floor(w/2)-1):1:floor(b/2)+floor(w/2), P(i,d)=Vstrip; end % Begin iterations to solve potential % --------------------------------for i=1:600, % i is the iteration step % the larger this number is the more accurate the potential % solution for j=2:1:a-1, % sweep each column of y-values % if j is not a unit on the microstrip conductor if j ~= d for i=2:b-1, % sweep each row of x-values % this equation solves for the potential by % discretizing Laplace's equation on the % rectangular grid P(i,j)=0.25*(P(i+1,j)+P(i-1,j)+P(i,j+1)+P(i,j-1)); end % else we are at y = d, in the axis of the strip else for i=2:b-1, % sweep each row of x-values if (i < (floor(b/2)-(w/2-1))) | (i > (floor(b/2)+w/2)) % this equation solves for the potential by % discretizing Laplace's equation on the rectangular % grid P(i,j)=(1/(4*epsave))*(epsave*(P(i+1,j)+... P(i-1,j))+epstop*P(i,j+1)+epsbottom*P(i,j-1)); end end % end the if conditional end end end 14_Sadiku_Ch14.indd 819 15/11/17 8:35 PM 820 CHAPTER 14 NUMERICAL METHODS % ----------------------------------% Plot potential distribution % Create the vector of voltage contours v=[0.005,0.01,0.05,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1]*... Vstrip; figure contour(P',v); % P' is the transposed matrix to meet the requirements colorbar % add the colorbar as a legend for the colors which % define the equipotential lines xlabel('Horizontal position') ylabel('Vertical position') title('Equipotential Curves For Microstrip Line') SUMMARY REVIEW QUESTIONS 1. Electric field lines and equipotential lines due to coplanar point sources can be plotted by using the numerical technique presented in this chapter. The basic concept can be extended to plotting magnetic field lines. 2. An EM problem in the form of a partial differential equation can be solved by using the finite difference method. The finite difference equation that approximates the differential equation is applied at grid points spaced in an ordered manner over the whole solution region. The field quantity at the free points is determined using a suitable method. 3. An EM problem in the form of an integral equation is conveniently solved by using the moment method. The unknown quantity under the integral sign is determined by matching both sides of the integral equation at a finite number of points in the domain of the quantity. 4. While the finite difference method is restricted to problems with regularly shaped solution regions, the finite element method can handle problems with complex geometries. This method involves dividing the solution region into finite elements, deriving equations for a typical element, assembling all elements in the region, and solving the resulting system of equations. 5. The finite difference has been applied to determine the characteristic impedance of a microstrip transmission line. Typical examples on how to apply each method to some practical problems have been shown. Computer programs for solving the problems are provided wherever needed. 14.1 At the point 1 1, 2, 0 2 in an electric field due to coplanar point charges, E 5 0.3 ax 2 0.4 ay V/m. A differential displacement of 0.05 m on an equipotential line at that point will lead to point (a) 1 1.04, 2.03, 0 2 (b) 1 0.96, 1.97, 0 2 14_Sadiku_Ch14.indd 820 (c) 1 1.04, 1.97, 0 2 (d) 1 0.96, 2.03, 0 2 15/11/17 8:07 PM Review Questions 821 14.2 Which of the following is not a correct finite difference approximation to dV/dx at xo if h 5 Dx?4 V 1 xo 1 h 2 2 V 1 xo 2 h V 1 xo 2 2 V 1 xo 2 h 2 (b) h V 1 xo 1 h 2 2 V 1 xo 2 h 2 (c) h (a) V 1 xo 1 h 2 2 V 1 xo 2 h 2 2h V 1 xo 1 h/2 2 2 V 1 xo 2 h/2 2 (e) h (d) 14.3 The triangular element of Figure 14.41 is in free space.

The approximate value of the potential at the center of the triangle is (a) 10 V (c) 5 V (b) 7.5 V (d) 0 V 14.4 For finite difference analysis, a rectangular plate measuring 10 by 20 cm is divided into eight subregions by lines 5 cm apart parallel to the edges of the plates. How many free nodes are there if the edges are connected to some source? (a) 15 (c) 9 (e) 3 (b) 12 (d) 6 14.5 Using the difference equation Vn 5 Vn21 1 Vn11 with Vo 5 V5 5 1 and starting with initial values Vn 5 0 for 1 # n # 4, the value of V2 after the third iteration is (a) 1 (c) 9 (e) 25 (b) 3 (d) 15 14.6 T he coefficient matrix [A] obtained in the moment method does not have one of these properties: (a) It is dense (i.e., has many nonzero terms). (b) It is banded. (c) It is square and symmetric. (d) It depends on the geometry of the given problem.

FIGURE 14.41 For Review Questions 14.3 and 14.10. 4 The formula in (a) is known as a forward-difference formula, that in (b) as a backward-difference formula, and that in (d) or (e) as a central-difference formula. 14_Sadiku_Ch14.indd 821 14/11/17 2:12 PM 822 CHAPTER 14 NUMERICAL METHODS 14.7 A major difference between the finite difference and the finite element methods is that (a) Using one, a sparse matrix results in the solution. (b) In one, the solution is known at all points in the domain. (c) One applies to solving partial differential equation. (d) One is limited to time-invariant problems. 14.8 If the plate of Review Question 14.4 is to be discretized for finite element analysis such that we have the same number of grid points, how many triangular elements are there? (a) 32 (c) 12 (b) 16 (d) 9 14.9 Which of these statements is not true about shape functions? (a) They are interpolatory in nature. (b) They must be continuous across the elements. (c) Their sum is identically equal to unity at every point within the element. (d) The shape function associated with a given node vanishes at any other node. (e) The shape function associated with a node is zero at that node.

14.10 The area of the element in Figure 14.41 is (a) 14 (c) 7 (b) 8 (d) 4 Answers: 14.1a, 14.2c, 14.3a, 14.4e, 14.5c, 14.6b, 14.7a 14.8b, 14.9e, 14.10d. PROBLEMS Section 14.2—Field Plotting 14.1 Use the program developed in Example 14.1 or your own equivalent code to plot the electric field lines and equipotential lines for the following cases: (a) Three point charges of 21 C, 2 C, and 1 C placed at 1 21, 0 2 , 1 0, 2 2 , and 1 1, 0 2 , respectively. (b) Five identical point charges of 1 C located at 1 21, 21 2 , 1 21, 1 2 , 1 1, 21 2 , 1 1, 1 2 , and 1 0, 0 2 , respectively. Section 14.3—The Finite Difference Method 14.2 A boundary-value problem is defined by d2V 5 x 1 1, 0,x,1 dx2 14_Sadiku_Ch14.indd 822 Subject to V(0) 5 0 and V(1)5 1. Use the finite difference method to find V(0.5). You may take D 5 0.25 and perform 5 iterations. Compare your result with the exact solution. 14/11/17 2:12 PM Problems 823 14.3 (a) Obtain dV d2V and 2 at x 5 0.15 from the following table. dx dx x 0.1 0.15 0.2 0.25 V 1.0017 1.5056 2.0134 2.5261 3.0452 0.3 (b) The data in the table are obtained from V 5 10 sinh x. Compare your result in part (a) with the exact values. 14.4 Show that the finite difference equation for Laplace's equation in cylindrical coordinates, V 5 V 1 r, z 2 , is V 1 ro, zo 2 5 1 h c V 1 ro, zo 1 h 2 1 V 1 ro, zo 2 h 2 1 a1 1 b 4 2ro V 1 ro 1 h, zo 2 1 a1 2 h b V 1 ro 2 h, zo 2 d 2ro where h 5 Dz 5 Dr. 14.5 Using the finite difference representation in cylindrical coordinates 1 r, f 2 at a grid point P shown in Figure 14.42, let r 5 m Dr and f 5 n Df so that V 1 r, f 2 0 P 5 V 1 mDr, nDf 2 5 Vnm. Show that =2V 0 m,n 5 1 1 1 bVnm21 2 2Vnm 1 a1 1 bVnm11 2 c a1 2 2m 2m Dr 1 1 1 Vn21 2 2 Vnm 1 Vn11 m 2d 1 m Df 2 2 m 14.6 The four sides of a square trough are maintained at potentials 10 V, 40 V, 50 V, and 80 V. Determine the potential at the center of the trough. 14.7 For the potential problem in Figure 14.43, use the finite difference method to determine V1 to V4.

Five iterations are enough. 60 V Gap 1 2 20 V 40 V 3 FIGURE 14.42 Finite difference grid in cylindrical coordinates; for Problem 14.5. 4 0V FIGURE 14.43 For Problem 14.7. 14_Sadiku_Ch14.indd 823 14/11/17 2:12 PM 824 CHAPTER 14 NUMERICAL METHODS y Vo a 0V a 100 V b c 0V FIGURE 14.44 For Problem 14.8. Vo 0V 0 0V x a Gap FIGURE 14.45 For Problem 14.9. 14.8 Use the finite difference method to obtain the potential at points a, b, and c in Figure 14.44. Five iterations are enough. 14.9 The cross section of a long conducting pipe is shown in Figure 14.45. For the indicated boundary conditions: (a) Find the potential distribution V(x, y) by solving Laplace's equation. (b) Find the potential at the center of the region using the finite difference method. Take a = 1 m and Vo = 50 V. 14.10 For the rectangular region shown in Figure 14.46, the electric potential is zero on the boundaries and the charge distribution rv is 50 nC/m3. Although there are six free nodes, there are only four unknown potentials (V1V4) because of symmetry. Solve for the unknown potentials. 14.11 Apply the band matrix technique to set up a system of simultaneous difference equations for each of the problems in Figure 14.47. Obtain matrices [A] and [B]. 0V 1 2 3 4 0V 0V 1 cm 0V FIGURE 14.46 For Problem 14.10. 14_Sadiku_Ch14.indd 824 FIGURE 14.47 For Problem 14.11. 15/11/17 8:35 PM Problems 825 14.12 (a) How would you modify matrices [A] and [B] of Example 14.3 if the solution region had charge density rv? (b) Write a program to solve for the potentials at the grid points shown in Figure 14.48 3 assuming a charge density rv 5 x 1 y 2 1 2 nC/m . Use the iterative finite difference method and take er 5 1.0. 14.13 Use the finite difference method to find the potentials at nodes 1 to 7 in the grid shown in Figure 14.49. 14.14 The two-dimensional wave equation is given by 1 '2ϕ '2ϕ '2ϕ 5 1 c2 't2 'x2 'z2 By letting ϕjm,n denote the finite difference approximation of ϕ 1 xm, zn, tj 2 , show that the finite difference scheme for the wave equation is j11 j21 j j j j 21 ϕ,n 5 2 ϕm,n 2 ϕ,n 1 a 1 ϕm11,n 1 ϕ21,n 2 2 ϕm,n j j j 2 a 1 ϕm,n11 1 ϕ,n21 2 2 ϕm,n where h 5 Dx 5 Dz and a 5 1 cDt/h 2 2. 14.15 Write a program that uses the finite difference scheme to solve the one-dimensional wave equation '2V '2V , 2 5 'x 't2 0 # x # 1, t.0 given boundary conditions V 1 0, t 2 5 0, V 1 1, t 2 5 0, t . 0 and the initial condition 'V/'t 1 x, 0 2 5 0, V 1 x, 0 2 5 sin px, 0 , x , 1. Take Dx 5 Dt 5 0.1. Compare your solution with the exact solution V 1 x, t 2 5 sin px cos pt for 0 , t , 4. y 0V Gap 4 mm 1 2 3 50 V 0V 4 5 Gaps 6 0 7 4 mm 0V FIGURE 14.48 For Problem 14.12. 14_Sadiku_Ch14.indd 825 100 V 100 V x Gap FIGURE 14.49 For Problem 14.13. 14/11/17 2:13 PM 826 CHAPTER 14 NUMERICAL METHODS Section 14.4—The Moment Method 14.16 The electric field due to a circular loop placed at the origin with charge r per unit length at a distance x from origin is given by Ex 5 p r r (u 2 cos u)du 5 F(u) 3 2 3/2 2peoa 0 (1 1 u 2 2u cosu) 2peoa where F(u) 5 3 p 0 (u 2 cos u)du (1 1 u 2 2u cosu) 2 3/2 and u = x/a. Use MATLAB to plot F(u) for 0 , u , 2.

14.17 A transmission line consists of two identical wires of radius a, separated by distance d as shown in Figure 14.50. Maintain one wire at 1 V and the other at 21 V and use the method of moments to find the capacitance per unit length. Compare your result with exact formula for C in Table 11.1. Take a 5 5 mm, d 5 5 cm, , 5 5 m, and e 5 eo. 14.18 Two conducting wires of equal length L and radius a are separated by a small gap and inclined at an angle as shown in Figure 14.51. Find the capacitance between the wires by using the method of moments for cases u 5 10º, 20º, . . . , , 180°. Take the gap as 2 mm, a 5 1 mm, L 5 2 m, er 5 1. 14.19 Given an infinitely long thin strip transmission line shown in Figure 14.52(a), use the moment method to determine the characteristic impedance of the line. We divide each strip into N subareas as in Figure 14.52(b) so that on subarea i, Vi 5 a Aij rj 2N j51 where 2D, ln Rij, 2peo Aij 5 µ 2D, 3 ln D, 2 1.5 4 , 2peo i2j i5j Rij is the distance between the ith and jth subareas, and Vi 5 1 or 21 depending on whether the ith subarea is on strip 1 or 2, respectively. Write a program to find the characteristic impedance of the line using the fact that Zo 5 "moeo C FIGURE 14.50 For Problem 14.17. l 2a d 14_Sadiku_Ch14.indd 826 14/11/17 2:13 PM Problems 827 G FIGURE 14.51 For Problem 14.18. FIGURE 14.52 Analysis of strip transmission line using moment method.

For Problem 14.19. where C is the capacitance per unit length and a ri D, Q i51 C5 5 Vd Vd N and Vd 5 2 V is the potential difference between strips. Take H 5 2 m, W 5 5 m, and N 5 20. 14.20 Consider an -shaped thin wire of radius 1 mm as shown in Figure 14.53. If the wire is held at a potential V 10 V, use the method of moments to find the charge distribution on the wire. Take 0.1. 14.21 Consider the coaxial line of the arbitrary cross section shown in Figure 14.54(a). Using the moment method to find the capacitance C per length involves dividing each conductor into N strips so that the potential on the jth strip is given by Vj 5 a ri Aij 2N i51 where 2D, Rij ln , ro 2pe Aij 5 µ D,i 2D, 2 1.5 d , c ln ro 2pe i2j i5j and Vj 5 21 or 1 depending on whether Di lies on the inner or outer conductor, respectively.

Write a MATLAB program to determine the total charge per length on a coaxial cable of elliptical cylindrical cross section shown in Figure 14.54(b) by using Q 5 a ri N i51 14_Sadiku_Ch14.indd 827 and the capacitance per unit length with C 5 Q/2. (a) As a way of checking your program, take A 5 B 5 2 cm and a 5 b 5 1 cm (coaxial line with circular cross section), and compare your result with the exact value of C 5 2pe/ln 1 A/a 2 .

14/11/17 2:13 PM 828 CHAPTER 14 NUMERICAL METHODS y 1m A B x 1m FIGURE 14.53 For Problem 14.20. FIGURE 14.54 For Problem 14.21; coaxial line of (a) arbitrary cross section, (b) elliptical cylindrical cross section. (b) Take A 5 2 cm, B 5 4 cm, a 5 1 cm, and b 5 2 cm. Hint: For the inner ellipse of Figure 14.54(b), for example, r5 a 2 "sin f 1 v2cos2 f where v 5 a/b, d, 5 r df.

Take ro 5 1 cm. 14.22 A conducting bar of rectangular cross section is shown in Figure 14.55. By dividing the bar into N equal segments, we obtain the potential at the jth segment as Vj 5 a qiAij N i51 14_Sadiku_Ch14.indd 828 where 1 , 4peoRij Aij 5 µ 1 , 2eo !phD i2j i5j and D is the length of the segment. If we maintain the bar at 10 V, we obtain 3 A 4 3 q 4 5 10 3 I 4 where 3 I 4 5 3 1 1 1. . .1 4 T and qi 5 rvthD. (a) Write a program to find the charge distribution rv on the bar and take , 5 2 m, h 5 2 cm, t 5 1 cm, and N 5 20. 14/11/17 2:13 PM Problems 829 h FIGURE 14.55 For Problem 14.22. t (b) Compute the capacitance of the isolated conductor by using C5 Q 5 1 q1 1 q2 1 c1 qN 2 /10 V Section 14.5—The Finite Element Method 14.23 Another way of defining the shape functions at an arbitrary point 1 x, y 2 in a finite element is using the areas A1, A2, and A3 shown in Figure 14.56. Show that ak 5 Ak , A k 5 1, 2, 3 where A 5 A1 1 A2 1 A3 is the total area of the triangular element.

FIGURE 14.56 For Problem 14.23. 3 2 (7, 6) (1.5, 2.5) 1 (2, 0.5) 1 (4, 2) 2 (7, 2) 3 (0.5, 1) (a) (b) FIGURE 14.57 For Problem 14.24. 14_Sadiku_Ch14.indd 829 14/11/17 2:13 PM 830 CHAPTER 14 NUMERICAL METHODS 14.24 Determine the element coefficient matrices of the triangular elements in Figure 14.57. 14.25 The nodal potential values for the triangular element of Figure 14.58 are V1 5 100 V, V2 5 50 V, and V3 5 30 V. (a) Determine where the 80 V equipotential line intersects the boundaries of the element. (b) Calculate the potential at 1 2, 1 2 . 14.26 The triangular element shown in Figure 14.59 is part of a finite element mesh. If V1 5 8 V, V2 5 12 V, and V3 5 10 V, find the potential at (a) (1, 2) and (b) the center of the element. 14.27 Determine the global coefficient matrix for the two-element region shown in F igure 14.60. 14.28 Calculate the global coefficient matrix for the two-element region shown in Figure 14.61. 14.29 Find the global coefficient matrix of the two-element mesh of Figure 14.62. 14.30 For the two-element mesh of Figure 14.62, let V1 5 10 V and V3 5 30 V.

Find V2 and V4. 14.31 Use the MATLAB code in Figure 14.33 to determine the potentials at node 5 of the system shown in Figure 14.63. FIGURE 14.58 For Problem 14.25. FIGURE 14.59 For Problem 14.26. y (2, 2) (0, 2) 1 2 (0, 0) FIGURE 14.60 For Problem 14.27. 14_Sadiku_Ch14.indd 830 (4, 0) x FIGURE 14.61 For Problem 14.28. 14/11/17 2:13 PM Problems 831 y 100V 4cm 7 8 8 6 2cm 4 5 1 0 5 7 2 0V FIGURE 14.62 For Problems 14.29 and 14.30. 9 1 6 4 2 2cm 3 0V 3 4cm x 0V FIGURE 14.63 For Problem 14.31. 14.32 Use the program in Figure 14.33 to solve Laplace's equation in the problem shown in Figure 14.64, where Vo 5 100 V. Compare the finite element solution to the exact solution in Example 6.5; that is, V 1 x, y 2 5 4Vo ` sin np x sinh npy , a p k50 n sinh np n 5 2k 1 1 FIGURE 14.64 For Problem 14.32.

14_Sadiku_Ch14.indd 831 14/11/17 2:13 PM 832 CHAPTER 14 NUMERICAL METHODS 14.33 Repeat Problem 14.32 for Vo 5 100 sin px. Compare the finite element solution with the theoretical solution [similar to Example 6.6(a)]; that is, V 1 x, y 2 5 100 sin p x sinh p y sinh p 14.34 Show that when a square mesh is used in the finite difference method, we obtain the same result in the finite element method when the squares are cut into triangles. Section 14.6—Application Note—Microstrip Lines 14.35 Consider the shielded microstrip problem shown in Figure 14.65. Use the finite difference method to find the potential at points 1 to 6.

Five iterations are sufficient. 14.36 Refer to the square mesh in Figure 14.66.

By setting the potential values at the free nodes equal to zero, find (by hand calculation) the potentials at nodes 1 to 4 for five or more iterations. 14.37 Determine the characteristic impedance of the microstrip line shown in Figure 14.67. Take a 2.02, b 7.0, h 1.0 w, t 0.01. 14.38 The cross section of a transmission line is shown in Figure 14.68. Use the finite difference method to compute the characteristic impedance of the line. 14.39 Half a solution region is shown in Figure 14.69 so that the y-axis is a line of symmetry. Use finite difference to find the potential at nodes 1 to 9. Five iterations are sufficient if you use an iterative method. 14.40 The potential system in Figure 14.70 is symmetric about the y-axis. Set the initial values at the free nodes equal to zero and calculate the potentials at nodes 1 to 5 for five iterations. 100 V 0V 1 2 3 100 V 4 5 ε1 = εo 1 2 3 4 ε2 = 3 εo 6 0V FIGURE 14.65 For Problem 14.35. 14_Sadiku_Ch14.indd 832 FIGURE 14.66 For Problem 14.36. 14/11/17 2:13 PM Problems 833 «1 «o « «o a 1 cm w 3 cm t 1 cm «2 9.6«o h 3 cm b FIGURE 14.67 For Problem 14.37. y FIGURE 14.68 For Problem 14.38. y 1 100 V 2 3 4 5 8 3 4 100 V 5 9 0V Gap x FIGURE 14.69 For Problem 14.39. 14_Sadiku_Ch14.indd 833 2 6 100 V 7 1 x 0V FIGURE 14.70 For Problem 14.40. 14/11/17 2:13 PM 14_Sadiku_Ch14.indd 834 14/11/17 2:13 PM APPENDIX A MATHEMATICAL FORMULAS A.1 TRIGONOMETRIC IDENTITIES tan A 5 sin A , cos A cot A 5 1 tan A sec A 5 1 , cos A csc A 5 1 sin A sin2 A 1 cos2 A 5 1, 1 1 tan2 A 5 sec2 A 1 1 cot2 A 5 csc2 A sin 1 A 6 B 2 5 sin A cos B 6 cos A sin B cos 1 A 6 B 2 5 cos A cos B 7 sin A sin B 2 sin A sin B 5 cos 1 A 2 B 2 2 cos 1 A 1 B 2 2 sin A cos B 5 sin 1 A 1 B 2 1 sin 1 A 2 B 2 2 cos A cos B 5 cos 1 A 1 B 2 1 cos 1 A 2 B 2 sin A 1 sin B 5 2 sin A1B A2B cos 2 2 sin A 2 sin B 5 2 cos A1B A2B sin 2 2 cos A 1 cos B 5 2 cos A1B A2B cos 2 2 cos A 2 cos B 5 22 sin A1B A2B sin 2 2 cos 1 A 6 90° 2 5 7sin A sin 1 A 6 90° 2 5 6cos A tan 1 A 6 90° 2 5 2cot A cos 1 A 6 180° 2 5 2cos A sin 1 A 6 180° 2 5 2sin A 835 15_Sadiku_APP-A.indd 835 15/11/17 5:37 PM 836 APPENDIX A MATHEMATICAL FORMULAS tan 1 A 6 180° 2 5 tan A sin 2A 5 2 sin A cos A cos 2A 5 cos2 A 2 sin2 A 5 2 cos2 A 2 1 5 1 2 2 sin2 A tan 1 A 6 B 2 5 tan 2A 5 sin A 5 e jA 2 e2jA , 2j tan A 6 B 1 7 tan A tan B 2 tan A 1 2 tan2 A cos A 5 e jA 1 e2jA 2 e jA 5 cos A 1 j sin A p < 3.1416 1 rad < 57.296° A.2 COMPLEX VARIABLES A complex number may be represented as z 5 x 1 jy 5 rlu 5 re ju 5 r 1 cos u 1 j sin u 2 i where x 5 Re z 5 r cos u, y 5 Im z 5 r sin u y r 5 0 z 0 5 "x2 1 y2, u 5 tan21 x 1 j 5 "21, 5 2j, j2 5 21 j The complex conjugate of z 5 z* 5 x 2 jy 5 rl2u 5 re2ju i 5 r 1 cos u 2 j sin u 2 1 e ju 2 n 5 e jnu 5 cos nu 1 j sin nu 1 de Moivre's theorem 2 If z1 5 x1 1 jy1 and z2 5 x2 1 jy2, then z1 5 z2 only if x1 5 x2 and y1 5 y2. z1 6 z2 5 1 x1 1 x2 2 6 j 1 y1 1 y2 2 or z1z2 5 1 x1x2 2 y1y2 2 1 j 1 x1y2 1 x2y1 2 z1z2 5 r1r2 e j1u11u22 5 r1r2l u 1 1 u 2 iii i 15_Sadiku_APP-A.indd 836 15/11/17 5:37 PM APPENDIX A MATHEMATICAL FORMULAS 837 or 1 x1 1 jy1 2 1 x2 2 jy2 2 x1x2 1 y1y2 x2y1 2 x1y2 z1 # 5 5 1j 1 x2 1 jy2 2 1 x2 2 jy2 2 z2 x22 1 y22 x22 1 y22 z1 r1 r1 5 e j1u12u22 5 l u 1 2 u 2 r2 r2 iii z2 "z 5 "x 1 jy 5 "r e ju/2 5 "r lu/2 ii zn 5 1 x 1 jy 2 n 5 rn e jnu 5 rn lnu i 1 n 5 integer 2 z1/n 5 1 x 1 jy 2 1/n 5 r1/n eju/n 5 r1/n lu/n 1 2pk/n 1 k 5 0, 1, 2, . . . , n 2 1 2 iiiiii ln 1 re ju 2 5 ln r 1 ln e ju 5 ln r 1 ju 1 j2kp 1 k 5 integer 2 A.3 HYPERBOLIC FUNCTIONS sinh x 5 ex 2 e2x , 2 cosh x 5 ex 1 e2x 2 tanh x 5 sinh x , cosh x coth x 5 1 tanh x csch x 5 1 , sinh x sech x 5 1 cosh x sin jx 5 j sinh x, sinh jx 5 j sin x, cos jx 5 cosh x cosh jx 5 cos x sinh 1 x 6 y 2 5 sinh x cosh y 6 cosh x sinh y cosh 1 x 6 y 2 5 cosh x cosh y 6 sinh x sinh y sinh 1 x 6 jy 2 5 sinh x cos y 6 j cosh x sin y cosh 1 x 6 jy 2 5 cosh x cos y 6 j sinh x sin y tanh 1 x 6 jy 2 5 sin 2y sinh 2x 6j cosh 2x 1 cos 2y cosh 2x 1 cos 2y cosh2 x 2 sinh2 x 5 1 sech2 x 1 tanh2 x 5 1 sin 1 x 6 jy 2 5 sin x cosh y 6 j cos x sinh y cos 1 x 6 jy 2 5 cos x cosh y 7 j sin x sinh y 15_Sadiku_APP-A.indd 837 15/11/17 5:37 PM 838 APPENDIX A MATHEMATICAL FORMULAS A.4 LOGARITHMIC IDENTITIES log xy 5 log x 1 log y x 5 log x 2 log y y log log xn 5 n log x log10 x 5 log x 1 common logarithm 2 loge x 5 ln x 1 natural logarithm 2 If 0 x 0 V 1, ln 1 1 1 x 2 . x A.5 EXPONENTIAL IDENTITIES ex 5 1 1 x 1 x2 x3 x4 1 1 1... 2! 3! 4! where e < 2.7183 exey 5 ex1y 3 ex 4 n 5 enx ln ex 5 x A.6 APPROXIMATIONS FOR SMALL QUANTITIES If 0 x 0 V 1, 1 1 6 x 2 n . 1 6 nx ex . 1 1 x ln 1 1 1 x 2 . x sin x . x or lim xS0 sin x 51 x cos x . 1 tan x .

x If a W b, (a 1 b)n an 1 nan21b 15_Sadiku_APP-A.indd 838 15/11/17 5:37 PM APPENDIX A MATHEMATICAL FORMULAS 839 A.7 DERIVATIVES If U 5 U 1 x 2 , V 5 V 1 x 2 , and a 5 constant, d dU 1 aU 2 5 a dx dx d dV dU 1 UV 2 5 U 1V dx dx dx d U c d 5 dx V V dU dV 2U dx dx 2 V d 1 aUn 2 5 naUn21 dx loga e dU d loga U 5 dx U dx d 1 dU ln U 5 dx U dx d U dU a 5 aU ln a dx dx d U dU e 5 eU dx dx d V dU dV U 5 VUv21 1 UV ln U dx dx dx d dU sin U 5 cos U dx dx d dU cos U 5 2sin U dx dx d dU tan U 5 sec2 U dx dx d dU sinh U 5 cosh U dx dx d dU cosh U 5 sinh U dx dx d dU tanh U 5 sech2 U dx dx 15_Sadiku_APP-A.indd 839 15/11/17 5:37 PM 840 APPENDIX A MATHEMATICAL FORMULAS A.8 INDEFINITE INTEGRALS If U 5 U 1 x 2 , V 5 V 1 x 2 , a and b are constants, and C is an arbitrary constant, 3 a dx 5 ax 1 C 3 U dV 5 UV 2 3 V dU n 3 U dU 5 3 Un11 1 C, n11 1 integration by parts 2 n 2 21 dU 5 ln U 1 C U U 3 a dU 5 aU 1 C, ln a a . 0, a 2 1 U U 3 e dU 5 e 1 C ax ax 3 e dx 5 a e 1 C 1 ax 3 xe dx 5 2 ax 3 x e dx 5 eax 1 ax 2 1 2 1 C a2 eax 2 2 1 a x 2 2ax 1 2 2 1 C a3 3 ln x dx 5 x ln x 2 x 1 C 3 sin ax dx 5 2 a cos ax 1 C 1 3 cos ax dx 5 a sin ax 1 C 1 3 tan ax dx 5 a ln sec ax 1 C 5 2 a ln cos ax 1 C 1 1 3 sec ax dx 5 a ln 1 sec ax 1 tan ax 2 1 C 1 2 3 sin ax dx 5 2 3 cos ax dx 5 3 x sin ax dx 5 15_Sadiku_APP-A.indd 840 x sin 2ax 2 1C 2 4a x sin 2ax 1 1C 2 4a 1 1 sin ax 2 ax cos ax 2 1 C a2 15/11/17 5:37 PM APPENDIX A MATHEMATICAL FORMULAS 841 3 x cos ax dx 5 1 1 cos ax 1 ax sin ax 2 1 C a2 ax 3 e sin bx dx 5 ax 3 e cos bx dx 5 eax 1 a sin bx 2 b cos bx 2 1 C a 2 1 b2 eax 1 a cos bx 1 b sin bx 2 1 C a 2 1 b2 3 sin ax sin bx dx 5 sin 1 a 2 b 2 x sin 1 a 1 b 2 x 2 1 C, 21a 2 b2 21a 1 b2 3 sin ax cos bx dx 5 2 3 cos ax cos bx dx 5 a 2 2 b2 cos 1 a 2 b 2 x cos 1 a 1 b 2 x 2 1 C, 21a 2 b2 21a 1 b2 sin 1 a 2 b 2 x sin 1 a 1 b 2 x 1 1 C, 21a 2 b2 21a 1 b2 a 2 2 b2 a 2 2 b2 3 sinh ax dx 5 a cosh ax 1 C 1 3 cosh ax dx 5 a sinh ax 1 C 1 3 tanh ax dx 5 a ln cosh ax 1 C 1 3 3 3 dx 1 5 ln 0 ax 1 b 0 a ax 1 b dx 1 x tan21 1 C 2 5 a a x 1a 2 xdx 1 ln 1 x2 1 a2 2 1 C 2 5 2 x 1a 2 x2 dx 21 x 1C 3 2 2 5 x 2 a tan a x 1a 1 x2a ln 1 C, dx 2a x 1 a 5 µ 3 2 1 a2x x 2 a2 ln 1 C, 2a a 1 x dx x 5 sin 21 1 C 3 2 2 a "a 2 x 3 3 15_Sadiku_APP-A.indd 841 dx 2 "x 6 a2 xdx "x2 1 a2 x2 . a2 x2 , a2 5 ln 1 x 1 "x2 6 a2 2 1 C 5 "x2 1 a2 1 C 15/11/17 5:37 PM 842 APPENDIX A MATHEMATICAL FORMULAS 5 1C 3 1 2 x 1 a2 2 3/2 "x2 1 a2 x/a2 dx 52 1C 3 1 2 x

1 a2 2 3/2 "x2 1 a2 xdx 1 5 ln a 3 1 2 x 1 a2 2 3/2 x2dx x x "x2 1 a2 1 b2 1C 2 a a "x 1 a2 dx 1 x 1 x tan21 b 1 C 3 1 2 2a 2 2 1 2 2 5 a a 2a x 1 a x 1a 2 A.9 DEFINITE INTEGRALS If m and n are integers, a, b, and c are constants, p p 0, 3 sin mx sin nx dx 5 3 cos mx cos nx dx 5 e p/2, 0 0 p 3 sin mx cos nx dx 5 0 • 2p 0 m 1 n 5 even 2m , m2 2 n2 m 1 n 5 odd p sin mx sin nx dx 5 3 3 0, 2p p/2, sin ax dx 5 • 0, 3 x 0 2p/2, ` sin mx sin nx dx 5 e 0, p, m2n m5n m2n m5n a.0 a50 a,0 ` sin 2 x p 3 2 dx 5 2 x 0 ` 3 0 sin 2 ax p dx 5 0 a 0 , a ¬ 0 2 2 x In the following, a ¬ 0 for the integrals to converge. ` n 2ax dx 5 3 xe 0 ` 2 2ax dx 5 3 e 0 ` 3 2` ` 3 2` 15_Sadiku_APP-A.indd 8 2 e2ax dx 5 n! an11 1 p 2 Åa p Åa e21ax 1bx1c2 dx 5 2 p 1b224ac2/4a e Åa 03/10/17 10:24 AM APPENDIX A MATHEMATICAL FORMULAS 843 2ax cos bx dx 5 3 e ` 0 2ax sin bx dx 5 3 e ` 0 a a 2 1 b2 b a 2 1 b2 A.10 VECTOR IDENTITIES If A and B are vector fields while U and V are scalar fields, then = 1 U 1 V 2 5 =U 1 =V = 1 UV 2 5 U =V 1 V =U =c V 1 =U 2 2 U 1 =V 2 U d 5 V V2 = Vn 5 n Vn21 =V 1 n 5 integer 2 = 1A # B2 5 1A # =2 B 1 1B # =2 A 1 A 3 1= 3 B2 1 B 3 1= 3 A2 = # 1A 1 B2 5 = # A 1 = # B = # 1A 3 B2 5 B # 1= 3 A2 2 A # 1= 3 B2 = # 1 VA 2 5 V = # A 1 A # =V = # 1 =V 2 5 =2V = # 1= 3 A2 5 0 = 3 1A 1 B2 5 = 3 A 1 = 3 B = 3 1A 3 B2 5 A 1= # B2 2 B 1= # A2 1 1B # =2A 2 1A # =2B = 3 1 VA 2 5 =V 3 A 1 V 1 = 3 A 2 = 3 1 =V 2 5 0 = 3 1 = 3 A 2 5 1 = = # A 2 2 =2A # # C A dl 5 3 = 3 A dS L S L S S v C Vdl 5 23 =V 3 dS # # C A dS 5 3 = A dv C VdS 5 3 =V dv S v C A 3 dS 5 23 = 3 A dv S 15_Sadiku_APP-A.indd 843 v 15/11/17 5:37 PM 15_Sadiku_APP-A.indd 844 15/11/17 5:37 PM APPENDIX B MATERIAL CONSTANTS TABLE B.1 Approximate Conductivity* of Some Common Materials at 20°C Material Conductors Silver Copper (standard annealed) Gold Aluminum Tungsten Zinc Brass Iron (pure) Lead Mercury Carbon Water (sea) Semiconductors Germanium (pure) Silicon (pure) Insulators Water (distilled) Earth (dry) Bakelite Paper Glass Porcelain Mica Paraffin Rubber (hard) Quartz (fused) Wax Conductivity (siemens/meter) 107 107 107 107 107 107 107 107 5 106 106 3 104 4 6.1 5.8 4.1 3.5 1.8 1.7 1.1 2.2 4.4 1024 1024 1025 10210 10211 10212 10212 10215 10215 10215 10217 10217 * The values vary from one published source to another because there are many varieties of most materials, and conductivity is sensitive to temperature, moisture content, impurities, and the like. 845 16_Sadiku_APP-B.indd 845 16/11/17 4:19 PM 846 APPENDIX B MATERIAL CONSTANTS TABLE B.2 Approximate Dielectric Constant or Relative Permittivity 1 er 2 and Strength of Some Common Materials* Material Dielectric Constant εr (Dimensionless) Dielectric Strength E (V/m) Barium titanate 1200 7.5 106 Water (sea) 80 Water (distilled) 81 Nylon 8 Paper 7 12 106 Glass 5–10 35 106 Mica 6 70 106 Porcelain 6 Bakelite 5 20 106 Quartz (fused) 5 30 106 Rubber (hard) 3.1 25 106 Wood 2.5–8.0 Polystyrene 2.55 Polypropylene 2.25 Paraffin 2.2 30 106 Petroleum oil 2.1 12 106 Air (1 atm) 1 3 106 * The values given are only typical; they vary from one published source to another because of the different varieties of most materials and the dependence of er on temperature, humidity, and the like. TABLE B.3 Relative Permeability (mr) of Some Materials* Material Diamagnetic Bismuth Mercury Silver Lead Copper Water Hydrogen (STP) Paramagnetic Oxygen (STP) Air Aluminum Tungsten Platinum Manganese Ferromagnetic Cobalt Nickel Soft iron Silicon–iron mr 0.999833 0.999968 0.9999736 0.9999831 0.9999906 0.9999912 . 1.0 0.999998 1.00000037 1.000021 1.00008 1.0003 1.001 250 600 5000 7000 * The values given are only typical; they vary from one published source to a nother owing to different varieties of most materials. 16_Sadiku_APP-B.indd 846 16/11/17 4:19 PM APPENDIX C MATLAB MATLAB has become a powerful tool of technical professionals worldwide. The term MATLAB is an abbreviation for Matrix Laboratory, implying that MATLAB is a computational tool that employs matrices and vectors/arrays to carry out numerical analysis, signal processing, and scientific visualization tasks. Because MATLAB uses matrices as its fundamental building blocks, one can write mathematical expressions involving matrices just as easily as one would on paper. MATLAB is available for Macintosh, Unix, and Windows operating systems. A student version of MATLAB is available for PCs. A copy of MATLAB can be obtained from The Mathworks, Inc. 3 Apple Hill Drive Natick, MA 01760-2098 Phone: (508) 647-7000 Website: The brief introduction on MATLAB presented in this appendix is sufficient for solving problems in this book. Additional information about MATLAB can be found in MATLAB books and from online help. The best way to learn MATLAB is to learn the basics and start working with the program right away. C.1 MATLAB FUNDAMENTALS The Command window is the primary area in which the user interacts with MATLAB. A little later, we will learn how to use the text editor to create M-files, which allow executing sequences of commands. For now, we focus on how to work in the Command window. We will first learn how to use MATLAB as a calculator.

We do so by using the algebraic operators in Table C.1. TABLE C.1 Basic Operations Operation MATLAB Formula Addition Division (right) Division (left) Multiplication Power Subtraction a+b a/b (means a b) a\b (means b a) a*b a^b a–b 847 17_Sadiku_APP-C.indd 847 16/11/17 4:19 PM 848 APPENDIX C MATLAB To begin to use MATLAB, we use these operators. Type commands to the MATLAB prompt “>>” in the Command window (correct any mistakes by backspacing) and press the key.

For example, » a=2; b=4; c=-6; » dat = b^2 - 4*a*c dat = 64 » e=sqrt(dat)/10 e = 0.8000 The first command assigns the values 2, 4, and –6 to the variables a, b, and c, respectively. MATLAB does not respond because this line ends with a semicolon. The second command sets dat to b2 4ac, and MATLAB returns the answer as 64. Finally, the third line sets e equal to the square root of dat and divides by 10. MATLAB prints the answer as 0.8. As function sqrt is used here, other mathematical functions listed in Table C.2 can be used. Table C.2 provides just a small sample of MATLAB functions. Others can be obtained from the online help. To get help, type >> help [a long list of topics comes up] TABLE C.2 Typical Elementary Math Functions Function Remark abs(x) acos, acosh(x) acot, acoth(x) angle(x) asin, asinh(x) atan, atanh(x) conj(x) cos, cosh(x) cot, coth(x) exp(x) fix imag(x) log(x) log2(x) log10(x) real(x) sin, sinh(x) sqrt(x) tan, tanh(x) 17_Sadiku_APP-C.indd 848 Absolute value or complex magnitude of x Inverse cosine and inverse hyperbolic cosine of x in radians Inverse cotangent and inverse hyperbolic cotangent of x in radians Phase angle (in radians) of a complex number x Inverse sine and inverse hyperbolic sine of x in radians Inverse tangent and inverse hyperbolic tangent of x in radians Complex conjugate of x Cosine and hyperbolic cosine of x in radians Cotangent and hyperbolic cotangent of x in radians Exponential of x Round toward zero Imaginary part of a complex number x Natural logarithm of x Logarithm of x to base 2 Common logarithms (base 10) of x Real part of a complex number x Sine and hyperbolic sine of x in radians Square root of x Tangent and hyperbolic tangent of x in radians 16/11/17 4:19 PM

APPENDIX C MATLAB 849 and for a specific topic, type the command name.

For example, to get help on log to base 2, type >> help log2 [a help message on the log function follows] Note that MATLAB is case sensitive, so that sin(a) is not the same as sin(A). Try the following examples: >> 3^(log10(25.6)) >> y=2* sin(pi/3) >>exp(y+4-1) In addition to operating on mathematical functions, MATLAB easily allows one to work with vectors and matrices.

A vector (or array) is a special matrix with one row or one column. For example, >> a = [1 -3 6 10 -8 11 14]; is a row vector. Defining a matrix is similar to defining a vector. For example, a 3 3 matrix can be entered as >> A = [1 2 3; 4 5 6; 7 8 9] or as >> A = [1 2 3 4 5 6 7 8 9] In addition to the arithmetic operations that can be performed on a matrix, the operations in Table C.3 can be implemented. We can use the operations in Table C.3 to manipulate matrices as follows. » B = A' B = 1 4 7 2 5 8 3 6 9 » C = A + B TABLE C.3 Matrix Operations Operation Remark A’ det(A) inv(A) eig(A) diag(A) exp(A) 17_Sadiku_APP-C.indd 849 Finds the transpose of matrix A Evaluates the determinant of matrix A Calculates the inverse of matrix A Determines the eigenvalues of matrix A Finds the diagonal elements of matrix A Exponential of matrix A 16/11/17 4:19 PM 850 APPENDIX C MATLAB C = 2 6 10 6 10 14 10 14 18 » D = A^3 - B*C D = 372 432 492 948 1131 1314 1524 1830 2136 » e= [1 2; 3 4] e = 1 2 3 4 » f=det(e) f = -2 » g = inv(e) g = -2.0000 1.0000 1.5000 -0.5000 » H = eig(g) H = -2.6861 0.1861 Note that not all matrices can be inverted. A matrix can be inverted if and only if its determinant is nonzero. Special matrices, variables, and constants are listed in Table C.4. For example, type >> eye(3) ans= 1 0 0 0 1 0 0 0 1 to get a 3 3 identity matrix. TABLE C.4 Special Matrices, Variables, and Constants Matrix/Variable/Constant Remark eye Identity matrix ones An array of ones zeros An array of zeros i or j Imaginary unit or sqrt(–1) pi 3.1416 NaN Not a number inf Infinity eps A very small number, 2.2e16 rand Random element 17_Sadiku_APP-C.indd 850 16/11/17 4:19 PM APPENDIX C MATLAB 851 C.2 USING MATLAB TO PLOT To plot using MATLAB is easy. For a two-dimensional plot, use the plot command with two arguments as >> plot(xdata,ydata) where xdata and ydata are vectors of the same length containing the data to be plotted. For example, suppose we want to plot y=10*sin(2*pi*x) from 0 to 5*pi, we will proceed with the following commands: >> x = 0:pi/100:5*pi; % x is a vector, 0 y = 10*sin(2*pi*x); % create a vector y >> plot(x,y); % create the plot With this, MATLAB responds with the plot in Figure C.1. MATLAB will let you graph multiple plots together and distinguish them with different colors.

This capability is obtained with the command plot(xdata, ydata, ‘color’), where the color is indicated by using a character string from the options listed in Table C.5. For example, >> plot(x1, y1, ‘r’, x2,y2, ‘b’, x3,y3, ‘--’); will graph data (x1, y1) in red, data (x2, y2) in blue, and data (x3, y3) in dashed lines, all on the same plot. MATLAB also allows for logarithm scaling. Rather than the plot command, we use: loglog log(y) versus log(x) semilogx y versus log(x) semilogy log(y) versus x 10 8 6 4 2 0 2 4 6 8 10 0 2 4 6 8 10 12 14 16 FIGURE C.1 MATLAB plot of y=10*sin(2*pi*x). 17_Sadiku_APP-C.indd 851 16/11/17 4:19 PM 852 APPENDIX C MATLAB TABLE C.5 Various Color and Line Types y Yellow . Point m Magenta o Circle c Cyan x -mark r Red + Plus g Green - Solid b Blue * Star w White : Dotted k Black -. Dashdot -- Dashed Three-dimensional plots are drawn by using the functions mesh and meshgrid (mesh domain). For example, to draw the graph of z = x*exp(-x^2–y^2) over the domain -1 xx = -1:.1:1; » yy = xx; » [x,y] = meshgrid(xx,yy); » z=x.*exp(-x.^2 -y.^2); » mesh(z); (The dot symbol used in x. and y. allows element-by-element multiplication.) The result is shown in Figure C.2. Other plotting commands in MATLAB are listed in Table C.6. The help command can be used to find out how each of these is used. 0.5 0 0.5 30 20 10 0 0 5 10 15 20 25 FIGURE C.2 A three-dimensional plot.

17_Sadiku_APP-C.indd 852 16/11/17 4:19 PM APPENDIX C MATLAB 853 TABLE C.6 Other Plotting Commands Command Comments bar(x,y) contour(z) errorbar (x,y,l,u) hist(x) plot3(x,y,z) polar(r, angle) stairs(x,y) stem(x) subplot(m,n,p) surf(x,y,x,c) A bar graph A contour plot A plot with error bars A histogram of the data A three-dimensional version of plot (c) A polar coordinate plot A stairstep plot Plots the data sequence as stems Multiple (m n) plots per window A plot of three-dimensional colored surface C.3 PROGRAMMING WITH MATLAB So far MATLAB has been used as a calculator. You can also use MATLAB to create your own program.

The command line editing in MATLAB can be inconvenient if one has several lines to execute. To avoid this problem, one creates a program that is a sequence of statements to be executed. If you are in the Command window, click File/New/M-files to open a new file in the MATLAB Editor/Debugger or simple text editor. Type the program and save the program in a file with an extension .m, say filename.m (it is for this reason such files are called M-files). Once a program has been saved as an M-file, exit the Debugger window. You are now back in the Command window. Type the file without the extension .m to get results. For example, the plot that was made earlier in Figure C.1 can be improved by adding a title and labels and typing as an M-file called example1.m as follows: x = 0:pi/100:5*pi; % x is a vector, 0 example1 in the Command window, and hit to obtain the result shown in Figure C.3. To allow flow control in a program, certain relational and logical operators are necessary. These are shown in Table C.7. Perhaps the most commonly used flow control statements are for and if.

The for statement is used to create a loop or a repetitive procedure and has the general form for x array [commands] end 17_Sadiku_APP-C.indd 853 16/11/17 4:19 PM 854 APPENDIX C MATLAB A sine function 10 8 6 10*sin(2*pi*x) 4 2 0 2 4 6 8 10 0 2 4 6 8 10 x (in radians) 12 14 16 FIGURE C.3 MATLAB plot of y=10*sin(2*pi*x) with title and labels. The if statement is used when certain conditions must be met before an expression is executed. It has the general form if expression [commands if expression is True] else [commands if expression is False] end TABLE C.7 Relational and Logical Operators Operator Remark < less than greater than >= greater than or equal == equal ~= not equal & and | or ~ not 17_Sadiku_APP-C.indd 854 16/11/17 4:19 PM APPENDIX C MATLAB 855 For example, suppose we have an array y(x) and we want to determine the minimum value of y and its corresponding index x. This can be done by creating an M-file as follows. % example2.m % This program finds the minimum y value % and its corresponding index x = [1 2 3 4 5 6 7 8 9 10]; %the nth term in y y = [3 9 15 8 1 0 -2 4 12 5]; min1 = y(1); for k=1:10 min2=y(k); if(min2 < min1) min1 = min2; xo = x(k); else min1 = min1; end end diary min1, xo diary off Note the use of for and if statements. When this program is saved as example2.m, we execute it in the Command window and obtain the minimum value of y as –2 and the corresponding value of x as 7, as expected: » example2 min1 = -2 xo = 7 If we are not interested in the corresponding index, we could do the same thing using the command >> min(y) The following tips are helpful in working effectively with MATLAB: • Comment your M-file by adding lines beginning with a % character. • To suppress output, end each command with a semicolon (;).

You may remove the semicolon when debugging the file. • Press up and down arrow keys to retrieve previously executed commands. • If your expression does not fit on one line, use an ellipse (. . .) at the end of the line and continue on the next line.

For example, MATLAB considers y = sin(x + log10(2x + 3)) + cos(x + ... log10(2x+3)); as one line of expression. • Keep in mind that variable and function names are case sensitive. 17_Sadiku_APP-C.indd 855 16/11/17 4:19 PM 856 APPENDIX C MATLAB C.4 SOLVING EQUATIONS Consider the general system of n simultaneous equations as a11x1 1 a12x2 1 c1 a1nxn 5 b1 a21x1 1 a22x2 1 c1 a2nxn 5 b2 (an1x1 1 an2x2 1 c1 annxn 5 bn or in matrix form AX 5 B where a11 a12 c a1n x1 b1 a21 a22 c a2n x2 b2 A 5 ≥ c c c c¥ , X 5 ≥ c¥ , B 5 ≥ c¥ an1 an2 an3 ann xn bn The square matrix A is known as the coefficient matrix, while X and B are vectors. We are seeking X, the solution vector. There are two ways to solve for X in MATLAB. First, we can use the backslash operator (\) so that X 5 A\B Second, we can solve for X as X A1B which in MATLAB is the same as X inv(A)*B We can also solve equations by using the command solve. For example, given the quadratic equation x2 1 2x 2 3 5 0, we obtain the solution by using the MATLAB command >> [x]=solve('x^2 + 2*x - 3 =0') x = [-3] [1] indicating that the solutions are x 3 and x 1. Of course, we can use the command solve for a case involving two or more variables. We will see that in the following example.

17_Sadiku_APP-C.indd 856 16/11/17 4:19 PM APPENDIX C MATLAB 857 EXAMPLE C.1 Use MATLAB to solve the following simultaneous equations: 25x1 2 5x2 2 20x3 5 50 25x1 1 10x2 2 4x3 5 0 25x1 2 4x2 1 9x3 5 0 Solution: We can use MATLAB to solve this in two ways: Method 1: The given set of simultaneous equations could be written as 25 £ 25 25 25 220 x1 50 10 24 § £ x2 § 5 £ 0 § or AX 5 B 24 9 x3 0 We obtain matrix A and vector B and enter them in MATLAB as follows: » A = [25 -5 -20; -5 10 -4; -5 -4 9] A = 25 -5 -20 -5 10 -4 -5 -4 9 » B = [50 0 0]' B = 50 0 0 » X = inv(A)*B X = 29.6000 26.0000 28.0000 » X=A\B X = 29.6000 26.0000 28.0000 Thus, x1 29.6, x2 26, and x3 28. Method 2: Since the equations are not many in this case, we can use the command solve to obtain the solution of the simultaneous equations as follows: [x1,x2,x3]=solve('25*x1 - 5*x2 - 20*x3=50', '-5*x1 + 10*x2 - 4*x3 =0', '-5*x1 - 4*x2 + 9*x3=0') x1 = 17_Sadiku_APP-C.indd 857 16/11/17 4:19 PM 858 APPENDIX C MATLAB 148/5 x2 = 26 x3 = 28 The same result obtained by Method 1 appears again. PRACTICE PROBLEM C.1 Use MATLAB to solve the problem in the following simultaneous equations. 3x1 2 x2 2 2x3 5 1 2x1 1 6x2 2 3x3 5 0 22x1 2 3x2 1 6x3 5 6 Answer: x1 5 3 5 x3, x2 5 2. C.5 PROGRAMMING HINTS A good program should be well documented, of reasonable size, and capable of performing computations with reasonable accuracy within a reasonable amount of time. The following hints may make writing and running MATLAB programs easier. • Use the fewest commands possible and avoid execution of extra commands. This is particularly applicable for loops.

• Use matrix operations directly as much as possible and avoid for, do, and/or while loops if possible. • Make effective use of functions for executing a series of commands over several times in a program. • When unsure about a command, take advantage of the help capabilities of the software. • Start each file with comments to help you remember what it is all about.

• When writing a long program, save frequently. If possible, avoid a long program; break it down into smaller subroutines. 17_Sadiku_APP-C.indd 858 16/11/17 4:19 PM APPENDIX C MATLAB 859 C.6 OTHER USEFUL MATLAB COMMANDS Table C.8 lists some common useful MATLAB commands that may be used in this book. TABLE C.8 Other Useful MATLAB Commands Command Explanation diary mean min (max) grid poly roots sort sound std sum 17_Sadiku_APP-C.indd 859 Saves screen display output in text format Mean value of a vector Minimum (maximum) of a vector Adds a grid mark to the graphic window Converts a collection of roots into a polynomial Finds the roots of a polynomial Sorts the elements of a vector Plays vector as sound Standard deviation of a data collection Sum of elements of a vector 16/11/17 4:19 PM APPENDIX D THE COMPLETE SMITH CHART 860 18_Sadiku_APP-D.indd 860 16/11/17 1:44 PM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS CHAPTER 1 1.1 1.3 1.5 1.7 1.9 1.11 0.8704ax 2 0.3482ay 2 0.3482az 2ax 1 9ay 1 az (a) ax 2 ay 1 8az (b) 232 (c) 86.69o (a) T(4,6,21) and S(10,12,8) (b) 6ax 1 6ay 1 9az (c) 12.37 (a) 2ax 1 10az (b) 22 (c) 100.14o (a) 7.3485 (b) 4 (c) 24 (d) 206 (e) 16ax 1 12ay 1 8az (f) 162.3o (g) 86.45o 1.13 1.15 1.17 1.19 1.21 1.23 1.25 213, 221ax 2 2ay 1 6az, 120.66o (a) (B • A)A 2 (A • A)B (b) 2A2(A 3 B) (a) 7.6811, (b) 22ay 25az, (c) 42.57o, (d) 11.023, (e) 17.31 8.646 218ax 25ay 1 4az (a) 22.8577 (b) 20.2857ax 1 0.8571ay 1 0.4286az (c) 65.91o (a) 10, (b) 15ax 2 30ay 2 15az, (c) 5ax 1 5az 861 19_Sadiku_APP-E_NEW1.indd 861 20/11/17 8:47 AM 862 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 1.27 (a) 0.8242ax 1 0.1374ay 1 0.5494az (b) 100 5 4x2y2 1 x2 1 2xz 1 z2 1 z4 1.29 1.31 (a) (b) (c) (a) (b) (c) 2.1 2.3 2.5 (a) P(5.3852, 68.2°, 1), P(5.477, 79.48°, 68.2°) (b) Q(5, 306.88°, 0), Q(5.90°, 306.88°) (c) R(6.325,18.438, 24), R (7.483,122.318, 18.438) (a) (6.324, 71.56o , 24) (b) (7.483, 147.69o, 71.56o) T(x, y, z) 5 (7.5, 4.33, 5) T(r, f, z) 5 (8.66, 30o, 5) 1 4 4 (a) (rar 1 4az), asin2 u 1 sin ubar 1 sin u acos u 2 bau r r "r2 1 z2 2.9 23ax 1 2ay 1 4az G 5 6.403, H 5 7.348 218 u GH 5 112.5° 6.403 1.286ax 2 2.571ay 1 3.857az 6ax CHAPTER 2 2.7 2.11 r2 "r2 1 z2 (rar 1 zaz), r2 sin2 uar 4 3 xax 1 yay 1 zaz 4 (x2 1 y2 1 z2) 2.13 (a) B 5 r cosfaz (b) B 5 0.5r sin2(u) cosfar 2 r sin2u cosfau 2.15 2.17 2.19 Proof (a) 1, (b) 1, (c) 20.4472 (a) 8ap 1 2af 2 7az (b) 7 (c) 216ar 1 29af 2 10az (d) 78.56o 2.21 rzsin 2far 1 rzcos 2faf 1 0.5r2 sin2faz 2.23 (a) rsinu 3 sinf cosu(rsinu 1 cosf)ar 1 sinf(rcos2u 2 sin u cosf)au 1 3cosfaf 4 , 19_Sadiku_APP-E_NEW1.indd 862 (b) 5au 2 21.21af (b) "r2 1 z2 arar 1 r af 1 zaz b, 4.472ar 1 0.8944af 1 2.236az r 1 z2 2 20/11/17 8:47 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 863 2.25 2.27 2.29 2.31 64 (a) 9 (a) (b) (c) (d) (e) (f) 2.33 2.5 3.1 3.3 3.5 3.7 3.9 3.11 3.13 3.15 3.17 (a) 2.356 (b) 0.5236 (c) 4.189 (a) 6 (b) 110 (c) 4.538 11.502 2047.5 (a) 250 (b) 239.5 21.5 2 484.58 (a) (6y 2 2z)ax 1 6xay 1 (1 2 2x)az (b) (10 cosf 2 z)ar 2 10 sinfaf 2 raz 2sinf 2 (c) 2 2 cos far 2 2 af r r sinu Proof 0.4082ax 2 0.8165ay 1 0.4082az 240.42 2.36, 0.5413ax 1 1.624ay 1 1.624az (a) 3y 2 x (b) 2z2 1 sin2f 1 2rsin2f (c) 3 Proof 50.07o, (b) 0.9428ar 1 0.2357af 2 0.2357az An infinite line parallel to the z-axis.

Point (2,21,10). A circle of radius rsinu 5 5, i.e., the intersection of a cone and a sphere. An infinite line parallel to the z-axis. A semi-infinite line parallel to the x2y plane. A semi-circle of radius 5 in the y2z plane.

CHAPTER 3 3.19 3.21 3.23 3.25 3.27 3.29 19_Sadiku_APP-E_NEW1.indd 863 20/11/17 8:47 AM 864 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 3.31 (a) 6YZax 1 3XY2ay 1 3X2YZaz (b) 4YZax 1 3XY2ay 1 4X2YZaz (c) 6XYZ 1 3XY3 1 3X2YZ2 (d) 2(X2 1 Y2 1 Z2) 5 2r2 3.33 3.35 3.37 3.39 (a) 209.44 (b) 209.44 24 37.7 198.97 3.41 (a) 2y2aX 1 2zay 2 x2aZ, (b) (r 2 3z)af 1 4r az, 0 (c) 2 3.43 3.45 3.47 3.49 3.51 3.53 3.55 Proof 29.4956 0 (a) 4p (b) 4p (c) 24p (d) 2.2767 (e) 7.2552 (f) 9.532 (a) 2z 1 5x 1 8, (b) 8ax 1 2xay 1 5yaz Proof (a) 6(x 1 y 1 z) (b) a 2 2 sinf cosf cosf a 1 a 1 3 af, 0 r3sinu r r3sinu u r 3.59 23z2 1 2rbsin2f r cosu sinf (c) 6 1 4 cosu sinf 2 sin2u r 1 (a) 2 , (b) 2 r r 2rz cos far 2 rz sin faf 1 r2 cosfaz, 3z cosf 3.61 3.63 3.65 3.67 Proof Proof (a) G is irrotational, (b) 8, (c) 26 Proof 3.69 Proof 3.57 19_Sadiku_APP-E_NEW1.indd 864 0 2 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 865 CHAPTER 4 4.1 21.8291ax 2 0.5226ay 1 1.3065az mN 4.3 (a) (b) (c) 4.5 4.7 4.9 4.11 (a) 0.5 C, (b) 1.206 mC, (c) 1579.1 C 10.472 mC 288 mC (a) 4 nC, (b) 3.6 31.025az V/m 4.13 19_Sadiku_APP-E_NEW1.indd 865 Q ax 2peoa2 2Q ax 4"2peoa2 2Q 10"5pe0a 2 ax 1 Q 1 d az 2 c1 2 4pe0a 5!5 rsh 1 1 2 c d az 2 2 2 4pe0 !a 1 h !b 1 h2 4.15 2151.7ax 2 303.5ay 4.17 4.19 (a) 565.5ax 2 1131ay 2 1696.5az kV/m (b) 2565.5ax 2 1131ay 2 1696.5az kV/m (c) 565.5ax 1 1131ay 1 1696.5az kV/m Proof 4.21 4.23 (a) 0.32ax mC/m2 (b) 251.182 mC 21.326 nC 4.25 (a) 8y C/m2 (b) 6sinf 1 4z C/m3 (c) 0 4.27 22.7ar mC/m2 4.29 4.31 (a) 2y C/m3 (b) 1/3 C (c) 1 C Proof 4.33 (a) 125.7 mC, 377 mC, (b) 0, 1.2ar mC/m2 4.35 4.37 4.39 1.325 V (a) 135 kV, (b) 135 kV, (c) 135 kV 215.38ar 2 7.688af 1 30.75az V/m 20/11/17 8:48 AM 866 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 4.41 4.43 8p 3 ar 15 o 2a3ro 2a3ro (b) 2 ar, 15eor 15eor 3 ro r ro r4 a2ro r r2 (c) 2 1 a 2 2 bar, a b eo 3 eo 20a2 6 4eo 5a (d) Proof 230 C/m3 (a) 4.47 4.49 4.51 4.53 2eoEo , 0,r,a a (a) 0.1592 nC/m2, (b) 480 mJ 28 V 45.24 mJ (a) 2e2z sinfar 2 e2z cosfaf 1 re2z sinfaz (b) Proof 4.55 2 4.45 4.57 4.59 Q 1 1 a 2 b a 4peo b Not a genuine EM field; 0.5858 C (a) 21.136ay kV/m (b) (ax 1 0.2ay) 3 107 m/s 4.61 Proof, 4.63 4.65 Qd (2sinusinfar 2 cosu sinfau 2 cosfaf) 4peor3 54.748, 125.268 1.886 nJ 4.67 Q2 8peoa 4.69 (a) 2e2z sinfar 2 e2z cosfaf 1 re2z sinfaz (b) 8.512 pJ 5.1 5.3 5.5 5.7 5.9 20.1172 A 100 A 25.13 A (a) 33.95 mV (b) 265.1 A (c) 2.386 kW The silver wire is longer. CHAPTER 5 19_Sadiku_APP-E_NEW1.indd 866 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 867 5.11 5.13 5.15 5.17 5.19 (a) 0.27 mV (b) 50.3 A (copper), 9.7 A (steel) (c) 0.322 mV 130.86 A 3.6 3 1028 ax C/m2, 1.0407 2.261arkV/m, 70arnC/m2 (a) 286.86 3 10218 C (b) 883.5 3 10218 C 5.21 (c) 2796.61 3 10218 C 2.16 kV, 2.16ax 1 0.432ay 1 2.16az kV/m, 133.69ax 1 26.74ay 1 133.69az nC/m2 5.23 For r , a, E 5 0 5 D 5 P a ar a For a , r , b, D 5 r2 , E 5 , P5 r2 pr 2eopr2 2pr 2ar 2ar 24ar , E5 , P5 For b , r , c, D 5 2pr2 10eopr2 10pr2 For r . c, D 5 5.25 5.27 1.733 (a) roa2(2er 1 1) 6eoer (b) roa2 3eo 5.29 (a) (b) (c) (d) 5.31 19_Sadiku_APP-E_NEW1.indd 867 2ar 2ar , P50 2, E 5 pr eopr2 Possible Not possible Not possible Possible 100 (a) 3 C/m3.s r 5.33 (b) 314.16 A Proof 5.35 5.37 (a) 2.741 3 104s, (b) 5.305 3 104s, (c) 7.07 mS Proof 5.39 (a) 79.6ax 2 265.3ay 1 212.2az pC/m2, 53.05ax 2 88.42ay 1 70.74az pC/m2, (b) 3.0593 nJ/m3, 1.7684 nJ/m3 20/11/17 8:48 AM 868 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 5.41 (a) 6.667ax 2 13.33ay 1 6.667az V/m, 13.3ax 2 23.3ay 1 33.3az V/m (b) 16ax 2 18ay 1 36az V/m 5.43 5.45 5.47 5.49 (a) 387.8ax 2 452.4ay 1 678.6az V/m, 12ax 2 14ay 1 21az nC/m2 (b) 4ax 2 2ay 1 3az nC/m2, 0 (c) 12.62 mJ/m2, 9.839 mJ/m2 30 sinuar 1 5 cosuau, eo(60 sinuar 1 10 cosuau) 0.476 pC/m2 49.118, 608, 308 6.1 6.3 6.5 (a) 2270ax 1 540ay2135az V/m (b) 14.324 nC/m3 27.25 V 157.08y4 2 942.5y2 1 30.374 kV 6.7 a0.3142 2 CHAPTER 6 6.9 6.11 6.13 6.15 6.17 6.21 127.58 V 23xy 436.14 nC/m2 36.91 V, 9.102ar kV/m, 161ar nC/m2, 322 nC/m2, 2107.3 nC/m2 (a) Proof (b) 2Vo sin far 2 Vo cos faf 100 100 V52 1 150, E 5 2 2 ar V/m r r (a) V(r5 15 mm) 5 12.4 V, (b) u 5 8.93 3 106 m/s 6.23 (a) (b) (c) 6.25 4Vo ` a exp(2npx/a) 3 sin(npy/a)ax 2 cos(npy/a)ay 4 a n5odd 6.19 6.27 19_Sadiku_APP-E_NEW1.indd 868 66.51 bar r 4Vo ` sin(npx/b)sinh(np(a 2 y)/b) a p n5odd nsinh(npa/b) 4Vo ` sin(npy/a)sinh(npx/a) a p n5odd nsinh(npb/a) 4Vo ` sin(npy/a)sinh(np(b 2 x)/a) a p n5odd nsinh(npb/a) Proof 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 869 6.29 6.31 6.33 6.35 6.37 6.39 6.41 6.43 6.45 6.47 6.49 6.51 6.53 6.55 Proof 1.75 1/3 Co, 1/3 Qo, 1/3 Eo, 1/3 Wo (a) 25 pF, (b) 63.66 nC/m2 1 1 1 1 1 1 a 2 b 4peoc 4peo a b 0.8665 mF Proof 2pe1e2 e2 In(b/a) 1 e1In(c/b) 6.57 Proof 6.59 (a) 6.61 6.63 6.65 6.67 6.69 19_Sadiku_APP-E_NEW1.indd 869 Proof Proof Derivations 2psV2 b In a 2.122 pF 6 pF Vo x 1 d Vo In ,2 a In2 d (x 1 d)In2 x eoxVo (b) 2 a d(x 1 d)In2 x eV (c) 0, 2 o o 2dIn2 eo S (d) d In 2 Proof 0.326 nF (a) 212.107 pF/m2, (b) 210 nC 20.1092(ax 1 ay 1 az) N (a) 2138.2ax 2 184.3ay V/m, (b) 21.018 nc/m2 20/11/17 8:48 AM 870 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS CHAPTER 7 7.1 7.3 7.5 7.7 7.9 7.11 7.13 7.15 (a) See text.

(b) 0.7433ax 1 0.382ay 1 0.1404az A/m 20.1592ax 20.234ax 1 0.382ay 1 0.1404az A/m (a) 28.471ay mA/m (b) 13(2ax 1 ay) mA/m (c) 25.1ax 1 1.7ay mA/m (d) 5.1ax 1 1.7ay mA/m (a) 20.6792az A/m (b) 0.1989az A/m (c) 0.1989(ax 1 ay) A/m (a) 1.964az A/m (b) 1.78az A/m (c) 20.1178az A/m (d) 20.3457ax 2 0.3165ay 1 0.1798az A/m Proof Ifo 1 1 a 2 b Oz r2 4p r1 7.17 (a) 69.63 A/m, (b) 36.77 A/m 7.19 (a) See text. 0, r , a I r 2 a2 a,r,b a b, 2pr b2 2 a2 I , r.b 2pr (b) Hf 5 g 7.21 94.25 A 7.23 Jo, 0 , r , a Hf 5 c Jo a , r.a r 7.25 (a) 7.27 2ko a, r,a a z a (b) ko a baf, r . a r (b) 50.265 kA 19_Sadiku_APP-E_NEW1.indd 870 2 (a) 3r 3 103az A/m2 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 871 7.29 7.31 7.33 7.35 7.37 7.39 7.41 7.43 7.45 7.47 7.49 7.51 7.53 7.55 7.57 (a) 80af nWb/m2, (b) 1.756 mWb 1 (a) pa2Jo, 2 Jor r2 a2 2 2 2 b, r , a 4 a (b) Hf 5 µ aJo , r.a 4r moI az 4a Proof, 1.37 3 1028 Wb 2.854 Wb 12.53 Wb/m2 (a) magnetostatic field, (b) magnetostatic field, (c) neither electrostatic nor magnetostatic field. p p2 (10sinpyax 1 cospxaz) A/m2 asinpxay 2 10cospyaz b A/m, mo mo (a) B 5 (26xz 1 4x2y 1 3xz2)ax 1 (y 1 6yz 2 4xy2)ay 1 (y2 2 z3 2 2x2 2 z)az Wb/m2 (b) 8 Wb (c) Proof 1.011 Wb Proof Ao (2cos uar 1 sin uau) Wb/m2 r3 20r 40 (a) 2 a mA/m, 2 az mA/m2 mo f mo (b) 2 400 A 1A Proof CHAPTER 8 8.1 8.3 8.5 19_Sadiku_APP-E_NEW1.indd 871 8.7 8.9 2152.8ax 2 170.4ay 2 189.6az mN (a) 20.05ax 2 0.25ay N (b) 50ax 1 250ay V/m (0.2419, 1, 1.923), 4.071ax 2 2.903az m/s. The particle gyrates in a circle in the y 5 1 plane with center at (0,1,19/12). 26.67 nC 85.714 Wb/m2 8.11 100 mN 20/11/17 8:48 AM 872 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 8.13 8.15 8.17 8.19 8.21 8.23 8.25 8.27 (a) 2.197az, (b) 0.575ar 15.71az mN/m 540(ay + az) N.m 627.5 MA 24 mN.m Proof 7.5, 1.477az mA/m, 0.1477az A/m2 4.54 8.29 8.31 8.33 8.35 8.37 8.39 8.41 8.43 8.45 8.47 8.49 8.51 8.53 8.55 8.57 8.59 8.61 (a) Proof, (b) Proof 1.6ax 2 4ay 1 12az mWb/m2, 32.34 J/m3 5ax 2 6az A/m (a) 4ar 1 15af 2 8az mWb/m2 (b) 60.68 J/m3, 57.7 J/m3 mo(22ar 1 0.05625af) Wb/m2 (a) 25ay A/m, 26.28ay m Wb/m2 (b) 235ay A/m, 2110ay mWb/m2 (c) 5ay A/m, 6.283ay m Wb/m2 Proof 2 mH 11.82 mH Proof 304.1 pJ 101.86 J 190.8 A.t, 19080 A/m 88.5 mWb/m2 7.2643 3 104 A.t/Wb (a) 37 mN (b) 1.885 mN Proof 9.1 0.4738 sin377t V 9.3 9.5 9.7 9.9 212.57 cos 104 t A 20 sin 1.6b b 9.888 mV, point A at higher potential 21.8 V CHAPTER 9 19_Sadiku_APP-E_NEW1.indd 872 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 873 9.11 9.13 9.15 9.17 9.19 9.21 9.23 9.25 9.27 9.29 (a) = 3 Es 5 jvmHs, = 3 Hs 5 (s 2 jve)Es 'Dy 'Dz (b) = ? D 5 rv S 1 1 5 rv 'x 'y 'z 'By 'Bx 'B =?B 5 0 S 1 1 z50 'x 'y 'z 'E 'B 'E 'B S z2 y5 2 x =3E52 't 'y 'z 't 'By 'Ex 'Ez 2 52 'z 'x 't 'Ey 'Ex 'Bz 2 52 'x 'y 't 'Hy 'D 'Hz 'Dx S =3H5J1 2 5 Jx 1 't 'y 'z 't 'Dy 'Hx 'Hz 2 5 Jy 1 'z 'x 't 'Hy 'Hx 'Dz 2 5 Jz 1 'x 'y 't Proof 0.3z2 cos 104 t mC/m3 40b sin(vt 1 bx)az V/m, veo 0.333 rad/m 9.31 10b sinu cos(vt 2 br)af A/m vmor 9.33 9.35 Derivation (a) Yes, (b) yes, (c) no, (d) no. 9.37 2.636 cos(12px) sin(1010t 2 25.66y)ax 1 3 sin(12px) cos(1010t 2 25.66y)ay mA/m, 33/2 9.39 9.41 9.43 19_Sadiku_APP-E_NEW1.indd 873 0.97 mV 6.33 A, counterclockwise 277.8 A/m2, 77.78 mA (a) 0.444 3 1023 (b) 5.555 (c) 7.2 3 1024 600 kHz bHo sin (vt 2 bz)ay 12b sinu cos(vt 2 br)af, v 5 2p 3 108 veor Derivation (a) V5 constant, E 5 2Ao v cos(vt 2 bz)ax (b) b 5 v Èmo «o 2 20/11/17 8:48 AM 874 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 9.45 (a) 210ejp/3ax (b) 4cos(4y)e 2j2xaz (c) 5e 2jp/6ax 2 8e 2jp/4ay 9.47 (a) 1 2j3z e af r 3 (b) cos(vt 2 3z)ar reov (c) 9 3 108 rad/s 9.49 9.51 3.333, 24.533ej3.33x ay Proof 10.1 (a) along ax (b) 1 ms, 1.047 m, 1.047 3 106 m/s (c) See Figure E.1. CHAPTER 10 25 25 0 x /2 25 t 25 x /2 25 0 t T/8 25 25 0 0 x /2 0 25 t T/4 x /2 t T/2 FIGURE E.1 For Problem 10.1(c). 10.3 10.5 10.7 10.9 19_Sadiku_APP-E_NEW1.indd 874 (a) 108 rad/s (b) 0.333 rad/m (c) 18.85 m (d) Along 2ay (e) 20.1665 A/m Derivations (a) 5.41 1j6.129 /m, (b) 1.025 m, (c) 5.125 3 107 m/s, (d) 101.4 o (e) 259.16e2j41.44 e2gZ ay mA/m (a) 6.85 m, (b) 0.73 3 1023 Np 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 875 10.11 10.13 10.15 10.17 10.19 10.21 10.23 10.25 10.27 10.29 10.31 10.33 10.35 10.37 10.39 10.41 10.43 19_Sadiku_APP-E_NEW1.indd 875 Proof (a) 20 ns, (b) 3 m, (c) 2.257e20.1y sin (p 3 108t 2 2.088y 1 2.74o)az V/m, (d) 2.74° (a) 168.8 Np/m (b) 210.5 rad/m (c) 263.38/38.73° V, (d) 2.985 3 107 m/s 5.224 (a) lossless (b) 12.83 rad/m, 0.49 m (c) 25.66 rad (d) 4.62 kV 14.175, 1.0995 (a) 5.662 V/m (b) 5.212 dB (c) 36.896/3.365° V (a) No, nonconducting (b) No, nonconducting (c) Yes, conducting (a) 2.287 V (b) 207.61 V (c) 12.137 kHz 1.038 kHz 2.723 Np/m, 2.723 rad/m, 2.306 m, 2.768 3 107 m/s 2.94 31026 m (a) Linearly polarized along az (b) 10 MHz (c) 205.18 (d) 20.456 sin (2p 3 107 t 2 3y)ax A/m (a) Circular polarization (b) Elliptical polarization (a) Elliptically polarized.

(b) 2159.2 sin(vt 2 bz)az 1 106 cos(vt 2 bz)ay mA/m (a) Linearly polarized (b) Elliptically polarized (c) Linearly polarized (a) 5.76 (b) 157.1 V (c) 1.25 3 108 m/s (d) 0.955 cos (109 t 1 8x)ay A/m (e) 2143.25 cos2 (109t 1 8x)ax W/m2 20/11/17 8:48 AM 876 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 10.45 10.47 10.49 10.51 10.53 10.55 10.57 10.59 p2 5 24.46 cos2(vt 2 b2z)ax W/m2, b2 5 300 rad/m 10.61 10.63 89.51%, 10.84% (a) 9 3 108 rad/s (b) 2.094 m (c) 6.288, 16.71/40.74° V (d) Er 5 9.35 sin(vt 2 3z 1 179.7o)ax V/m Er 5 0.857 e43.94z sin(9 3 108 t 1 51.48z 1 38.89o)ax V/m 10.65 10.67 10.69 ut1 5 19.47o, ut2 5 28.13o 0.8146/174.4° Proof 10.71 1.235, 258 MHz, 0.491 3 (12.6 1 j16.8)ax 1 (10.2 1 j13.6)ay 2 8.59az 4 e 2j3.4x14.2y mA/m 10.73 10.75 10.77 19_Sadiku_APP-E_NEW1.indd 876 1 sinue 2j3raf A/m 12pr (b) 7.145 mW 0.225 (a) 2.828 3 108 rad/s, sin(vt 2 2z)af A/m r 9 (b) 2 sin2(vt 2 2z)az W/m2 r (c) 11.46 W 12 3 109rad/s, 27.539 sin(vt 2 40x)ay 2 3.775 sin(vt 2 40x)az kV/m, 94.23ax kW/m2 Proof Vo Io (a) az 4pr2 In(b/a) 1 (b) Vo Io 2 2 20sin(vt 1 10z)ax 2 10sin(vt 1 10z 1 p/6)ay V/m, 40sin(vt 2 10z)ax 1 20sin(vt 2 10z 1 p/6)ay V/m (a) 21.508 sin(vt 2 5x)az 1 0.503 sin (vt 1 5x)az kV/m (b) 2.68ax kW/m2 (c) 2 p1 5 38.197 cos2(vt 2 b1z)ax 1 13.75cos2(vt 1 b1z)ay W/m2, b1 5 1200 rad/m, (a) Proof (a) ui 5 36.87o (b) 106.1ay 1 79.58az mW/m2 (c) E r 5 2(1.518ay 1 2.02az) sin(vt 1 4y 2 3z), Et 5 (1.279ay 2 5.968az) sin(vt 2 9.539y 23z) V/m (a) 5cos(vt20.5px 2 0.866pz)ay, (4ax 2 3az)cos(vt20.5px 2 0.866pz) (b) 36.87o 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 877 10.79 (a) 1.45, (b) 9, (c) 1.643 10.81 (a) T11 5 1/S21, T12 5 2S22 /S21, T21 5 S11 /S21, T22 5 S12 2 S11S22 /S21 (b) T 5 c 10.83 2.5 20.5 d 0.5 0.3 The microwave wavelengths are of the same magnitude as the circuit components. The physical dimension of the lumped element must be in this range, which is not physically realizable. CHAPTER 11 11.1 11.3 11.5 11.7 11.9 11.11 11.13 11.15 11.17 11.19 11.21 11.23 11.25 11.27 11.29 11.31 11.33 11.35 19_Sadiku_APP-E_NEW1.indd 877 0.0354 V/m, 50.26 nH/m, 221 pF/m, 0 S/m Proof 0.2342 pF, 1.4 3 1022 V 'V 'I 'I 'V 5 2L , 5 2C 'z 't 'z 't (a) 8.159 3 1022 1 j2.384 3 103 /m, (b) 8.159 3 1022 1 j2.384 3 103 0.2305 V/m, 0.3316 mH/m, 0.1326 nF/m, 92.2 mS/m (a) 644.3 2 j97 V, (5.415 1 j33.96) 3 1023 /mi (b) 1.85 3 105 mi/s (c) 185.02 mi 0.46 mm 0.655 mH/m, 59.4 pF/m, 105 V (a) 0.2898 1 j3.278 /m, 2.3 3 108 m/s, 69V (b) 5.554 m (c) 0.3051m (1 1 j10) 3 1023 /m, 6.283 3 106 m/s 55.12 1 j45.85 V, 0.1783 ms 57.44 1 j48.82 V (a) Proof, (b) 62.6 1 j82.93 V 0.0136 Np/m, 38.94 rad/m, 129.1 –j0.045 V, 6.452 3 107 m/s 0.7071/45° (a) j29.375 V, 5.75/102° V (b) j40 V, 12.60/0° V (c) 2j3471.88, 22.74/0° V (d) 2j18.2 V, 6.607/180° V (a) 1.6 (b) 1.2 1 j0.8 (c) 0.6 2 j2.4 See Figure E.2 for their locations on the Smith chart. 20/11/17 8:48 AM .8 x=0 878 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS r = 1.6 r = 1.2 r = 0.6 B A x= 2.4 C FIGURE E.2 For Problem 11.35. 11.37 11.39 11.41 11.43 11.45 11.47 11.49 11.51 11.53 19_Sadiku_APP-E_NEW1.indd 878 (0.2 2 j0.4)Zo 10.25 1 j12.3, 5.19 46 1 j26 V, 0.4167/144° (a) 0.475/42° (b) 2.8 (c) 27.5 1 j32.5V (d) At 0.05833l (e) Same as in (d), i.e., 0.05833l 7.4 2 j4.4 mS 42 V (a) 0.5543/25°, 296 V, 21.622 V (b) 184 1 j124 V, 3.7, 38.4 1 j60.4 V (c) 3Zin,max and 2Zin,min (a) 0.4714l (b) s 5 ` , GL 5 1/106.26° (a) 24.5 V (b) 55.33 V, 67.74 V 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 879 11.55 11.57 11.59 11.61 11.63 11.65 10.25 W 20 1 j15 mS, 2j10 mS, 6.408 1j5.189 mS, j10 mS, 2.461 1 j5.691 mS (a) 34.2 1 j41.4 V (b) 0.38l, 0.473l (c) 2.65 0.0723l, 0.4093l 2.11, 1.764 GHz, 0.357/244.5°, 70 2 j40 V See Figure E.3. V(0,t) 9V 5V 5.444V 5.395V 1/9 1/3 4 8 −1 V 12 t(µs) −3 V I(0,t) 180mA 215.5 mA 220mA 216.05 mA 60mA 4 12 FIGURE E.3 For Problem 11.65.

11.67 11.69 9.57 V 17.6 V, 600m Vg Zo , where Vo 5 Vg 2 Zo 1 Zg 11.73 11.75 11.77 t(µs) −20mA 11.71 19_Sadiku_APP-E_NEW1.indd 879 8 (a) 14.9 mm, (b) 2.52 mm 127.64 V 13.98 dB 20/11/17 8:48 AM 880 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS CHAPTER 12 12.1 12.3 (a) 2.5 GHz (b) 7 TE modes and 2 TM modes See Table E.1 TABLE E.1 For Problem 12.3 Mode TE01 9.901 TE10 4.386 TE11 10.829 TE02 19.802 TE22 21.66 TM11 10.829 TM12 20.282 TM21 13.228 12.5 12.7 12.9 12.11 12.13 12.15 12.17 12.19 12.21 12.23 12.25 A design could be a 5 9 mm, b 5 3 mm (a) 3 cm, 1.25 cm (b) 10, 12, 13 GHz (c) 8.67 GHz 254.15 rad/m, 3.09 3 108, 388.3 V 2.325 ms (a) 5.391 GHz (b) 2.62 3 108 m/s (c) 4.368 cm 375.1 V, 1.5 mW (a) TE23 (b) j400.7 /m (c) 985.3 V Proof (a) 122.32 rad/m, 3.185 3 108 m/s, 2.826 3 108 m/s, 400.21 V, (b) 189.83 rad/m, 2.052 3 108 m/s, 1.949 3 108 m/s, 257.88 V, 3up 4.927 cm 12.27 12.29 12.31 12.33 19_Sadiku_APP-E_NEW1.indd 880 fc (GHz) vmba2 2 2 Ho sin (py/b)az 2p2 (a) 0.012682 Np/m, 0.0153 Np/m (b) 0.02344 Np/m, 0.0441 Np/m 6.5445 m (a) 2.165 3 1022 Np/m (b) 4.818 3 1023 Np/m 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 881 12.35 12.37 12.39 12.41 12.43 12.45 12.47 12.49 12.51 12.53 12.55 12.57 12.59 12.61 0.1339 dB/m 199.94 rad/m, 1.481 3 1022 Np/m, 2.183 3 1022 Np/m, 2.357 3 108 m/s, 1.689 3 108 m/s, 5 cm Proof 1 Proof, 2 (np/b)(rp/c)Eo sin(mpx/a) cos(npy/b) cos(rpz/c) V/m h (a) 2.606 GHz, (b) 4727.7 5.701 GHz (TE011), 7.906 GHz (TE012), 10 GHz (TE101 and TE021), 10.61 GHz (TE013 or TM110), 11.07 GHz (TE111 or TM111) (a) 16.77 GHz, (b) 6589.51 (a) a 5 b 5 c 5 3.788 cm, (b) a 5 b 5 c 5 2.646 cm (a) a 5 b 5 c 5 1.77 cm, (b) 9767.61 1.4286 (a) 0.2271, (b) 13.13o, (c) 6 modes (a) 29.23o, (b) 63.1% a12 5 8686a10 3.1698 mW CHAPTER 13 50hb sin(vt 2 br) (sin faf 1 cosu cos fau) V/m, mr 250 b sin(vt 2 br) (sin faf 2 cosu cos fau)A/m mr 13.3 Hs 5 13.5 13.7 13.9 (a) Proof (b) ø 5 0.05l 1.447 W (a) Proof (b) For l 5 l, f(u) 5 See Figure E.4. 13.1 19_Sadiku_APP-E_NEW1.indd 881 j0.1667 j62.83 sinue 2jpr/3af, Es 5 sinue 2jpr/3au r r cos(p cosu) 1 1 sinu 20/11/17 8:48 AM 882 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 0 30 30 l=λ 60 2 60 1.5 90 1 0.5 90 120 120 150 150 180 For l 5 3l , f(u) 5 2 cosa 3p cosub 2 sinu 0 30 30 l = 3l/2 60 1.5 60 1 90 0.5 90 120 150 19_Sadiku_APP-E_NEW1.indd 882 150 180 For l 5 2l, f(u) 5 cosu sin(2p cosu) sinu 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 883 0 30 30 l = 2λ 60 2.5 2 90 60 1.5 1 0.5 90 120 120 150 150 FIGURE E.4 For Problem 13.9. 180 13.11 13.13 13.15 13.17 13.19 13.21 13.23 (a) 65.22 m (b) 0.8333 m (c) 0.9375 m (d) 0.125 m (a) 9.071 mA (b) 0.25 mW (a) 1.26 mV (b) 1.575 W (c) 1.933% 33.3% cos 2u 2jbr (a) Hs 5 e af A/m, (b) 7.778 mW, (c) 76.78% 120pr (a) 12.73ar mW/m2, (b) 0.098 V/m See Figure E.5. 0 30 30 60 2.5 2 90 60 1.5 1 0.5 90 120 120 150 150 180 19_Sadiku_APP-E_NEW1.indd 883 l = 2λ FIGURE E.5 For Problem 13.23. 20/11/17 8:48 AM 884 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 1.5l2 sin2u (a) 1.5 sin2 u, (b) 1.5, (c) , (d) 3.084 4p (a) 62.5 m, (b) 36.5 V, (c) 3.282 (a) 2.546 sin u sin2 f, 2.546 (b) 2.25p sin2 u sin3 f, 7.069 (c) 0.75(1 1 sin2u sin2f), 1.5 2.121 A a2 30 jhbIodl 1 sinu cosa bd cosubau 2p 2 13.25 13.27 13.29 13.31 13.33 13.35 13.37 See Figure E.6. (a) (b) 0 30 30 d = λ, α = π/2 60 0 d = λ/4, α = 3π/4 60 1 90 90 120 120 150 150 90 60 0.8 0.6 0.4 0.2 90 120 120 150 180 (c) 30 60 1 0.8 0.6 0.4 0.2 30 150 180 0 30 30 d = 3λ/4, α = 0 60 1 90 60 0.8 0.6 0.4 0.2 90 120 120 150 150 180 FIGURE E.6 For Problem 13.37. 19_Sadiku_APP-E_NEW1.indd 884 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 885 13.39 See Figure E.7. 90 4 120 Polar plot 60 3 150 30 2 1 ψ 180 210 330 240 19_Sadiku_APP-E_NEW1.indd 885 0 300 270 FIGURE E.6 For Problem 13.39. 13.41 13.43 13.45 13.47 13.49 13.51 13.53 13.55 7.162 3 1023 m2 115.384 dB 21.29 pW 19 dB 272.1 pW The transmitted power must be increased 16 times. (a) 8.164 dB, (b) 55.05 dB (a) 17.1 mV/km, (b) 51.93 V 20/11/17 8:48 AM 886 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS CHAPTER 14 14.1 See Figure E.8 FIGURE E.8 For Problem 14.1. 14.3 14.5 14.7 (a) 10.117, 1.56 (b) 10.113, 1.506 Proof V1 5 39.93, V2 5 34.97, V3 5 24.96, V4 5 19.98 V 14.9 4Vo ` (a) a p n5odd 14.11 19_Sadiku_APP-E_NEW1.indd 886 (b) 25 V (a) 24 1 1 24 0 1 F 1 0 0 1 0 0 sina 0 1 24 0 0 1 npy npy npx npx sina b sinh a b b sinh a b a a a a 4Vo ` 1 a p n5odd n sinh(np) n sinh(np) 1 0 0 Va 2200 0 1 0 2100 Vb 0 0 1 2100 Vc V VF V5F 24 1 0 2100 Vd 1 24 1 0 Ve 0 1 24 0 Vf [A] [B] 20/11/17 8:48 AM APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS 887 (b) 24 1 0 1 H 0 0 0 0 1 24 1 0 1 0 0 0 0 1 24 0 0 1 0 0 1 0 0 0 24 1 0 1 0 14.13 1 5 10.97, V2 5 26.25, V3 5 44.06, V4 5 17.97, V5 5 11.05, V V6 5 26.28, V7 5 44.07 V The numerical and analytical solutions are plotted in Figure E.9. Analytical solution V Analytical solution V 14.15 0 0 0 0 V1 230 1 0 0 0 215 V2 0 1 0 0 230 V3 1 0 1 0 27.5 V4 XH X5H X 24 1 0 1 0 V5 1 24 0 0 27.5 V6 0 0 24 1 0 V7 1 0 1 24 V8 0 [A] [B] 1 0 –1 1 spatia 0.5 l dom ain of x 0 0 1 3 2 ain time dom 0 1 3 2 ain m o d e tim 4 1 0 –1 1 spatia 0.5 l dom ain of x 0 4 FIGURE E.9 For Problem 14.15. 14.17 14.19 14.21 12.77 pF/m 100 V (a) Exact C 5 80.26 pF/m, Zo 5 41.56 V; for numerical solution, see Table E.2. TABLE E.2 For Problem 14.21(a) 19_Sadiku_APP-E_NEW1.indd 887 N C (pF/m) Zo (V) 10 20 40 100 82.386 80.966 80.438 80.025 40.486 41.197 41.467 41.562 20/11/17 8:48 AM 888 APPENDIX E ANSWERS TO ODD-NUMBERED PROBLEMS (b) For numerical result, see Table E.3 TABLE E.3 For Problem 14.21(b) N C (pF/m) Zo (V) 10 20 40 100 109.51 108.71 108.27 107.93 30.458 30.681 30.807 30.905 14.23 Proof 14.25 (a) (1.5, 0.5) along 12 and (0.9286, 0.9286) along 13 (b) 56.67 V 14.27 14.29 14.31 14.33 0.8802 20.2083 D 0 20.6719 20.2083 1.5333 21.2 20.125 0.8333 20.667 20.6667 1.4583 D 0 20.375 20.1667 20.4167 25 V See Table E.4. 0 21.2 1.4083 20.2083 0 20.375 0.625 20.25 20.6719 20.125 T 20.2083 1.0052 20.1667 20.4167 T 20.25 0.833 TABLE E.4 For Problem 14.33 14.35 14.37 14.39 19_Sadiku_APP-E_NEW1.indd 888 Node no. FEM Solution Exact Solution 8 9 10 11 14 15 16 17 20 21 22 23 26 27 28 29 3.635 5.882 5.882 3.635 8.659 14.01 14.01 8.659 16.99 27.49 27.49 16.99 31.81 51.47 51.47 31.81 3.412 5.521 5.521 3.412 8.217 13.30 13.30 8.217 16.37 26.49 26.49 16.37 31.21 50.5 50.5 31.21 V1 5 V3 5 V4 5 V6 5 35.71 V, V2 5 V5 5 42.85 40.587 V 92.01, 74.31, 82.87, 53.72, 61.78, 78.6, 30.194, 36.153, 53.69 V 20/11/17 8:48 AM

 16/11/17 1:46 PM 20_Sadiku_Index.indd 896 16/11/17 1:46 PM POWERS OF TEN Power Prefix Symbol 1024 yotta Y 1021 zetta Z 1018 exa E 1015 peta P 1012 tera T 109 giga G 106 mega M 103 kilo k 102 hecto h 101 deka da 101 deci d 102 centi c 103 milli m 106 micro m 109 nano n 1012 pico p 1015 femto f 1018 atto a 1021 zepto z 1024 yocto y THE GREEK ALPHABET Uppercase Lowercase Name A B G D E Z H U I K L M 20_Sadiku_BACKCOVER_A.indd 867 Uppercase Lowercase Name a Alpha N b Beta J g Gamma O d Delta P « Epsilon R z Zeta S h Eta T u Theta i Iota k Kappa l Lambda m Mu n Nu j Xi o Omicron p Pi r Rho s, Sigma t Tau Upsilon f Phi x Chi c Psi v Omega 16/11/17 2:12 PM VEC TOR DERIVATIVES Cartesian Coordinates (x, y, z) A 5 Ax a x 1 A y a y 1 A z a z 'V 'V 'V ax 1 ay 1 a 'x 'y 'z z 'Ay 'Ax 'Az = #A 5 1 1 'x 'y 'z =V 5 ax ' =3A5 ∞ 'x Ax 5 c = 2V 5 ay ' 'y Ay az ' ∞ 'z Az 'Ay 'Ay 'Az 'Ax 'Az 'Ax 2 2 2 d ax 1 c d ay 1 c da 'y 'z 'z 'x 'x 'y z '2V '2V '2V 1 1 'x2 'y2 'z2 Cylindrical Coordinates (, , z) A 5 Arar 1 Afaf 1 Az az =V 5 'V 1 'V 'V ar 1 af 1 a r 'r 'f 'z z =#A 5 'Az 1 ' 1 'Af 1 rAr 2 1 1 r 'r r 'f 'z ar 1 ' =3A5 ∞ r 'r Ar 5 c =2V 5 raf ' 'f rAf az ' ∞ 'z Az 'Ar 'Ar 'Af 'Az 1 'Az 1 ' 1 rAf 2 2 2 2 da 1 c da 1 c da r 'f 'z r 'z 'r f r 'r 'f z 1 ' 'V 1 '2V '2V ar b1 2 21 2 r 'r 'r r 'f 'z 20_Sadiku_BACKCOVER_B.indd 1 16/11/17 2:15 PM VEC TOR DERIVATIVES (cont.) Spherical Coordinates (r, ,) A 5 Ar ar 1 A u au 1 A faf =V 5 =#A5 'V 1 'V 1 'V ar 1 au 1 a r 'r 'u r sin u 'f f 1 ' 2 1 ' 1 'Af 1 r Ar 2 1 1 Au sin u 2 1 2 r sin u 'u r sin u 'f r 'r ar 1 ' =3A5 2 ∞ r sin u 'r Ar 5 20_Sadiku_BACKCOVER_B.indd 2 1 r sin u 2 af ' ∞ 'f 1 r sin u 2 Af 'Au 1 ' 1 1 'Ar ' 1 Af sin u 2 2 1 rAf 2 d au 2 c d ar 1 c r sin u 'f r sin u 'u 'f 'r 1 =2V 5 rau ' 'u rAu 'Ar 1 ' c 1 rAu 2 2 da r 'r 'u f 1 ' 2 'V 1 ' 'V 1 '2V 1 u 1 ar b asin b 'r 'u r2 'r r2 sin u 'u r 2sin2u 'f2 16/11/17 2:15 PM