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Abstract Mathematical Morphology is a general theory that studies the decompositions of mappings between complete lattices in terms of some families of simple mappings: dilations, erosions, anti-dilations and anti-erosions. Nowadays, this theory is largely used in Image Processing and Computer Vision to extract information from images. The KHOROS system is an open and general environment for Image Processing and Visualization that has become very popular. One of the main characteristics of KHOROS is its flexibility, since it runs on standard machines, supports several standard data formats, uses a visual programming language, and has tools to help the user to build and install his own programs. A set of new programs can be organized as a subsystem, called Toolbox. This paper presents a fast and comprehensive Mathematical Morphology Toolbox for the KHOROS system, that deals with binary, gray-scale and multiple band images. Each program has specialized algorithms for binary and gray-scale images, that are chosen automatically according to the input data. These implemented algorithms running on current general purpose workstations are as fast as the equivalent ones running on specialized hardware with 1986 technology.

1 Introduction

Mathematical Morphology is a solid theoretical body to study mappings between complete lattices (Serra, 1988) and an extremely powerful tool to extract image information (Serra, 1982).

Under a theoretical point of view, Mathematical Morphology studies the decomposition of mappings between complete lattices in terms of some families of simple mappings: dilations, erosions, anti–dilations and anti–erosions. These mappings are called the elementary mappings of Mathematical Morphology.

The mappings are built by combining the elementary mappings through the supremum, infimum and composition operations. Once a mapping was built, it can also be used as a primitive in order to build other mappings and so on. The set of primitive mappings and operations is called the *Mathematical Morphology toolbox.*

Under a practical point of view, the mappings and operations of the Mathematical Morphology toolbox are the tools to extract image information.

Usually, a goal is broken heuristically into subgoals, that are achieved by primitive mappings. The right composition of these primitives gives the mapping that achieves the desired goal. For example, in order to reduce the stripe effect on SPOT images, Banon and Barrera (1989) localized the stripe pixels and, then, interpolated new values just for these pixels. In the same way, in order to segment microscopic cell images, Barrera (1991) got a marker for each cell and regions containing groups of cells, before arriving to the image segmentation.

Thus, a good system to perform Mathematical Morphology applications must have two main characteristics: fast algorithms for the elementary mappings and a suitable interface to prototype new mappings.

The KHOROS system is a portable environment for Image Processing and Visualization that has become very popular. It runs on existing standards, has a visual programming language for user interface, and provides tools to build and install new programs.

Once the original set of morphological tools in KHOROS is not satisfactory, we decided to implement a toolbox dedicated to Image Processing by Mathematical Morphology.

Section 2 gives a formal specification of the toolbox implemented. Section 3 presents the main characteristics of the KHOROS system. Section 4 tells some aspects of the toolbox architecture, resumes section 2 as a set of practical tables and discusses the main algorithms implemented. Section 5 presents an application example. Section 6 gives some conclusions and directions for future works.

2 Morphological image processing

The mappings of the Mathematical Morphology toolbox can be organized hierarchically, based on their decomposition in terms of the elementary mappings. Thus, taking an increasing complexity order, we defined the following families of operations and mappings: basic image operations and transformations; first, second and third level image transformations.

2.1 Basic image operations and transformations

It has been shown (Banon and Barrera, 1990, 1991, 1993) that any transformation between complete lattices can be described in terms of the four classes of elementary transformations of Mathematical Morphology: dilations, erosions, anti–dilations and anti–erosions.

We recall that, in its algebraic sense, the words "dilations, erosions, anti–dilations and anti–erosions" means mappings ψ from a complete lattice $\mathcal{L}_1$ to another one $\mathcal{L}_2$ that have the following properties:

$$\psi(\bigvee \mathfrak{X}) = \bigvee \psi(\mathfrak{X}) \quad (\mathfrak{X} \in \mathcal{L}_1)$$

for *dilations*,

$$\psi(\bigwedge \mathfrak{X}) = \bigwedge \psi(\mathfrak{X}) \quad (\mathfrak{X} \in \mathcal{L}_1)$$

for *erosions*,

$$\psi(\bigvee \mathfrak{X}) = \bigwedge \psi(\mathfrak{X}) \quad (\mathfrak{X} \in \mathcal{L}_1)$$

for *anti–dilations* and

$$\psi(\bigwedge \mathfrak{X}) = \bigvee \psi(\mathfrak{X}) \quad (\mathfrak{X} \in \mathcal{L}_1)$$

for *anti–erosions.*

In the following, we present some classes of elementary mappings and operations.

Let Z be the set of integers. Let E be a rectangle of Z^2 and let K be an interval $[0, k]$ of Z, with $k > 0$. The collection of functions from E to K will represent the *gray-scale images* of interest. We denote such a collection by K^E and by f, g, f_1 and f_2 generic elements of K^E.

We first recall some useful *local* operations definitions on images. These definition are based on the structural properties of the interval $[0, k]$ of Z

The *intersection of* f_1 *and* f_2, denoted $f_1 \wedge f_2$ is the function in K^E given by, for any x in E,

$$(f_1 \wedge f_2)(x) = \min\{f_1(x), f_2(x)\}, \tag{1}$$

the *union of* f_1 *and* f_2, denoted $f_1 \vee f_2$ is the function in K^E given by, for any x in E,

$$(f_1 \vee f_2)(x) = \max\{f_1(x), f_2(x)\}. \tag{2}$$

The two binary operations $\wedge$ and $\vee$ from $K^E \times K^E$ to K^E are called, respectively, *intersection* and *union*. Actually, these operations applied to f_1 and f_2 produced, respectively, the infimum and the supremum of f_1 and f_2 with respect to the partial ordering $\leq$ given by

$$f_1 \leq f_2 \Leftrightarrow (f_1(x) \leq f_2(x) \quad (x \in E)).$$

For this reason, the two binary operations $\wedge$ and $\vee$ are also called, respectively, *infimum* and *supremum operations* (or, simply, *infimum* and *supremum*).

The *complementary* (or *inverse*) *of* f, denoted $\sim f$, is the function in K^E given by, for any x in E,

$$(\sim f)(x) = k - f(x). \tag{3}$$

The unary operation $\sim$ from K^E to K^E is called *complementary operation* (or *inversion*). This unary operation is both an anti-dilation and an anti-erosion.

The *difference between* f_1 *and* f_2, denoted $f_1 \sim f_2$ is the function in K^E given by, for any x in E,

$$(f_1 \sim f_2)(x) = \begin{cases} f_1(x) - f_2(x) & \text{if } f_2(x) \leq f_1(x) \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The binary operation $\sim$ from $K^E \times K^E$ to K^E is called *difference operation* (or *subtraction*). Actually, we have $f_1 \sim f_2 \leq f_1 \wedge (\sim f_2)$ and we get the equality for *binary images*, that is, for $f_1(E) = f_2(E) = \{0, k\}$.

Given a function $f \in K^E$, the unary operation $f \sim \cdot$ is both an anti–dilation and an anti–erosion, and the unary operation $\cdot \sim f$ is both a dilation and an erosion.

The *comparison between* f_1 *and* f_2, denoted $f_1 \leqq f_2$, is the function in K^E given by, for any x in E,

$$(f_1 \leqq f_2)(x) = \begin{cases} k & \text{if } f_1(x) \leq f_2(x) \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The binary operation $\leqq$ from $K^E \times K^E$ to K^E is called *comparison operation*. The unary operations $\cdot \leqq f$ and $f \leqq \cdot$ from K^E to K^E are called *adaptive thresholds with respect to f*. These unary operations are, respectively, an anti–dilation and an erosion.

The *equality between* f_1 *and* f_2, denoted $f_1 = f_2$, is the function in K^E given by, for any x in E,

$$(f_1 = f_2)(x) = \begin{cases} k & \text{if } f_1(x) = f_2(x) \\ 0 & \text{otherwise.} \end{cases}$$

The binary operation $=$ from $K^E \times K^E$ to K^E is called *equality operation*. Actually, we have

$$(f_1 = f_2) = (f_1 \leqq f_2) \wedge (f_2 \leqq f_1).$$

Let $f_1 \leq f_2$ the *toggle transform of f with respect to* f_1 *and* f_2, denoted $f_1[f]f_2$, is the function in K^E given by, for any x in E,

$$(f_1[f]f_2)(x) = \begin{cases} f_1(x) & \text{if } (f \sim f_1)(x) \leq (f_2 \sim f)(x) \\ f_2(x) & \text{otherwise.} \end{cases} \tag{6}$$

The transformation $\cdot [\cdot] \cdot$ from $K^E \times K^E \times K^E$ to K^E is called *toggle transformation*. Actually, we have

$$(f_1[f]f_2) = (g \wedge f_1) \vee ((\sim g) \wedge f_2),$$

where

$$g = (f \sim f_1) \leqq (f_2 \sim f).$$

The toggle transformation is both an erosion and a dilation.

We now recall the definitions of two important subclasses of dilations and erosions. These definitions are based on the Abelian group property of $(\mathbb{Z}^2, +)$.

Let B be a subset of $\mathbb{Z}^2$, called *structural set* (or, *structural element*). We denote by B_h the *translate of B by* any vector h in $\mathbb{Z}^2$, that is,

$$B_h = \{x + h: x \in B\}.$$

We denote by B^t the *transpose of B*, that is,

$$B^t = \{-x: x \in B\}.$$

We denote by B^c the *complement of B*, that is,

$$B^c = \{x: x \notin B\}.$$

The *dilation of f by B* is the function $\delta_B(f)$ in K^E, given by, for any x in E,

$$\delta_B(f)(x) = \max\{f(y): y \in B^t_x \cap E\}; \tag{7}$$

the *erosion of f by B* is the function $\varepsilon_B(f)$ in K^E, given by, for any x in E,

$$\varepsilon_B(f)(x) = \min\{f(y): y \in B_x \cap E\}. \tag{8}$$

In the above expressions, we recall that $\max(\emptyset) = 0$ and $\min(\emptyset) = k$.

The two transformations δ_B and ε_B from K^E to K^E are called, respectively, *dilation* and *erosion by B*.

The anti-dilation and anti-erosion by B can be built, respectively, from the dilation and erosion by B (see section 2.2).

The transformation ι from K^E to K^E, given by, for any $f \in K^E$,

$$\iota(f) = f$$

is called *identity transformation*. This transformation is both an erosion and a dilation by the set $\{o\}$, where o is the origin of $\mathbb{Z}^2$, that is, $o = (0, 0)$.

Now, we present some useful properties of dilations and erosions by structural elements.

Property 1 - *The dilation (erosion) by a structural set B is equivalent to the supremum (infimum) of dilations (erosions) by subsets in a family whose supremum is B, that is,*

$$\delta_B = \bigvee \{\delta_{B_i} : \bigvee B_i = B\}$$

$$(\ \varepsilon_B = \bigwedge \{\varepsilon_{B_i} : \bigvee B_i = B\}\ )$$

The *Minkowski addition* of two subsets A and B of Z^2 is the subset $A \oplus B$ of Z^2, given by

$$A \oplus B = \bigcup \{A_b : b \in B\}.$$

Property 2 - *The dilation (erosion) by the Minkowski addition of two subsets A and B is equivalent to the composition of the dilation (erosion) by A and B, that is,*

$$\delta_{A \oplus B} = \delta_A \delta_B$$

$$(\varepsilon_{A \oplus B} = \varepsilon_A \varepsilon_B).$$

A particular consequence of Properties 1 and 2 is that a dilation and an erosion by any subset B can be built by composing, respectively, dilations and erosions by subsets of the set $\{-1, 0, 1\}^2$, called the *elementary square*. Some studies point that this decomposition can lead to algorithms for dilations and erosions more efficient than the direct ones (Maragos, 1985, p. 48).

Property 3 - *The dilation (erosion) of f by B can be computed by the following composition*

$$\delta_B(f) = \bigvee \{\delta_{\{y\}}(f) : y \in B\}$$

$$(\ \varepsilon_B(f) = \bigwedge \{\delta_{\{y\}}(f) : y \in B'\}\ ).$$

2.2 First level image transformations

These transformations are built by using only once each basic transformation.

Let B be a subset of Z^2, the two transformations δ^a_B and ε^a_B from K^E to K^E, given by the following compositions

$$\delta^a_B = \sim \delta_B \quad \text{and} \quad \varepsilon^a_B = \sim \varepsilon_B, \tag{9}$$

are called, respectively, *anti-dilation* and *anti-erosion by B*.

The transformation ψ_B from K^E to K^E, given by the following composition

$$\psi_B = \delta_B \sim \varepsilon_B, \tag{10}$$

is called *morphological gradient.*

Let g be an element of K^E, the transformations $\delta_{B,g}$ and $\varepsilon_{B,g}$ from K^E to K^E, given by

$$\delta_{B,g} = \delta_B \wedge g \quad \text{and} \quad \varepsilon_{B,g} = \varepsilon_B \vee g, \tag{11}$$

are called, respectively, *conditional* (or *geodesic*) *dilation* and *erosion by B given g.*

The transformations γ_B and ϕ_B from K^E to K^E, given by the following compositions

$$\gamma_B = \delta_B \varepsilon_B \quad \text{and} \quad \phi_B = \varepsilon_B \delta_B, \tag{12}$$

are called, respectively, *(morphological) opening* and *closing by B.*

Let A and B be two subsets of Z^2 such that $A \subset B$, the two transformations $\lambda_{A,B}$ and $\mu_{A,B}$ from K^E to K^E, given by the following compositions

$$\lambda_{A,B} = \varepsilon_A \wedge \delta^*_{B^c} \quad \text{and} \quad \mu_{A,B} = \delta_A \vee \varepsilon^*_{B^c}, \tag{13}$$

are called, respectively, *sup-generating* and *inf-generating transformations of parameters A and B*. The sup-generating mapping of parameters A and B^c is also called *Hit-Miss transformation of parameters A and B*.

The two transformations $\sigma_{A,B}$ and $\tau_{A,B}$ from K^E to K^E, given by the following compositions

$$\sigma_{A,B} = \iota \sim \lambda_{A,B} \quad \text{and} \quad \tau_{A,B} = \iota \vee \lambda_{A,B}, \tag{14}$$

are called, respectively, *thinning* and *thickening of parameters A and B.*

Let g be an element of K^E, the transformations $\sigma_{A,B,g}$ and $\tau_{A,B,g}$ from K^E to K^E, given by the following compositions

$$\sigma_{A,B,g} = \sigma_{A,B} \vee g \text{ and } \tau_{A,B,g} = \tau_{A,B} \wedge g, \tag{15}$$

are called, respectively, *conditional thinning* and *thickening by* (A, B) *given g.*

2.3 Second level image transformations

These transformations are built by using more than once each basic transformation.

Let B be a subset of Z^2, the two transformations δ_B^n and ε_B^n from K^E to K^E, given, for $n > 0$, by the following $n - 1$ successive compositions

$$\delta_B^n = \delta_B \ldots \delta_B \quad \text{and} \quad \varepsilon_B^n = \varepsilon_B \ldots \varepsilon_B \tag{16}$$

and, for $n = 0$,

$$\delta_B^n = \iota \quad \text{and} \quad \varepsilon_B^n = \iota,$$

are called, respectively, *n-dilation* and *n-erosion by B*. Actually, δ_B^n and ε_B^n are, respectively, equivalent to the dilation and erosion by nB, where nB is given by the following $n - 1$ successive compositions

$$nB = (B \oplus B) \ldots \oplus B$$

and, for $n = 0$,

$$nB = \{o\}.$$

Let g be an element of K^E, the transformations $\delta_{B,g}^n$ and $\varepsilon_{B,g}^n$ from K^E to K^E, given by the following $n - 1$ successive compositions

$$\delta_{B,g}^n = \delta_{B,g} \ldots \delta_{B,g} \text{ and } \varepsilon_{B,g}^n = \varepsilon_{B,g} \ldots \varepsilon_{B,g}, \tag{17}$$

are called, respectively, *n-conditional dilation* and *erosion by B given g*.

The two transformations γ_B^n and ϕ_B^n from K^E to K^E, given by the following compositions

$$\gamma_B^n = \delta_B^n \varepsilon_B^n \quad \text{and} \quad \phi_B^n = \varepsilon_B^n \delta_B^n, \tag{18}$$

are called, respectively, *n-opening* and *n-closing by B*. Actually, γ_B^n and ϕ_B^n are, respectively, equivalent to the opening and closing by nB.

The two transformations θ and ψ from K^E to K^E, given by the following compositions

$$\theta = \phi_B^n \gamma_B^n \quad \text{and} \quad \psi = \gamma_B^n \phi_B^n \tag{19}$$

are called, respectively, *n-$\phi\gamma$-filter* and *n-$\gamma\phi$-filter* (*by B*).

The two transformations θ and ψ from K^E to K^E, given by the following compositions

$$\theta = \gamma_B^n \phi_B^n \gamma_B^n \quad \text{and} \quad \psi = \phi_B^n \gamma_B^n \phi_B^n, \tag{20}$$

are called, respectively, *n-$\gamma\phi\gamma$-filter* and *n-$\phi\gamma\phi$-filter* (*by B*).

Let denote the *n-$\phi\gamma$, n-$\gamma\phi$, n-$\phi\gamma\phi$* and *n-$\gamma\phi\gamma$-filter* by B generically by ψ^n_B. Let $\mathfrak{B}$ be a finite sequences of N subsets in $\mathbb{Z}^2$, with elements B_i such that $B_i \subset B_{i+1}$. The transformation $\psi^N_{\mathfrak{B}}$ from K^E to K^E, given by the following composition

$$\psi^N_{\mathfrak{B}} = \psi^N_{B_N} \psi^{N-1}_{B_{N-1}} \dots \psi^1_{B_1},$$

is called an *N alternate sequential filter of parameter* $\mathfrak{B}$.

Let $\mathcal{A}$ and $\mathfrak{B}$ be two finite sequences of n subsets, in $\mathbb{Z}^2$, respectively, with elements A_i and B_i such that $A_i \subset B_i$. The two transformations $\sigma^n_{\mathcal{A},\mathfrak{B}}$ and $\tau^n_{\mathcal{A},\mathfrak{B}}$ from K^E to K^E, given by the following $n-1$ successive compositions

$$\sigma^n_{\mathcal{A},\mathfrak{B}} = \sigma_{A_1,B_1} \dots \sigma_{A_n,B_n} \quad \text{and} \quad \tau^n_{\mathcal{A},\mathfrak{B}} = \tau_{A_1,B_1} \dots \tau_{A_n,B_n} \tag{21}$$

are called, respectively, *n-thinning* and *n-thickening of parameters $\mathcal{A}$ and* $\mathfrak{B}$.

The two transformations $\psi_{\mathcal{A},\mathfrak{B}}$ and $\omega_{\mathcal{A},\mathfrak{B}}$ from K^E to K^E, given by the following $n-1$ operations

$$\psi_{\mathcal{A},\mathfrak{B}} = \bigvee \{\lambda_{A_i,B_i} : i = 1, \dots, n\} \quad \text{and} \quad \omega_{\mathcal{A},\mathfrak{B}} = \bigwedge \{\mu_{A_i,B_i} : i = 1, \dots, n\}, \tag{22}$$

are called, respectively, *n-canonical transformation* and *n-canonical dual transformation of parameters $\mathcal{A}$ and* $\mathfrak{B}$.

The transformation β_B, from K^E to K^E, given by

$$\beta_B = (\iota \wedge \phi_B\gamma_B\phi_B) \vee \gamma_B\phi_B\gamma_B, \tag{23}$$

is called the *primitive of the center filter*.

2.4 Third level image transformations

These transformations are built by using an a priori undefined number of basic transformations.

Let B be a subset of $\mathbb{Z}^2$ and let f be an element of K^E, the transformations $\gamma_{B,f}$ and $\phi_{B,f}$ from K^E to K^E, given by, for any $g \in K^E$,

$$\gamma_{B,f}(g) = \bigvee \{\delta^n_{B,g}(f) : n = 1, \dots\} \quad \text{and} \quad \phi_{B,f}(g) = \bigwedge \{\varepsilon^n_{B,g}(f) : n = 1, \dots\}, \tag{24}$$

are called, respectively, *opening* and *closing by reconstruction from the marker f*.

The following infinite successive compositions

$$\alpha_B = \beta_B\beta_B \dots \beta_B \dots, \tag{25}$$

where β_B is the last transformation defined in Subsection 2.3, is called the *center filter*.

Let $\mathcal{A}$ and $\mathfrak{B}$ be two infinite sequences of n subsets in Z^2, respectively, with elements, A_i and B_i such that $A_i \subset B_i$. The two transformations $\Sigma_{\mathcal{A},\mathfrak{B}}$ and $T_{\mathcal{A},\mathfrak{B}}$ from K^E to K^E, given by the following infinite successive compositions

$$\Sigma_{\mathcal{A},\mathfrak{B}} = \sigma_{A_1,B_1} \dots \sigma_{A_i,B_i} \dots \quad \text{and} \quad T_{\mathcal{A},\mathfrak{B}} = \tau_{A_1,B_1} \dots \tau_{A_i,B_i} \dots, \tag{26}$$

are called, respectively, *skeleton by thinning* and *exoskeleton by thickening of parameters $\mathcal{A}$ and $\mathfrak{B}$.*

Let g be an element of K^E. The transformations $\Sigma_{\mathcal{A},\mathfrak{B},g}$ and $T_{\mathcal{A},\mathfrak{B},g}$ from K^E to K^E, given by the following infinite successive compositions

$$\Sigma_{\mathcal{A},\mathfrak{B},g} = \sigma_{A_1,B_1,g} \dots \sigma_{A_i,B_i,g} \dots \quad \text{and} \quad T_{\mathcal{A},\mathfrak{B},g} = \tau_{A_1,B_1,g} \dots \tau_{A_i,B_i,g} \dots, \tag{27}$$

are called, respectively, *conditional skeleton by thinning* and *conditional exoskeleton by thickening of parameters $\mathcal{A}$ and $\mathfrak{B}$* given g.

Let B be a subset of Z^2, the transformation σ_B from K^E to K^E, given by

$$\sigma_B = \bigvee \{\varepsilon_B^i - \gamma_B \varepsilon_B^i : i = 0, 1, \dots\}, \tag{28}$$

is called *morphological skeleton of parameter B.*

The transformation ϱ_B from K^E to K^E, given by

$$\varrho_B = \bigvee \{\varepsilon_B^i - \gamma_{B,\varepsilon_B^{i+1}} \varepsilon_B^i : i = 0, 1, \dots\}, \tag{29}$$

is called *last erosion of parameter B.*

The transformation β_B^n from K^E to K^E, given by, for any $g \in K^E$,

$$\beta_B^n(g) = \bigvee \{\varepsilon_B^i(g) - \delta_{B,g}^n(\varepsilon_B^{i+1}(g)) : i = 0, 1, \dots\} \tag{30}$$

is called *n-order conditional bisector of parameter B.*

3 The KHOROS System

KHOROS (Rasure et al., 1990) is a software environment designed for research in image processing. It was created at the Department of Electrical and Computer Engineering at the University of New

Mexico, Albuquerque, USA, and has become very popular. According to recent statistics of the KHOROS group, it has near ten thousand users around the world, that can have support and exchange information by a very active mailing list.

Once image processing encompasses a wide spectrum of applications, it was designed from a very broad perspective. For example, it includes mechanisms for distributed computing, interactive visualization of many data types, and suitable user interfaces.

One of the most powerful features of KHOROS is CANTATA, its high-level abstract interface. CANTATA is a graphically expressed, data-flow oriented language that provides a visual programming environment for the system. Data flow is an approach in which a program is described as a directed graph, where each node represents an operation (or function) and each direct arc represents a path over which data tokens flow. A CANTATA program is also called a *workspace*. Figure 1 (d) is an example of a workspace.

KHOROS was designed to be portable and extensible. It relies on existing standards (X Windows and UNIX), incorporates tools for software development and maintenance (a high level user interface specification and a code generation tool set), a flexible data exchange format, tools to export and import standard data formats, and an algorithm library.

There are two main types of programs in the KHOROS system: the *vroutines* and the *xvroutines*. The main characteristic of xvroutines is that they have their own graphical user interfaces, while the vroutines do not.

The user programs (vroutines or xvroutines) can be organized as independent subsystems, called *toolboxes*, that can easily be integrated to the system. Usually, a user toolbox is deposited in a public account at University of New Mexico computer and can be accessed by the community of KHOROS users, via anonymous ftp.

KHOROS has been extensively used (Koechner et al., 1990; Sauer et al., 1990; Kluth et al., 1992; Rots and Herreld, 1992) to perform image processing research, algorithm development, and data visualization. In fact, the known applications cover a very broad spectrum: industrial inspection, medical diagnosis, optical measurement, remote sensing, semiconductor processing, optics, medical imaging, ecosystem analysis, cell biology, etc.

4 A Mathematical Morphology toolbox for KHOROS

We implemented the Mathematical Morphology toolbox for binary, gray-scale and multiple band images as a KHOROS toolbox, where each family of morphological mappings is presented as a submenu of the toolbox main menu.

4.1 Architecture

Following the theory of Mathematical Morphology, all the transformations are built by composition of the elementary transformations and operations.

The dilations and erosions are further decomposed, respectively, in terms of dilations and erosions by subsets of the elementary square.

As the elementary transformations for binary images have some additional properties than the corresponding ones for gray-scale images, different algorithms were chosen for each case.

In order to simplify its use, the system was designed to be oriented by data type, that is, when executing a given operation or transformation that makes sense on different data types, it chooses automatically the most efficient algorithm for the current input data.

When the input data are multiple band images, the gray-scale algorithms are applied sequentially for each band.

A special data structure, much simpler than the standard KHOROS format, was designed to support the structural elements that are subsets of the elementary square.

All the main programs implemented are KHOROS vroutines. Complex transformations can be built either as CANTATA or C programs, that use, respectively, vroutines or subroutines of the available primitives.

4.2 Contents

The toolbox is composed by five groups of programs: basic image operations and transformations; first, second and third level image transformations; other tools (Table 1).

Table 1. Toolbox content.

Name	Routine	Expression
Basic image operations and transformations		
infimum	vinf	1
supremum	vsup	2
inversion	vinv	3
subtraction	vsubm	4
threshold	vthreshad	5
toggle	vtoggle	6
dilation	vdil	7
erosion	vero	8
First level image transformations		
anti-dilation	vadil	9
anti-erosion	vaero	9
gradient	vgradm	10
cond. dilation	vcdil	11
cond.erosion	vcero	11
opening	vopen	12
closing	vclose	12
Name	**Routine**	**Expression**
sup-generating	vsupgen	13
inf-generating	vinfgen	13
thinning	vthin	14
thickning	vthick	14
cond. thinning	vcthin	15
cond. thickning	vcthick	15
Second level image transformations		
n-dilation	vndil	16
n-erosion	vnero	16
n-cond. dilation	vncdil	17
n-cond. erosion	vncero	17
n-opening	vnopen	18
n-closing	vnclose	18
n-open/close	vnocfilt	19

Name	Routine	Expression
n–close/open	vncofilt	19
n–op./cl./op.	vnocofilt	20
n–cl./op./cl.	vncocfilt	20
n–thinning	vnthin	21
n–thickening	vnthick	21
n–canonical	vncanon	22
n–can. dual	vncanond	22
center primitive	vcenterp	23
Third level image transformations		
open. by rec.	vopenrec	24
clos. by rec.	vclosrec	24
center filter	vcenter	25
skel. by thin.	vskelthin	26
exoskel. by thick.	vskelthick	26
cond. skel. by thin.	vcskelthin	27
cond. exoskel. by thick.	vcskelthick	27
morph. skel.	vskel	28
last erosion	vlastero	29
cond. bisector	vbisset	30

The basic image operations and transformations are: infimum, supremum, inversion, subtraction, adaptive threshold, toggle mapping, dilation and erosion by subsets of the elementary square.

The first level image transformations are: anti–dilation, anti–erosion, morphological gradient, conditional dilation and erosion, opening, closing, sup–generating and inf–generating mapping, thinning and thickening, conditional thinning and thickening.

The second level image transformations are: n–dilation, n–erosion, n–conditional dilation and erosion, n–opening, n–closing, alternate sequential filters ($n-\phi\gamma$, n-$\gamma\phi$, n-$\gamma\phi\gamma$, $n-\phi\gamma\phi$), n–thinning, n–thickening, canonical transformation, dual canonical transformation, and center filter primitive.

The third level image transformations are: opening and closing by reconstruction, center filter, skeleton by thinning, exoskeleton by thickening, conditional skeleton by thinning, conditional exoskeleton by thickening, morphological skeleton, last erosion and conditional bisector.

The other tools are: an interface for the definition of structural elements, clockwise step rotation of structuring elements, comparison between two images and draw of the image edges (extreme lines and columns).

The structural elements that appear as parameters of the transformations are subsets of the elementary square. The sequence of structural elements used in the thinning, thickening and canonical transformations are built from clockwise step rotation of a given pair of structural elements. For example, an homotopic skeleton can be built by taking the sequence ($\mathcal{A}$, $\mathfrak{B}$), with $\mathcal{A}$ and $\mathfrak{B}$ given by, respectively,

```
111  011  001        111
010  011  011   ...  010   ...
000  000  001        000
```

and

```
000  000  100        000
000  100  100   ...  000   ...,
111  110  100        111
```

as parameter for the skeleton by thinning transformation.

The right choice of the parameters for these transformations gives a large number of tools to extract image information: image sharpening and smoothing, threshold segmentation, elimination of particles that hit the image edges, closing of holes, size distributions, skeletons and their characteristic points (triple, end, etc.), geometrical segmentation and filtering, etc. For example, Table 2, that was adapted from (Serra, 1982, p.392) for the squaregrid, gives some useful pairs of structuring elements for the thinning, thickening and canonical transformations.

For each program of the toolbox, there is an on line help associated, that gives the formal definition of the transformation and a set of well known parameters to extract useful image information.

4.3 Algorithms of the elementary mappings

Once the elementary mappings and operations are the kernel of the system, a considerable effort was put on making them as fast as possible in current general purpose hardware. In order to achieve a better performance, different algorithms were chosen for binary and gray-scale images.

Table 2. Some useful pairs of structural elements for thinning, thickening and canonical transformations.

Struct. Element A	Struct. Element B	Thinning	Thickening	Canonical Transformation
000 010 111	111 000 000	Homotopic skeleton	Conditional Segmentation	-
000 011 000	100 100 100	Homotopic marking	Pseudo Convex Hull	-
100 100 100	000 010 000	-	Convex Hull	-
000 010 000	000 101 111	Skeleton Pruning	-	End Points
000 010 000	111 101 111	-	Homotopic Pruning	Isolated Points
100 011 100	011 100 011	-	-	Triple Points
101 010 100	010 101 011	-	-	Triple Points
100 010 101	011 101 010	-	-	Triple Points
111 111 111	000 000 000	Boundary	-	-

4.3.1 Dilation and erosion for binary images

The key factor used to implement fast dilation and erosion (by structural elements) algorithms for binary images is the inherent parallelism of the 32-bit bitwise operations, found in general purpose CPU instructions set.

Following Property 3, the implementation of binary dilation and erosion can be made by translating the image by all the points of the structural element and taking, respectively, the logical OR and the logical AND of the translated images. By using this formulation, the parallelism can be easily achieved.

To use the intrinsic parallelism of the logical 32-bit bitwise AND and OR operations, we pack the binary images in sets of 32 pixels in a 32-bit integer.

As the packed image is stored in a row by row basis, vertical translations are efficiently handled by adding the current packed pixel address by the width of the packed image. Horizontal translations are implemented by shifting and masking operations in order to shift across 32–bit integer boundaries.

Using this approach, we can compute 32 pixels in parallel with the additional benefit of reading and writing the image data in a more efficient packed binary pixel format.

In order to achieve added performance, the image is subdivided in nine image regions: one middle, four corners and four side regions. Each region is processed separately to avoid unnecessary tests for image boundaries.

The KHOROS has a bit format which already supports a packed image format. The implementation follows the 32–bit parallel algorithm described using structural elements that are subsets of the elementary square. Some optimizations were made for the limit cases, when the structural element is an empty set or the complete elementary square.

4.3.2 Dilation and erosion for gray–scale images

Following the definition, the implementation of dilations and erosions (by structural elements) can be made by translating the structural element over the input image and taking, respectively, the local maximum and minimum.

By this approach, the neighborhood of each pixel need to be accessed, that is, it is necessary to access nNM array elements, where n is the cardinality of B, and N and M are, respectively, the number of lines and columns of the image.

Taking as structural elements just subsets of the elementary square, this algorithm leads to good implementations for gray–scale dilation and erosion.

The implementation adopted is divided into ten cases, according to the cardinality of the structural element, from zero (empty set) to nine (the complete elementary square). In each case, the structural elements points (i.e., the values for local translations) are stored in a corresponding number of fast registers. The nested conditional expressions were open to avoid unnecessary steps.

In order to achieve a better performance, as in the binary case, the image was subdivided in nine subregions and some optimizations were made for the limit cases.

4.3.3 Performance evaluation

Table 3 shows the performance evaluation for some dilations and erosions, in the binary and gray-scale cases.

The time spent for each transformation, given in milliseconds (ms), was calculated from a measure of the time spent by a sequence of a thousand transformations. The machine used was a SUN SPARCstation-2 and the input data were $256 \times 256 \times 1$ (binary) or $256 \times 256 \times 8$ (gray-scale) images.

The speed-up of performing a dilation or an erosion of a binary image by the dedicate algorithm is between 8 to 10 times (Table 3).

Table 3. Performance of dilations and erosions.

Structural Element	Binary Image (ms)	Gray-scale Image (ms)	Speed-up
111 111 111	8.5	71.2	8.4
010 111 010	9.1	90.0	9.9
000 111 000	5.5	48.0	8.7

The performance of these algorithms are equivalent to the ones running on specialized hardwares built with 1986 technology (Bilodeau, 1986), that is, 8ms and 0.1s, respectively, for binary and gray-scale images.

5 Example of Application

To illustrate the use of the Mathematical Morphology toolbox we present a solution (Figure 1 (d)) to the problem of contour extraction in noisy images. The original image f_1 (Figure 1 (a)) is a gray-scale image of a disk corrupted by additive Gaussian noise. By using a sequential alternated filter, with three stages and two structural elements B_1 and B_2 (Figure 1 (e)), we get the filtered image f_2 (Figure 1 (b)):

$$f_2 = \gamma^3_{B_2}\phi^3_{B_2}\gamma^3_{B_1}\phi^3_{B_1}\gamma^2_{B_2}\phi^2_{B_2}\gamma^2_{B_1}\phi^2_{B_1}\gamma^1_{B_2}\phi^1_{B_2}\gamma^1_{B_1}\phi^1_{B_1}(f_1).$$

By using a threshold transformation, with threshold value 20, and an internal contour extractor (Figure 1 (f)), we get the contour image f_3 (Figure 1 (c)):

$$f_3 = f' - \varepsilon_{B_1}(f'),$$

where

$$f' = (20 \leq f_2)$$

(here 20 means a constant image).

Other examples (workspaces) are deposited in the subdirectory *workspaces* of the toolbox. Among these examples are: reduction of stripping noise in SPOT images (Banon and Barrera, 1989), segmentation of digits and symbols of a calculator image, extraction of elongated structures in microscopic images, recognition of disks and squares.

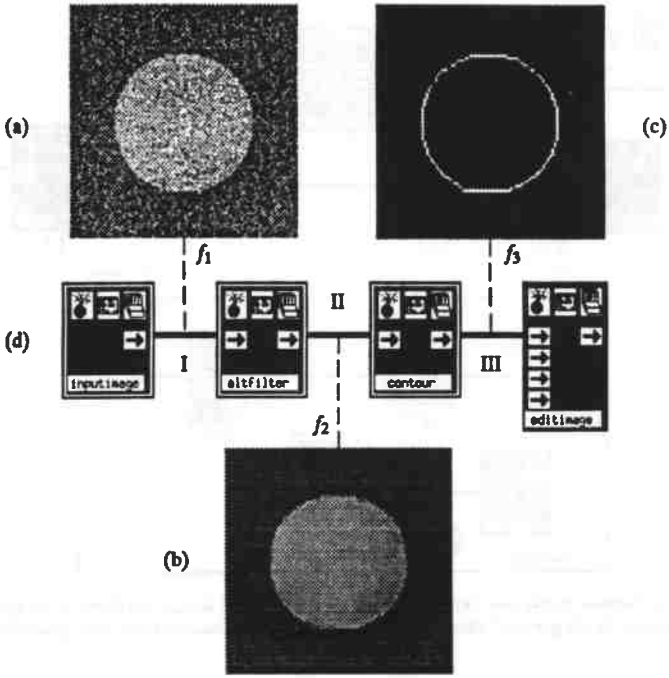
6 Conclusion

This paper presents a KHOROS toolbox for Image Processing by Mathematical Morphology. The implemented subsystem increases KHOROS potentiality by adding a set of high performance tools of multiple purpose use.

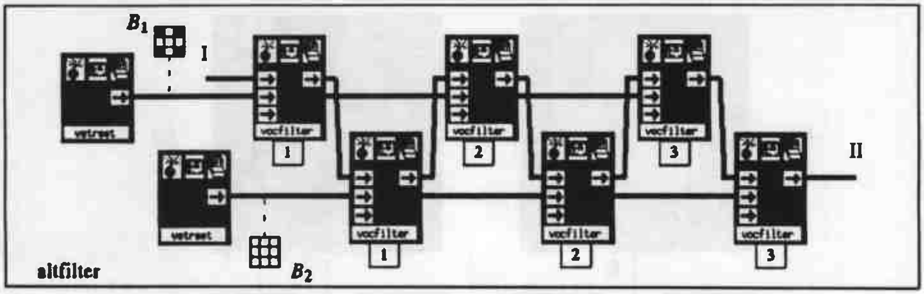
The implemented elementary transformations of Mathematical Morphology running on standard machines are as fast as the ones running on specialized hardwares built with 1986 technology. They perform a dilation or an erosion on a 256×256 image in about 8ms, in the binary case, and 0.1s, in the gray-scale case.

For each main program of the toolbox there is an on line help, that gives the formal definition of the transformation and a set of well known parameters useful to extract image information.

Since a high level transformation can be built either as a C program or a CANTATA workspace, the toolbox is useful for two main purposes: to solve real image processing problems and to didactic applications.


(a)
(c)
(d)
(b)
f_1
f_2
f_3
I
II
III
inputimage
altfilter
contour
editimage

:)



(f)

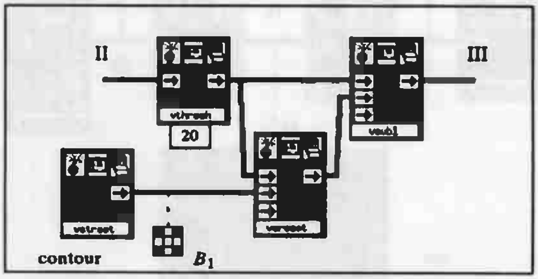


Figure 1. Contour extraction. (a) Original image. (b) Filtered image. (c) Contour image. (d) Algorithm. (e) Sequential alternated filter procedure. (f) Contour extraction procedure.

As a didactic application, it is possible, for example, to build a workspace that implement the thinning algorithm and gives an interesting animation of its dynamics.

Some workspaces that solve real image analysis problems (restoration, segmentation, pattern recognition, etc.) or implement high level morphological algorithms (skeleton, last erosion, etc.) are deposited in the subdirectory workspaces of the toolbox.

At the moment, we are implementing a complementary set of morphological algorithms (distance functions, labeling, watershed, change of the gradient homotopy, regional maxima and minima, etc.) that will be added to the toolbox.

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